The Relation of Particle Sequence to Atomic Sequence

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In this paper, we take the first steps of simplifying particles into a linear function that organizes particles based on their particle number, similar to how atoms are arranged by atomic number. This repeats the method that was used to organize atomic elements and create the Periodic Table of Elements in the 1800s. The solution to linearize particles into a predictable function is not as simple as atomic elements, but it does exist. We will introduce an equation that fits known particles into a linear function and enables the prediction of future particles based on missing energy levels. It also predicts an exact mass of the neutrino. To accomplish this, particles are first organized by particle numbers, similar to atomic numbers in the Periodic Table of Elements and then charted against their known Particle Data Group energy levels. The results show similarities between particles and atomic elements – in both total numbers in formation and also in numbers where both are known to be more stable.

Key words: neutrino, atomic numbers, periodic table, leptons, Mendeleev.

1. BACKGROUND – ATOMIC NUMBERS

In 1869, Dmitri Mendeleev presented *The Dependence Between the Properties of the Atomic Weights of the Elements* to the Russian Chemical Society [1], which included the first version of the Periodic Table of Elements. By relating atomic elements and their atomic mass, Mendeleev was able to predict undiscovered elements and their masses by arranging them into a periodic table [2].

Until the 1900s, atomic elements were considered to be fundamental. In 1911, the understanding of the atomic nucleus changed when Ernest Rutherford discovered the proton [3]. Following his discovery of the atomic nucleus structure, Antonius van den Broek proposed that the atomic number in the Periodic Table of Elements was the nuclear charge of the element [4]. Then, in 1913, Henry Moseley found a linear function between the atomic number and a measurable property of the atom's nucleus [5]. These events provided the foundation of a predictable sequence and a logical theory of the atomic nucleus. Now, it is commonly accepted that the proton is the particle that creates each of the atomic elements in the table. What seemed complex before the Periodic Table of Elements and the discovery of the proton is now simplified to basic atomic components: protons, neutrons and electrons [6].

The arrangement of the periodic table by Mendeleev and the linking to the proton as the atomic number by Moseley was possible because of a linear arrangement of atomic mass to atomic number. As elements were discovered, their atomic mass fit into a predictable sequence. This is shown in Fig. 1.

