# An Electrostatic Model of the Nuclear Binding Force

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***Abstract***- This paper carries out an electric field theory analysis of the interaction between two polar electric fields that are bounded at a fixed radius, and this model is applied to the neutron. This analysis demonstrates that the bounding of the electric fields creates a force that is push-pull in nature and tracks the nuclear binding force profile, further predicting how the force changes as the separation between the neutrons is reduced far below the push-pull balance point. As an adjunct to this analysis it shows that in order to bind to the nucleus the proton must have a strong neutron-like field at the very core of the positron-like field normally experienced in everyday interactions. Further, that a neutron will still interact with the proton's positron-like field after the separation between the neutron and the proton core increases so far that they no longer interact, but that this new interaction is weak and falls off very rapidly with distance and is inconsequential beyond a few neutron field radii. The consequences for the Lorentz Force Equation are also examined, and it is clear that the Lorentz equation applies only to electric fields generated by particles whose field radii are infinite in extent and as such cannot be applied to the neutron.

***Keywords*** *nuclear binding force, electromagnetism, field theory*

## I. INTRODUCTION

This paper is in three parts. In the first part the methodology for examining the interaction between two charged particles is developed. In the second part the interaction between two electrons is examined in detail, and the interaction is shown to have a structure. In the third part truncating the electric fields at a certain radius - as happens with the neutron - is shown to cause this same structure to create the push-pull force/distance relationship demonstrated by the nuclear binding force.

Neutrons are known to have a composite attraction/repulsion force between them [1]. They are also known to have a positively-charged electric core. Various models have added to this, for example a negative outer region of the core has been proposed [2]. A 2007 paper by G.A. Miller suggested a model for the neutron of a negatively charged exterior to attract protons, a positively charged middle to repel them after they approach sufficiently, and a negative core [3]. All these models raised issues over how the neutrons bonded to each other in the nucleus against their mutual electrostatic repulsion to other neutrons, and likewise how the protons bonded to each other, leading to the conclusion that the nuclear binding force cannot be electromagnetic. As will be shown below, a simple bounded positive electric field is a sufficient and effective mechanism that causes neutrons to attract each other as well as protons. The model's force-distance profile matches the nuclear binding force's profile.

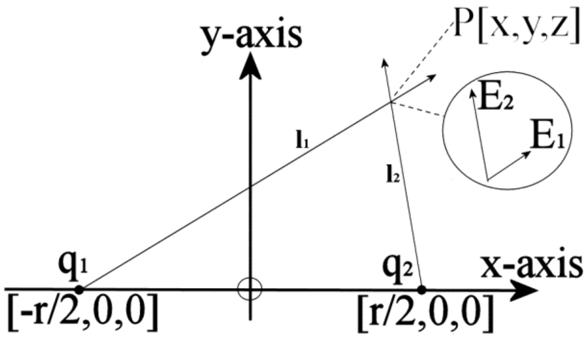


Figure 1. The potential energy density between two charges.

## II. CALCULATING THE FORCES BETWEEN TWO ELECTRONS

When there are multiple sources of electric field such as two electrons, the composite electric field vector created by them at a point in space must be calculated by the vector addition of the field from each source at that point, so if E is the electric field vector then

$$E_{resultant} = E_1 + E_2$$

Consider a pair of electric charges $q_1$ and $q_2$ whose electric fields extend to infinity, as is the case with electrons. Separate them by a distance 'r' and choose the co-ordinates so that one charge is at [-r/2,0,0] and the other at [r/2,0,0] as shown in Fig. 1. The inset shows the electric field vectors at point P[x,y,z]. The x-axis is drawn left to right on the sheet. The y-axis is vertical, and the z-axis points out of the page. $l_1$ and $l_2$ are the distances from the centers of charges $q_1$ and $q_2$ to the point P[x,y,z]. The total energy in the interaction is 'W' and the energy density dW/(dx dy dz) at point P[x,y,z] is given below [4].

