

A Bipolar Single Valued Neutrosophic Isolated Graphs: Revisited

Said Broumi

Laboratory of Information Processing, Faculty of
Science Ben M'Sik, University Hassan II, B.P
7955, Sidi Othman, Casablanca, Morocco
broumisaid78@gmail.com

Assia Bakali

Ecole Royale Navale ,Boulevard Sour Jdid,B.P
16303 Casablanca, Morocco
assiabakali@yahoo.fr

Mohamed Talea

Laboratory of Information Processing, Faculty
of Science Ben M'Sik, University Hassan II, B.P
7955, Sidi Othman, Casablanca, Morocco
taleamohamed@yahoo.fr

Florentin Smarandache

Department of Mathematics, University of New
Mexico, 705 Gurley Avenue, Gallup, NM,
87301, USA
fsmarandache@gmail.com; smarand@unm.edu

Mohsin Khan

Department of Mathematics, Abdul Wali Khan University,
Mardan 23200, Pakistan
mohsinkhan7284@gmail.com

ABSTRACT

In this research paper, the graph of the bipolar single-valued neutrosophic set model (BSVNS) is proposed. The graphs of single valued neutrosophic set models is generalized by this graph. For the BSVNS model, several results have been proved on complete and isolated graphs. Adding, an important and suitable condition for the graphs of the BSVNS model to become an isolated graph of the BSVNS model has been demonstrated.

KEYWORDS

Bipolar single valued neutrosophic graphs (BSVNG), complete-BSVNG, isolated-BSVNGs.

1 Introduction

The concept of 'Neutrosophic logic' was developed by Prof. Dr. F. Smarandache in 1995 and get published in 1998. "It is a branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra"[4]. The concepts of fuzzy sets [8] and intuitionistic fuzzy set [6] were generalized by adding an independent indeterminacy-membership. Neutrosophic logic is a powerful tool to deal

with incomplete, indeterminate, and inconsistent information, which is the main reason for widespread concerns of researchers. The concept of neutrosophic set(NS for short) is characterized by three independent degrees namely truth-membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree (F).To practice NSs in real life situations efficiently,The subclass of the neutrosophic sets called single-valued neutrosophic set (in short SVNS) was defined by Smarandache in [4]. In another paper [5], Wang et al. defined the various operations and operators for the SVNS model. In [11] Deli et al. proposed a new concept called bipolar neutrosophic sets. This concept appear as a generalization of fuzzy sets, intuitionistic fuzzy sets, bipolar fuzzy sets, bipolar intuitionistic fuzzy sets and single valued neutrosophic set. The benefits of applying the NSs have been addressed in [18].The theory of graphs is the mostly used tool for resolving combinatorial problems in diverse disciplines like computer science, algebra and topology, etc. In [2, 4] Smarandache proposed two kinds of neutrosophic graphs to deal with situations in which there exist inconsistencies and indeterminacies among the vertices which cannot be dealt with by fuzzy graphs and different hybrid structures including bipolar fuzzy graphs, intuitionistic fuzzy graphs, bipolar intuitionsitc

fuzzy graphs [1,7,9, 10], The first kind is based on literal indeterminacy (I) component, the second kind of neutrosophic graphs is based on numerical truth-values (T, I, F), Recently, a hybrid study by combining SVNS and classical graph theory was carried out and that concept is called Single valued neutrosophic graph (SVNG) was presented by Broumi et al [12, 13, 14, 17, 20, 22].In addition, the concept of bipolar neutrosophic set was combined with graph theory and new graph model was presented. This concept is called bipolar single valued neutrosophic graph (BSVNGs). In [15,16] Broumi et al. proposed the concept of bipolar single valued neutrosophic graph as a generalized the concept of fuzzy graph, intuitionistic fuzzy graph, bipolar fuzzy graph and single valued neutrosophic graph.

The objective of this article is to demonstrate the essential and satisfactory condition of BSVNGs to be an isolated-BSVNG.

2. Background of research

Some of the important background knowledge in this paper is presented in this section. These results can be found in [4, 5, 12,13,15, 21].