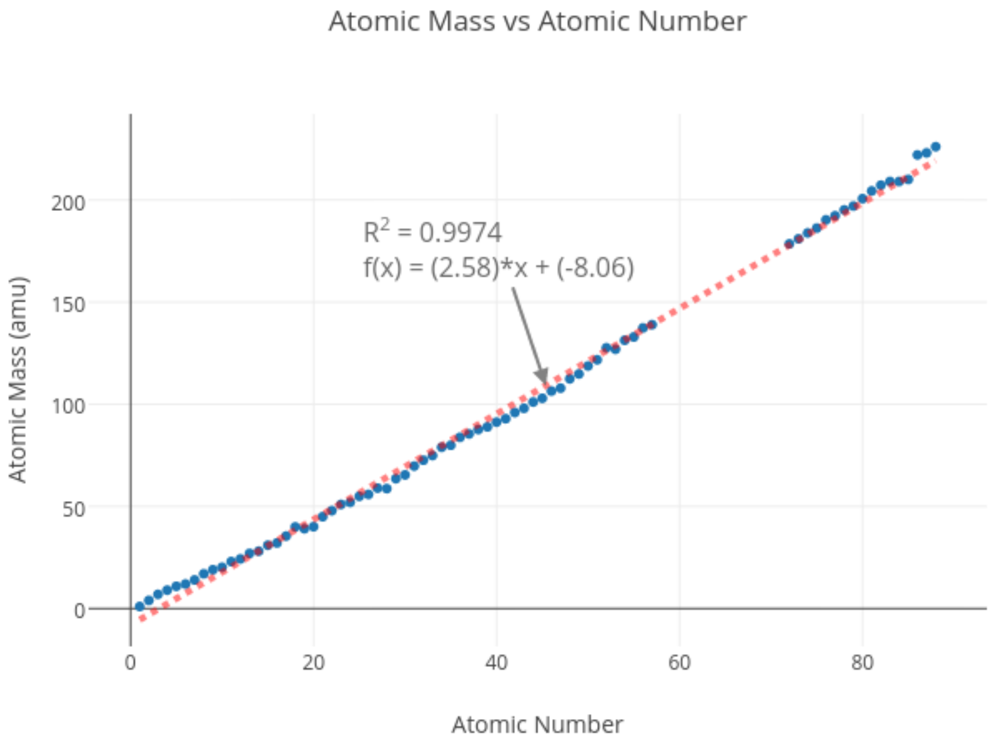

Fig. 1. Atomic number vs atomic mass. Hydrogen (Z=1) to ununoctium (Z=118) [7].

Hydrogen, for example, has an atomic number of one (Z=1) and an atomic mass of 1.008 amu [8]. Helium has an atomic number of 2 (Z=2) due to its two protons and an atomic mass of 4.003 amu (stable helium has two neutrons in addition to the protons). By plotting atomic numbers and mass, it yields a predictable, linear line (Fig. 1). What once seemed complex was simplified with a math function that allowed the prediction of undiscovered atomic elements.

2. SIMPLIFING PARTICLE ENERGY INTO A LINEAR EQUATION

Currently, the world of subatomic particles is nearly as complex as the discovery of new atomic elements in the 1800s. Atomic elements were simplified to be based on the number of protons in the nucleus, yet as we dig deeper into the constructs of the proton, it becomes a complex world again. A proton can be smashed into another proton at high energies to create various new particles. This leads to a question. Why would nature go from complex (dozens of elements) to simple (protons, neutrons and electrons) back to complex (dozens of subatomic particles) as we get smaller and smaller?

The search for a building block to unite subatomic particles, analogous to the proton as the building block for elements, begins with an approach similar to the one Mendeleev used in the 1800s when elements were organized by number and mass/energy [1]. If particles can be simplified like atomic elements in Fig. 1, they need to be organized into a linear equation [9], such as the following, where m is the slope of the line:

$$y = mx + b \tag{1}$$

Our objective is to find commonality in the masses of major particles. Unfortunately, plotting the currently known energies of particles ranging from 2.2 eV for the neutrino to 125 GeV for the Higgs boson [10] doesn't yield helpful results. It is far from linear. Thus, to find a linear solution, the first hint is in the quantized lepton mass equation from A.O. Barut [11], found in Eq. (2). Particle masses are not linear, but raised to a fourth power suggested in Barut's equation. This equation summarizes the electron, muon electron and tau electron masses:

$$M_n = M_e + \frac{3M_e}{2\alpha}\sum_{n=0}^{n} n^4 \tag{2}$$

To create a linear function for all particles, it was assumed that there is a fundamental particle responsible for the creation of all other particles, similar to the proton's role creating atomic elements. Whereas the proton is given the letter Z, for proton count in an atomic nucleus, the letter K was assigned to represent the particle number. K is represented in integers (1, 2, 3, etc.) for the number of fundamental particles that combine – like protons in the core of an atomic nucleus - to create the particles that are seen in nature and particle laboratories. The linear function from Eq. (1) is modified to contain this particle number, K:

$$y = mK \tag{3}$$

Eq. (3) was not expected to result in a linear function for particles, otherwise it would have been recognized years ago. However, reviewing A.O. Barut's equation [11], it is clear that a quantized number to the fourth power can yield the masses of the electron family. If the electron masses were governed by a quantum number to the fifth power (K^5), but divided by the same number to the fourth power (K^4), it would yield a linear solution. Mathematically, this is represented by:

$$y = m(\frac{K^5}{K^4}) = mK \tag{4}$$

The hypothesis presented in this paper is that particles, when divided by the fourth power of the quantum particle number (K^4), yields a linear solution because one quantum particle number (K) would remain in the equation, as described in Eq. (4).