$$\frac{dW}{dx\,dy\,dz} = \frac{\varepsilon\left(|\,E_{resultant}\,|^2\right)}{2}$$

$$= \frac{\varepsilon\left(|E_1 + E_2|^2\right)}{2}$$

$$= \frac{\varepsilon E_1 \bullet E_1}{2} + \varepsilon E_1 \bullet E_2 + \frac{\varepsilon E_2 \bullet E_2}{2}$$

Now the first term is the energy density that the first electric field vector would have at that point if the first charge was alone in the universe. The third term is the energy density the second electric field vector would have at that point if the second charge was alone in the universe. The second term is the change in energy density caused by bringing the two charges together to interact. If we subtract the energy density of each field $E_1$ and $E_2$ as it would be at infinite separation we are left with the second term as the "change" component. When this term is integrated over all of space, we get the potential energy 'U' between the two charges in moving from an infinite separation to their current separation. Hence this change term is the "potential energy density" at an arbitrary point P[x,y,z] for a charge separation of r, and 'dU/(dx dy dz)' is the potential energy density at a point.

$$\frac{dU}{dx\,dy\,dz} = \varepsilon\left(E_1 \bullet E_2\right)$$

To find the total potential energy U at a separation r, first find the potential energy density at a point.

$$E_1 = \frac{q_1}{4\pi\varepsilon l_1^2} = \frac{q_1}{4\pi\varepsilon\left(\left(x+\frac{r}{2}\right)^2 + y^2 + z^2\right)}$$

$$E_2 = \frac{q_2}{4\pi\varepsilon l_2^2} = \frac{q_2}{4\pi\varepsilon\left(\left(x-\frac{r}{2}\right)^2 + y^2 + z^2\right)}$$

Now integrate dU/(dx dy dz) over all space.

$$U = \varepsilon\iiint\left(E_1 \bullet E_2\right) dx\,dy\,dz$$

$$= \frac{q_1 q_2}{16\pi^2\varepsilon}\iiint \frac{\left(x+\frac{r}{2}\right)\left(x-\frac{r}{2}\right)+y^2+z^2}{\left(\left(\left(x+\frac{r}{2}\right)^2+y^2+z^2\right)\left(\left(x-\frac{r}{2}\right)^2+y^2+z^2\right)\right)^{\frac{3}{2}}}\,dx\,dy\,dz \qquad (1)$$

We will not be able to integrate this for the analysis of two intersecting neutron-charge spheres because the limits are difficult to handle, as will become apparent in due course, being the intersection volume of two spheres. Further, there is an infinity in the equation at the center of each charge. We know this cannot exist in reality as it would lead to the particle's electric field having an infinite energy so there must be an inner limit to the charge distribution. There is little point in continuing with the integration because of problems with the limits when we come to look at the interaction between two neutrons and we need to use Finite Element Analysis which allows us to carry out a summation instead [5].

First, we can simplify the equation by noting that there is perfect rotational symmetry around the x-axis as all particles have rotational symmetry, provided they are placed along the x-axis as we chose to do for this analysis. We rotate the y-axis around the x-axis through 180 degrees, thereby replacing the integration over z with the term πy, to eliminate the z-axis. This makes the computational load of the Finite Element calculation much lower. This gives

$$U = \frac{q_1 q_2}{16\pi^2\varepsilon}\iint \frac{y\left(\left(x+\frac{r}{2}\right)\left(x-\frac{r}{2}\right)+y^2\right)}{\left(\left(\left(x+\frac{r}{2}\right)^2+y^2\right)\left(\left(x-\frac{r}{2}\right)^2+y^2\right)\right)^{\frac{3}{2}}}\,dx\,dy$$

This can be taken a little further if desired, by recognizing that the mirror symmetry around the y-axis allows us to integrate that axis over the range 0 to infinity, and double the result.

We now convert the integration into a Finite Element summation. To avoid the inner limit on each charge where the equation goes to infinity, we can simply take about 1% of the charge separation as the inner radius limit without affecting the result significantly. The reason it does not affect the results very much is that the left charge in Fig. 1 has its fields to the left of it aligned with the field from the right charge and to the right the fields are in opposition to the same degree so as the gradient in the field from the right charge is sufficiently low over very short distances the effects from the right and the left will almost entirely balance out. These regions at the centers of the charges are eliminated from the summation.