Definition 2.1 [4] Let ζ be a universal set. The neutrosophic set A on the universal set ζ categorized into three membership functions called the true membership function $T_A(x)$, indeterminate membership function $I_A(x)$ and false membership function $F_A(x)$ contained in real standard or non-standard subset of $]0, 1^+[$ respectively and satisfy the following condition

$$0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3 \quad (1)$$

Definition 2.2 [5] Let ζ be a universal set. The single valued neutrosophic sets (SVNs) A on the universal ζ is denoted as following

$$A = \{ \langle x: T_A(x), I_A(x), F_A(x) \rangle \mid x \in \zeta \} \quad (2)$$

The functions $T_A(x) \in [0, 1]$, $I_A(x) \in [0, 1]$ and $F_A(x) \in [0, 1]$ are called “ degree of truth, indeterminacy and falsity membership of x in A”, satisfy the following condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \quad (3)$$

Definition 2.3 [12] A SVNG of $G^* = (V, E)$ is a graph $G = (A, B)$ where

a. The following memberships: $T_A: V \rightarrow [0, 1]$, $I_A: V \rightarrow [0, 1]$ and $F_A: V \rightarrow [0, 1]$ represent the truth, indeterminate and false membership degrees of $x \in V$ respectively and

$$0 \leq T_A(w) + I_A(w) + F_A(w) \leq 3 \quad (4)$$

$$\forall w \in V$$

b. The following memberships: $T_B: E \rightarrow [0, 1]$, $I_B: E \rightarrow [0, 1]$ and $F_B: E \rightarrow [0, 1]$ are defined by

$$T_B(v, w) \leq \min [T_A(v), T_A(w)] \quad (5)$$

$$I_B(v, w) \geq \max [I_A(v), I_A(w)] \quad (6)$$

$$F_B(v, w) \geq \max [F_A(v), F_A(w)] \quad (7)$$

Represent the true, indeterminate and false membership degrees of the arc $(v, w) \in (V \times V)$, where

$$0 \leq T_B(v, w) + I_B(v, w) + F_B(v, w) \leq 3 \quad (8)$$

$$\forall (v, w) \in E$$

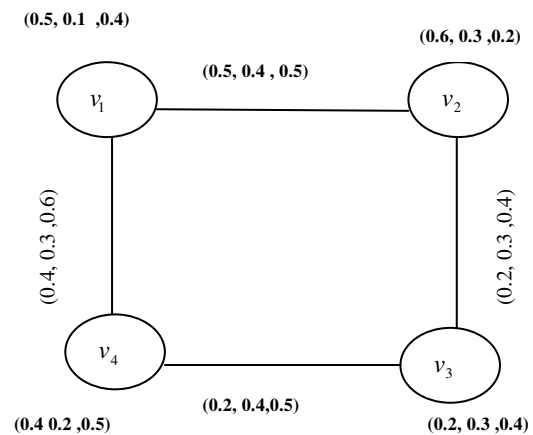


Fig.1.SVN-graph

Definition 2.4 [12]. A SVNG $G = (A, B)$ is named a complete-SVNG if

$$T_B(v, w) = \min [T_A(v), T_A(w)] \quad (9)$$

$$I_B(v, w) = \max [I_A(v), I_A(w)] \quad (10)$$

$$F_B(v, w) = \max [F_A(v), F_A(w)] \quad (11)$$

$\forall v, w \in V$

Definition 2.5[12]. Let $G = (A, B)$ be SVNG. Hence, the complement of SVNG G on G^* is a SVNG \bar{G} on G^* where

a. $\bar{A} = A$ (12)

b. $\bar{T}_A(w) = T_A(w), \bar{I}_A(w) = I_A(w), \bar{F}_A(w) = F_A(w)$
 $\forall w \in V$ (13)

c. $\bar{T}_B(v, w) = \min [T_A(v), T_A(w)] - T_B(v, w)$ (14)

$\bar{I}_B(v, w) = \max [I_A(v), I_A(w)] - I_B(v, w)$ (15)

$\bar{F}_B(v, w) = \max [F_A(v), F_A(w)] - F_B(v, w),$
 $\forall (v, w) \in E.$

Definition 2.6 [15]. A BSVNG $G = (A, B)$ of G^* $= (V, E)$ is a partner such that $A = (T_A^P, I_A^P, F_A^P, T_A^N, I_A^N, F_A^N)$ is a BSVNS in V and $B = (T_B^P, I_B^P, F_B^P, T_B^N, I_B^N, F_B^N)$ is a BSVNS in E such that