This hypothesis also requires a value and an explanation for the slope (m) in Eq. (3). An initial energy was chosen as the slope, which is the baseline particle number (K). Since the neutrino (v_e) is the lightest-known particle, the slope of the linear function in Eq. (4) was initially set to the high end of the estimated neutrino energy range – 2.2 eV [12]. This becomes the first attempt at a linear particle energy equation with m=2.2 eV (neutrino) and x=K (particle number):

$$y = 2.2K \tag{5}$$

For Eq. (5) to work properly, particle energies need to be modified by dividing by K^4, as described in Eq. (4). This is because particle energy is to the fifth power of K, but dividing by the fourth power allows a linear function that can be graphed.

2.1. PLOTTING PARTICLE NUMBERS VS REST ENERGY: y = 2.2×K

Many of the known subatomic particles were calculated and charted based on the proposed linear equation, Eq. (5), and on their particle number K. The neutrino was used as the baseline for the value K=1. It can be considered the equivalent of hydrogen in atomic elements, occupying the first position in the table. In the first iteration, a value of 2.2 eV is used for the rest energy of the neutrino. All calculations use Particle Data Group (PDG) rest energy values for subatomic particles [10].

Steps:

1. Using Eq. (5), a linear function was created by using y = 2.2×K, where m=2.2 is the neutrino mass in electron-volts (eV), and values of K range from K=1 to K=141. This is plotted as the trendline in Fig. 2 and the values are found in the third row of Table 1 (y=2.2×K).

2. The rest energies of many known particles, from the neutrino to the Higgs boson, were divided by the nearest K^4 value. The rest energies were compared to the closest value in Step 1 and placed into the appropriate column for K in Table 1 and then plotted in Fig. 2.

Step 1 example: Value of y when K=12 using Eq. (5).

$$y = 2.2(12) = 26.40 \tag{6}$$

Step 2 example: The electron's rest energy (5.11×10^5 eV) is divided by K^4. The best fit is when K=12 for the electron. The values from Step 1 and Step 2 are inserted into the column for K=12 in Table 1 and assigned to the electron particle.

$$\frac{5.11\times10^5}{(12)^4} = 24.64 \tag{7}$$

Particle Number (K)	**1**	**9**	**12**	**23**	**34**	**60**	**142**
Particle Name	Neutrino	Muon Neutrino	Electron	Tau Neutrino	Muon Electron	Tau Electron	Higgs Boson
Energy(eV) [a]	2.20	1.70×10^5	5.11×10^5	1.55×10^7	1.06×10^8	1.78×10^9	1.25×10^{11}
y=2.2×K [b]	2.20	19.80	26.40	50.60	74.80	132.00	312.40
En. (eV)/K^4 [c]	2.20	25.91	24.64	55.39	79.32	137.11	308.18

Table 1. Particle rest energy of leptons and the Higgs boson compared to linear particle energy equation y=2.2×K. Particles were placed into the column with the closest fit of K (particle number).

[a] Values obtained from Particle Data Group [10].
[b] Calculated value using Eq. 5.
[c] Calculated value using Particle Data Group energy [10] divided by particle number to the fourth degree (K^4).

The steps above were repeated for many of the known particles, beyond the lepton family of particles, and then charted in Fig. 2. Particle energies are nearly linearized when dividing their energies by the fourth power of the particle number (K^4). A relatively simple equation can be used to arrange particles into a chart that is similar to atomic elements (Fig. 1 - atomic numbers vs atomic mass).