In theory for electrons the outer limits of the integration stretch to infinity which would make the summation impossible in finite time. In fact, as we travel farther from the charges the potential energy density falls off rapidly as the inverse cube of the distance from the center of the system. Even allowing that the volume of space involved is growing at the square of the distance from the center of the system, it still means that only about 1% of the potential energy lies outside a sphere of radius 100 times the separation between the charges. For our purposes this is an acceptable error and we can use this as the summation's upper limit.

With these revised limits the above equation allows us to calculate the potential energy. However, we are looking for the force between the two charges rather than the potential energy. We use the equation F=dU/dr, where force 'F' is equal to the rate at which energy U changes divided by the rate at which separation 'r' changes. The technique in Finite Element Analysis is to compute the potential energy at a desired separation, then change the separation by a small amount (but not too small or quantization errors from the Finite Element grid start to creep into the results) and recalculate the potential energy. Then the difference in the potential energies, divided by the change in separation, gives us the force at that separation.

## III. USING THE FINITE ELEMENT SUMMATON METHOD FOR THE FORCE BETWEEN TWO ELECTRONS

When we use the above approach for the force between two electrons $q_1$ and $q_2$ the result agrees with the value provided by the Lorentz equation for the force between two charges, with minor computational errors from the Finite Element approach, namely

$$F = \frac{q_1 q_2}{4\pi\varepsilon r^2}$$

In the form where one point charge is conceptually seen as lying in the distributed electric field of another we have $F=q_1E_2$. Here $E_2$ is the electric field vector generated by charge $q_2$ as seen at the center of $q_1$. Conceptually, $q_1$ is a point charge and the distributed field from q2 permeates all space. We can equally choose $q_2$ to be a point charge in the distributed field from $q_1$ and get the same result. This does not mean that one charge has miraculously become a point charge while the other remains a distributed electric field; it is merely a shorthand form that allows a quicker calculation.

## IV. THE STRUCTURE OF THE ELECTROSTATIC INTERACTION BETWEEN TWO ELECTRONS

Let us look in more detail at the energy interaction between two electrons. Consider the numerator from (1).

$$\left(\left(x+\frac{r}{2}\right)\left(x-\frac{r}{2}\right)+y^2+z^2\right)$$

This expands to

$$\left(x^2+y^2+z^2-\frac{r^2}{4}\right)$$

Inspection shows that the numerator is zero on the circumference of a sphere whose diameter ‘r’ is the line joining the two charges. The sign of the function changes as we move from inside the sphere to the outside; this tells us that there are attractive forces inside this region and repulsive forces outside it. Between two electrons Finite Element analysis tells us that this “Sphere of Attraction” generates attractive forces whose strength is about 22% of the repulsive forces outside the sphere, leading to a net repulsion.

Between two opposite charges such as the electron and positron this sphere is a region of repulsion and in a similar way this repulsion is overcome by the stronger attractive forces outside the sphere.

To get an idea of the structure of attraction and repulsion in the interaction between two electrons, consider Fig. 2, which shows how electric field lines from each charge are orthogonal to each other on the sphere whose diameter joins the centers of the two charges. Being orthogonal to each other, there is no interaction on the surface of this sphere between the charges and hence no potential energy is generated on this surface. Inside the sphere, however, the field lines intersect at between +90 to +270 degrees, so the fields oppose each other at least in part and hence tend to cancel each other. This reduces the energy density inside the sphere, leading to attractive forces in this region. Outside the sphere, however, the field vector intersection is between -90 to +90 degrees and the fields augment each other so that the energy density increases and repulsive forces are generated. The net effect is one of repulsion between the electrons, as the overall strength of the attractive forces inside the sphere is a little over one fifth of the repulsive forces outside the sphere. This ratio applies at all separations because the electron’s fields extend to infinity.