(i) $T_B^P(v, w) \leq \min (T_A^P(v), T_A^P(w))$ and
 $T_B^N(v, w) \geq \max (T_A^N(v), T_A^N(w))$ (17)

(ii) $I_B^P(v, w) \geq \max (I_A^P(v), I_A^P(w))$ and
 $I_B^N(v, w) \leq \min (I_A^N(v), I_A^N(w))$ (18)

(iii) $F_B^P(v, w) \geq \max (F_A^P(v), F_A^P(w))$
 and $F_B^N(v, w) \leq \min (F_A^N(v), F_A^N(w)),$
 $\forall (v, w) \in E$ (19)

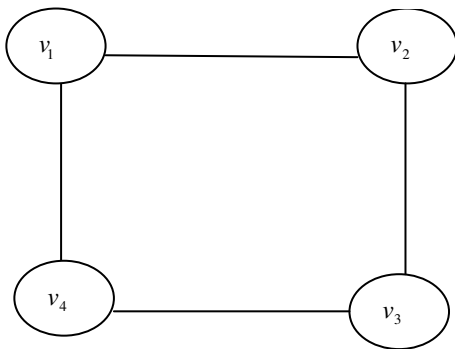


Fig.2 BSVNG

v_i	The values of vertex
v_1	(0.2, 0.2, 0.4, -0.4, -0.1, -0.4)
v_2	(0.1, 0.3, 0.5, -0.6, -0.2, -0.3)
v_3	(0.2, 0.3, 0.5, -0.3, -0.2, -0.1)
v_4	(0.3, 0.2, 0.4, -0.2, -0.3, -0.5)

Table1. The values of vertex of BSVNG

	The values of edge
v_{12}	(0.1, 0.3, 0.6, -0.2, -0.3, -0.1)
v_{23}	(0.1, 0.3, 0.6, -0.1, -0.6, -0.7)
v_{34}	(0.1, 0.5, 0.6, -0.1, -0.6, -0.5)
v_{14}	(0.2, 0.3, 0.5, -0.2, -0.3, -0.5)

Table2. The values of edge of BSVNG

Definition 2.7 [15]. The complement of BSVNG $G = (A, B)$ of $G^* = (V, E)$ is a BSVNG $\bar{G} = (\bar{A}, \bar{B})$ of $G^* = (V, E)$ such that

- (i) $\bar{A} = A = (T_A^P, I_A^P, F_A^P, T_A^N, I_A^N, F_A^N)$ and
- (ii) $\bar{B} = (T_B^P, I_B^P, F_B^P, T_B^N, I_B^N, F_B^N)$ on $E = V \times V$ is defined as

$T_B^P(v, w) = \min (T_A^P(v), T_A^P(w)) - T_B^P(v, w)$
 $T_B^N(v, w) = \max (T_A^N(v), T_A^N(w)) - T_B^N(v, w)$ (20)

$I_B^P(v, w) = \max (I_A^P(v), I_A^P(w)) - I_B^P(v, w)$
 $I_B^N(v, w) = \min (I_A^N(v), I_A^N(w)) - I_B^N(v, w)$ (21)

$F_B^P(v, w) = \max (F_A^P(v), F_A^P(w)) - F_B^P(v, w)$
 $F_B^N(v, w) = \min (F_A^N(v), F_A^N(w)) - F_B^N(v, w),$ (22)
 $\forall (v, w) \in E$

Definition 2.8 [15]. A BSVNG $G = (A, B)$ is called a complete-BSVNG if

$T_B^P(v, w) = \min (T_A^P(v), T_A^P(w)),$ (23)

$T_B^N(v, w) = \max (T_A^N(v), T_A^N(w)),$ (24)

$I_B^P(v, w) = \max (I_A^P(v), I_A^P(w)),$ (25)

$I_B^N(v, w) = \min (I_A^N(v), I_A^N(w))$ (26)

$F_B^P(v, w) = \max (F_A^P(v), F_A^P(w)),$ (27)

$F_B^N(v, w) = \min (F_A^N(v), F_A^N(w)) \quad \forall v, w \in V$ (28)