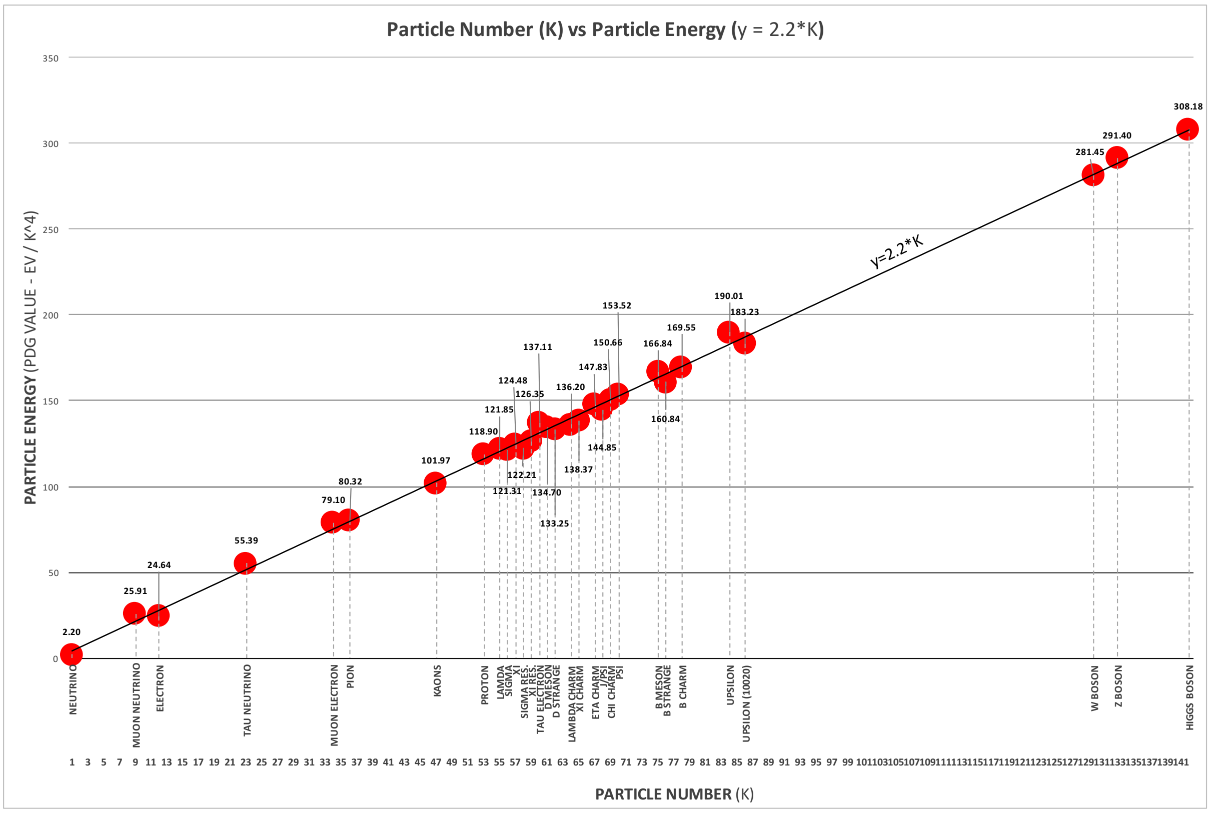

Fig. 2. Relation of particle rest energy to particle number (K) - similar to atomic number (Z) - based on a linear equation y=2.2×K. The Y axis is the Particle Data Group (PDG) rest energy value of the particle in electron-volts (eV) divided by a value that is based on the particle number to the fourth power (K^4).

Some particles were excluded due to overlapping particle numbers. When this occurred, the neutral charge particle was used as the energy value. For example, the neutral kaon has a rest energy of 497.6 MeV and the charged kaons have rest energies of 493.7 MeV [10]. Both would occupy the slot with particle number 47 (K=47), so the neutral charge value is used. Note that the same occurrence happens in atomic elements. Lithium, for example, has 3 protons (Z=3), yet it has atomic weight differences for ^{6}Li and ^{7}Li at 6.02 amu and 7.02 amu [13] respectively.

Observations:

- Despite the fact that particle energies from the lightest neutrino to the massive Higgs boson range from 2.2 eV to 1.25×10^{11} eV, they can be simplified to fit into a linear solution based on a new quantum number, called the particle number K in this paper.
- Beginning with the proton at K=53, until K=87, there is a cluster of subatomic particles. This is not surprising since many of these particles are found in particle accelerator experiments by smashing protons at high energies.
- There are similarities between particles and atomic elements in both the range (number of known atomic elements at 118 elements), and the sequence of stable atomic elements that start to become apparent in Fig. 2. This led to further iteration on the linear equation, explained in the next section.