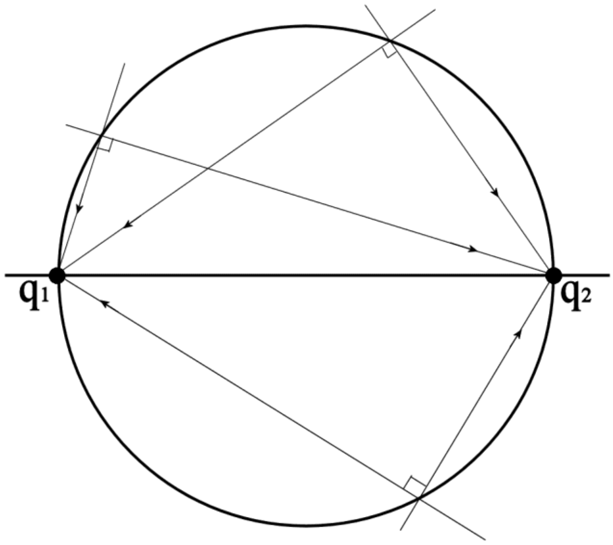


Figure 2. The orthogonal field vectors from each electron describe a sphere.

Fig. 3 shows the classical field pattern which results from

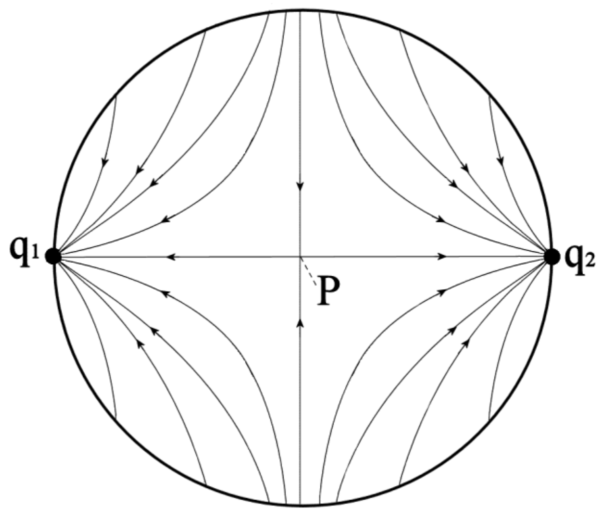


Fig 3. The composite field pattern between two electrons

this field cancellation within the Sphere of Attraction from electrons $q_1$ and $q_2$. This structure looks rather like a repulsion pattern between the two electrons but nothing could be further from the truth. At point P in Fig. 3, midway between the electrons, the fields from the electrons are equal and opposite, so the energy density is zero and the attractive force is at its highest. Field regions that are vertically up and down from this point have their horizontal component cancelled leaving only their vertical component. Other flux lines to the left and right of these in Fig. 3 are continuous, running from the periphery to the center of one or other electron, but the nearer they are to P the greater the cancellation and the weaker the field strength and the higher the attractive forces.

It can be seen that the flux lines between two electrons make a rather beautiful composite field pattern in space. More complex arrangements of charges create an art form of their own. No matter how complex the charge distribution, providing all the charges have fields that extend to infinity the net forces are perfectly described by the Lorentz Force Equation and its "point charge in a distributed field" approach works well.

It is clear that for two charges of opposite polarity such as an electron and a positron the sphere of attraction becomes the sphere of repulsion. The repulsive forces are much smaller than the attractive forces outside the sphere and so the attractive forces dominate.

## V. THE ELECTROSTATIC FORCES BETWEEN TWO NEUTRONS

Consider now what happens if the charges' fields do not extend to infinity but are suddenly truncated at some finite radius. Equation (1) above is still valid, but because the limits of the interaction are truncated the limits of the integration changes and the result is no longer the Lorentz value. The neutron is such a particle, known to have a positive electric field truncated at a radius of roughly $0.7 \times 10^{-15}$ meters. Consider Fig. 4, showing the interaction between two neutrons at three different separations. At each separation the black circles show the limits of the electric field of the two neutrons and the dashed circle indicates the limits of the Sphere of Attraction. The Finite Element summation limits cover only that region where the neutron fields overlap and therefore interact, namely that region which is simultaneously within the field boundary of both neutrons. We can reasonably assume that the field strength of the neutron follows the classic $1/r^2$ profile inside its charge radius. This region is shown dotted-in in Fig. 4 and the Finite Element summation limits cover those parts of the neutrons' fields that are inside the boundary radius of both fields.