Definition 2.9[7]. The complement of BIFG $G = (A, B)$ of $G^* = (A, B)$ is a BIFG $\bar{G} = (\bar{A}, \bar{B})$ of $\bar{G}^* = (V, V \times V)$ where $\bar{A} = A = (T_A^P, F_A^P, T_A^N, F_A^N)$ and $\bar{B} = (\bar{T}_B^P, \bar{F}_B^P, \bar{T}_B^N, \bar{F}_B^N)$ are defined as

$\bar{T}_B^P(v, w) = \min (T_A^P(v), T_A^P(w)) - T_B^P(v, w)$ (29)

$\bar{F}_B^P(v, w) = \max (F_A^P(v), F_A^P(w)) - F_B^P(v, w)$ (30)

$\bar{T}_B^N(v, w) = \max (T_A^N(v), T_A^N(w)) - T_B^N(v, w)$ (31)

$\bar{F}_B^N(v, w) = \min (F_A^N(v), F_A^N(w)) - F_B^N(v, w) \quad \forall v, w \in V, vw \in \bar{V}^2$ (32)

Theorem 2.10[13] Let $G = (A, B)$ be a SVNG, then the SVNG is called an isolated-SVNG if

and only if the complement of G is a complete-SVNG.

Theorem 2.11[21] Let $G=(A,B)$ be a FG, then the FG is called an isolated-FG if and only if the complement of G is a complete- FG

3. MAIN RESULTS

Theorem 3.1: A BSVNG (A,B) is an isolated-BSVNG iff the complement of BSVNG is a complete- BSVNG.

Proof: Let $G=(A, B)$ be a complete- BSVNG.

$$\text{Therefore } T_B^P(v, w) = \min(T_A^P(v), T_A^P(w)),$$

$$T_B^N(v, w) = \max(T_A^N(v), T_A^N(w)),$$

$$I_B^P(v, w) = \max(I_A^P(v), I_A^P(w)),$$

$$I_B^N(v, w) = \min(I_A^N(v), I_A^N(w)),$$

$$F_B^P(v, w) = \max(F_A^P(v), F_A^P(w)),$$

$$F_B^N(v, w) = \min(F_A^N(v), F_A^N(w)), \forall v, w \in V.$$

Hence in \bar{G} ,

$$\begin{aligned} \bar{T}_B^P(v, w) &= \min(T_A^P(v), T_A^P(w)) - T_B^P(v, w) \\ &= \min(T_A^P(v), T_A^P(w)) - \min(T_A^P(v), T_A^P(w)) \\ &= 0 \end{aligned}$$

and

$$\begin{aligned} \bar{I}_B^P(v, w) &= \max(I_A^P(v), I_A^P(w)) - I_B^P(v, w) \\ &= \max(I_A^P(v), I_A^P(w)) - \max(I_A^P(v), I_A^P(w)) \\ &= 0 \end{aligned}$$

In addition

$$\begin{aligned} \bar{F}_B^P(v, w) &= \max(F_A^P(v), F_A^P(w)) - F_B^P(v, w) \\ &= \max(F_A^P(v), F_A^P(w)) - \max(F_A^P(v), F_A^P(w)) \\ &= 0 \end{aligned}$$

We have for the negative membership edges

$$\begin{aligned} \bar{T}_B^N(v, w) &= \max(T_A^N(v), T_A^N(w)) - T_B^N(v, w) \\ &= \max(T_A^N(v), T_A^N(w)) - \max(T_A^N(v), T_A^N(w)) \end{aligned}$$

= 0 and

$$\begin{aligned} \bar{I}_B^N(v, w) &= \min(I_A^N(v), I_A^N(w)) - I_B^N(v, w) \\ &= \min(I_A^N(v), I_A^N(w)) - \min(I_A^N(v), I_A^N(w)) \\ &= 0 \end{aligned}$$

In addition

$$\begin{aligned} \bar{F}_B^N(v, w) &= \min(F_A^N(v), F_A^N(w)) - F_B^N(v, w) \\ &= \min(F_A^N(v), F_A^N(w)) - \min(F_A^N(v), F_A^N(w)) \\ &= 0 \end{aligned}$$

$$\text{So } (\bar{T}_B^P(v, w), \bar{I}_B^P(v, w), \bar{F}_B^P(v, w), \bar{T}_B^N(v, w), \bar{I}_B^N(v, w), \bar{F}_B^N(v, w)) = (0, 0, 0, 0, 0, 0)$$

Hence $G=(A, B)$ is an isolated-BSVNGs

Proposition 3.2: The notion of isolated-BSVNGs generalized the notion of isolated fuzzy graphs.