2.2. PLOTTING PARTICLE NUMBERS VS REST ENERGY: $y = 2.2 \times K \times \sum (n^3-(n-1)^3)/n^4$

A second chart was created by extending the previous linear equation with a summation of energy of spherical shells, decreasing in energy based on the number of shell wavelengths (n) and proportional to the particle number (K) [14].

$$\sum_{n=1}^{K} \frac{n^3-(n-1)^3}{n^4} \tag{8}$$

This summation, shown in Eq. (8), is then added to the linear equation in Eq. (9). It forms a new linear particle energy function where m=2.2 (neutrino) and x=K×∑. Then, the same steps from the previous section were repeated with the modified equation.

$$y = 2.2K\sum_{n=1}^{K} \frac{n^3-(n-1)^3}{n^4} \tag{9}$$

Step 1 example: Using Eq. (9), the value of y when K=10.

$$y = 2.2(10)\sum_{n=1}^{(10)} \frac{n^3-(n-1)^3}{n^4} = 47.05 \tag{10}$$

Step 2 example: The electron's rest energy (5.11×10^5 eV) is divided by K^4. The best fit is when K=10 for the electron. The values from Step 1 and Step 2 are inserted into the column for K=10 in Table 2 and assigned to the electron particle.

$$\frac{5.11\times10^5}{(10)^4} = 51.10 \tag{11}$$

Particle Number (K)	**1**	***8***	**10**	***20***	**29**	**51**	**119**
Particle Name	Neutrino	Muon Neutrino	Electron	Tau Neutrino	Muon Electron	Tau Electron	Higgs Boson
Energy(eV) [a]	2.20	1.70×10^5	5.11×10^5	1.55×10^7	1.06×10^8	1.78×10^9	1.25×10^{11}
y=2.2×K×∑ [b]	2.20	36.58	47.05	99.80	147.44	264.04	624.64
En. (eV)/K^4 [c]	2.20	41.50	51.10	96.88	149.87	262.67	624.83

Table 2. Particle rest energy of leptons and the Higgs boson compared to a linear particle energy equation y=2.2×K×∑. Particles were placed into the column with the closest fit for the value of K (particle number). 8 and 20 are magic numbers also seen in atomic elements.

[a] Values obtained from Particle Data Group [10].
[b] Calculated value using Eq. 9.
[c] Calculated value using Particle Data Group energy [10] divided by particle number to the fourth degree (K^4).

The same process was repeated for all of the particles in the previous section and charted against the revised linear equation from Eq. (9).