In Fig. 4a the neutron fields do not overlap, and at all

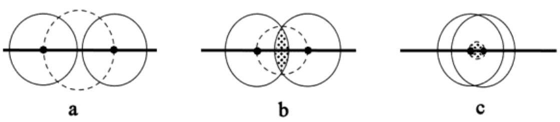


Figure 4. The interaction between two neutrons at different separations.

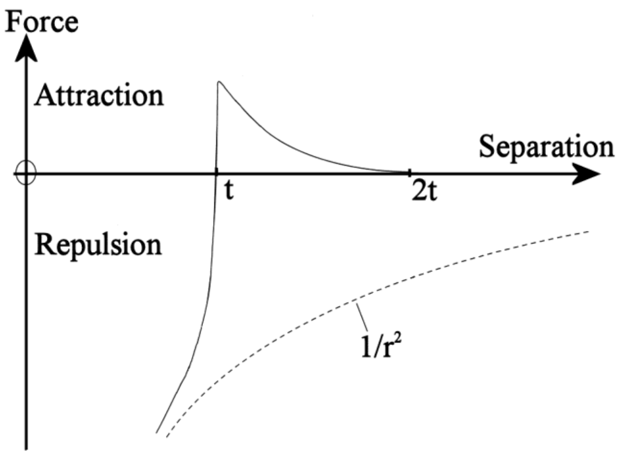


Figure 5. Plot of force by separation for two neutrons

separations greater than twice the truncated charge radius there is no overlap and hence no interaction. In Fig. 4b there is a partial overlap of the neutrons' fields, and so there is an interaction; however, this interaction is entirely contained within the Sphere of Attraction and so at this separation the forces are entirely attractive. In Fig. 4c the Sphere of Attraction is small compared with the total overlap, and bearing in mind that most of the repulsive forces are generated inside a radius of about five times the separation it can be seen that the region of repulsion dominates. Finite Element Analysis gives the force profile shown in Fig. 5. This figure compares the force/separation curve of two neutrons of radius 't'. For reference, the $1/r^2$ curve that would apply if the neutron's electric field extended to an infinite radius is shown as a dotted line. There is no interaction at a separation of more than twice the neutron's charge radius 't' as the fields do not overlap. Below this, as the neutrons come together, the attractive force rapidly climbs to a sharp peak at about 1.02t, then dramatically reverses into a very steep part of the curve, passing through zero force at a separation of 1.00t and continuing falling to converge with the $1/r^2$ curve. Two neutrons would therefore come to rest at a center-to-center separation of one neutron radius t where the force is zero.

Diagrams of how the neutron-neutron force changes with separation generally show a softer curve than that shown in Fig. 5, having a soft peak to right of the theoretical peak. There are two possibilities for this difference. The first possibility is that nuclear kinetics limits the accuracy of measurement, softening the curve as if viewed through a low-pass filter and essentially smoothing out the sharp peak and instead showing just the broad outline of the curve. The second possibility is that the neutron's electric field does not abruptly drop to zero at the limiting radius but fades to zero over a small shift in that radius (our calculation in Fig. 5 assumed a hard truncation of the electric field).

Existing studies of the force between two neutrons agree with the right-hand side of the plot in Fig. 5 but detailed studies of the force as it drops below zero do not seem to exist, especially in the region of convergence to a $1/r^2$ curve at separations significantly less than the charge radius 't'. This information would be a real test of this approach. However, with the information we have this theory gives a very good match to the attraction/repulsion forces between neutrons. This demonstrates that the electrostatic forces between neutrons are in fact likely to be the nuclear binding forces. It also confirms the inapplicability of the Lorentz Force Equation in this situation. For example, in Fig. 4b Lorentz predicts that there is no force because the center of one neutron is outside the electric field of the other so is in a region of zero external field strength, yet there is clearly an interaction in the region where the fields overlap. The Lorentz Force Equation works only for those particles whose electric fields extend to infinity.