Proof: If the value of $I_A^P(w) = F_A^P(w) = T_A^N(w) = I_A^N(w) = F_A^N(w) = 0$, then the notion of isolated-BSVNGs is reduced to isolated fuzzy graphs.

Proposition 3.3: The notion of isolated-BSVNGs generalized the notion of isolated-SVNGs.

Proof: If the value of $T_A^N(w) = I_A^N(w) = F_A^N(w) = 0$, then the concept of isolated-BSVNGs is reduced to isolated-SVNGs.

Proposition 3.4: The notion of isolated-BSVNGs generalized the notion of isolated-bipolar intuitionistic fuzzy graph.

Proof: If the value of $I_A^P(w) = I_A^N(w)$, then the concept of isolated-BSVNGs is reduced to isolated-bipolar intuitionistic fuzzy graphs

IV.COMPARTIVE STUDY

In this section, we present a table showing that the bipolar single valued neutrosophic graph generalized the concept of the crisp graph, fuzzy graph [9], intuitionistic fuzzy graph[1], bipolar fuzzy graph[10], bipolar intuitionistic fuzzy graph[7] and single valued neutrosophic graph[12].

For convenience we denote F-graph : Fuzzy graphs

IF-graph: Intuitionistic fuzzy graph
 BF-graph: Bipolar fuzzy graph
 BIF-graph: Bipolar intuitionistic fuzzy graph
 SVN-graph: Single valued neutrosophic graph
 BSVN-graph: Bipolar single valued neutrosophic graph

BSVNGs generalized the isolated-fuzzy graph and isolated- SVNGs. In addition, in future research, we shall concentrate on extending the idea of this paper by using the interval valued bipolar neutrosophic graph as a generalized form of bipolar neutrosophic graph.

Type of graphs	The membership values of vertex/ edge					
	$T_A^p(w)$	$I_A^p(w)$	$F_A^p(w)$	$T_A^n(w)$	$I_A^n(w)$	$F_A^n(w)$
crisp graph	1 or 0	0	0	0	0	0
FG	ϵ [0,1]	0	0	0	0	0
IFG	ϵ [0,1]	0	ϵ [0,1]	0	0	0
SVNG	ϵ [0,1]	ϵ [0,1]	ϵ [0,1]	0	0	0
BFG	ϵ [0,1]	0	0	ϵ [-1,0]	0	0
BIFG	ϵ [0,1]	0	ϵ [0,1]	ϵ [-1,0]	0	ϵ [-1,0]
BSVNG	ϵ [0,1]	ϵ [0,1]	ϵ [0,1]	ϵ [-1,0]	ϵ [-1,0]	ϵ [-1,0]

Table3. Different types of graphs

Neutrosophic graph is the generalization of crisp graph, fuzzy graph, intuitionistic fuzzy graph, bipolar fuzzy graph, bi-polar intuitionistic fuzzy graph and single-valued neutrosophic graph. In this table, we can see that by removing the indeterminacy and non-membership values from neutrosophic graph, the neutrosophic graph reduces to fuzzy graph. By removing the indeterminacy value from neutrosophic graph, the neutrosophic graph reduces to intuitionistic fuzzy graph. Similarly, by removing the positive and negative indeterminacy and non-membership values from bi-polar neutrosophic graph, the bi-polar neutrosophic graph reduces to bi-polar fuzzy graph. By removing the positive and negative indeterminacy values from bi-polar neutrosophic graph, the bi-polar neutrosophic graph reduces to bi-polar intuitionistic fuzzy graph. By the similar way, we can reduce a bi-polar single valued neutrosophic graph to a neutrosophic graph by removing the negative membership, indeterminacy and non-membership values.

5. CONCLUSION

In this article, we have proved necessary and sufficient condition under which BSVNGs is an isolated-BSVNGs. The notion of isolated-

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