## VI. A MODEL FOR THE PROTON

With the electron we know that the electric field continues inwards to a fixed radius at which it is truncated, because if it continued to zero radius the rest mass/energy of the electron would be infinite. We could perhaps model the proton electromagnetically as a sort of super-electron where the inner radius of the field continues to a smaller radius than that of the electron. However, it is impossible to make such an object hold position inside an atomic nucleus that contains other protons and such a model will not work. However, a model that does work well is where the proton has a neutron-like core with a high positive field strength similar to the neutron's, surrounded by a much weaker positron-strength positive field that extends from the inner core's outer periphery out to infinity. The interaction between such a proton core and a neutron would be similar to that between two neutrons at close range, and would also serve to help bind protons into the nucleus against the mutual repulsion of their positron-like outer fields provided that the field strengths of the neutron and the proton core are some orders higher than that of the proton's outer field strength. The attractive force is most effective when the neutron core and the proton core are of similar size. Two protons lying together would be forced slightly further apart than the zero-force rest point by the mutual repulsion of their outer fields.

Where the neutron and the proton core are close but do not overlap and hence do not interact, and provided that the inner boundary of the proton's outer field starts at the outer boundary of the proton core field, this methodology predicts that there will be a weaker close-range attractive force between the

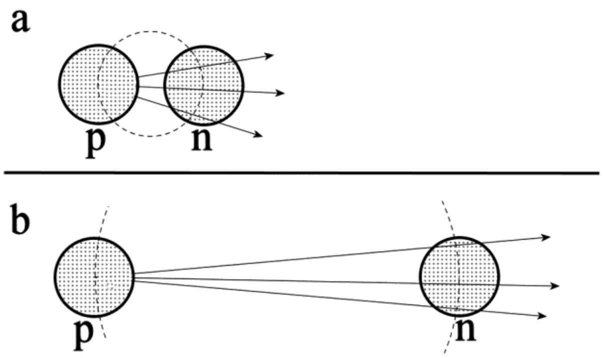


Figure 6. The interaction between a neutron and a proton's outer positron field.

neutron's field and the proton's outer positron-strength field. Consider Fig. 6, with 'p' marking the proton and 'n' the neutron. The neutron and strong proton core fields are shown dotted. The neutron sits in the positron-strength outer field of the proton, with which it interacts as indicated by the drawn electric field vectors of the proton's outer field. The Sphere of Attraction is shown as a dashed curve. In Fig. 6a the neutron and the proton core are close. The strong divergence of the proton's outer field here means that the field is significantly stronger in the region of attraction to the left of the neutron and weaker in the region of repulsion to the right, and this leads to a net attractive force despite the slightly reduced area on the left. In Fig. 6b the neutron is farther from the center of the proton, and the attractive forces on the left of the neutron more nearly match the repulsive forces on the right, almost balancing out and leading to a much-reduced attractive force. In addition, the outer field of the proton is weaker at this separation, and the two effects combine to cause the attractive force to drop rapidly with increasing separation. This interaction between the neutron and the proton's outer positronic field is weak compared to the nuclear binding force and falls off very quickly indeed. The core of one proton will also interact with the outer field of an adjacent one in the same way but except at small separations the effect is dwarfed by the mutual repulsion of the protons' positron-like outer fields. This mechanism extends the region of attraction outwards.

## VII. CONCLUSION

It is a popular tenet that the positive field of the neutron means that neutrons electrostatically repel each other and therefore the nuclear binding force cannot be electrostatic. This paper demonstrates that the reverse is true, in that bounded electric fields must exhibit a push-pull force profile. The models of the neutron and the proton used in this paper are exceedingly simple, and the match between this model of the nuclear binding force and actual experimental data are good. The simplicity of this model and its match with experimental data suggest that the nuclear binding force may well be electromagnetic in origin. The match is not perfect and there are undoubtedly other forces at work in the atomic nucleus but this is clearly a dominating mechanism.

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