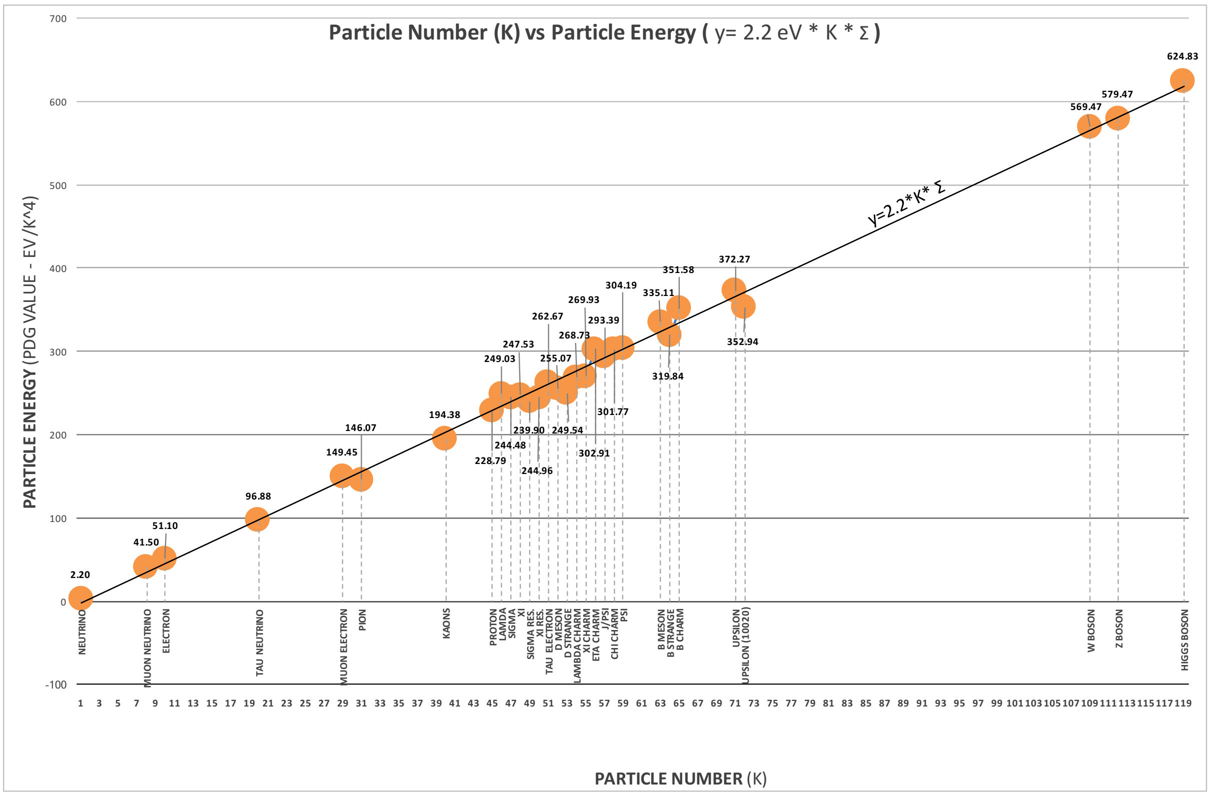

Fig. 3. Relation of particle rest energy to particle number (K) based on linear equation y=2.2×K×∑. The Y axis is the Particle Data Group (PDG) rest energy value of a particle in electron-volts (eV) divided by a value that is based on the particle number to the fourth power (K^4). With the inclusion of ∑ in the equation, the lepton sequence is similar to atomic element magic numbers.

Observations:

- In atomic elements, there are magic numbers of nucleons in an atom that lead to greater stability relative to other elements. The first five numbers in this magic number sequence are: 2, 8, 20, 28, and 50 [15]. In Fig. 3, the leptons (neutrino and electron family of particles), nearly fit into this sequence. They have K values of: neutrino (1), muon neutrino (8), tau neutrino (20), muon electron (29) and tau neutrino (51).

- The Higgs boson at K=119 is barely outside of the range of the Periodic Table of Elements which has 118 known elements.

Given the closeness of the lepton sequence and the Higgs boson value, the linear equation was further iterated upon.

2.3. PLOTTING PARTICLE NUMBERS VS REST ENERGY: $y = 2.38925 \times K \times \sum (n^3 - (n-1)^3)/n^4$

When plotting particles in Fig. 4, it is found that the trendline is y=2.3875x with an accuracy of R^2=0.99529. This is an indicator that the initially estimated fundamental particle mass (neutrino) is incorrect at 2.2 eV and that it is possibly closer to 2.3875 eV. Thus, for the third attempt, the slope of the linear equation was first set to m=2.3875. However, it was recognized that a slight variation of this value led to an exact calculation of the electron particle. Since the electron's rest energy has been accurately measured, the final slope was set to m=2.38925 to match this result (x remains the same – a function of K×∑).

$$y = 2.38925K\sum_{n=1}^{K}\frac{n^3-(n-1)^3}{n^4} \tag{12}$$

The same steps from the previous section were repeated with the final equation, Eq. (12). An example of the calculation for the electron is as follows:

Step 1 example: Using Eq. (12), the value of y when K=10.

$$y = 2.38925(10)\sum_{n=1}^{(10)} \frac{n^3 - (n-1)^3}{n^4} = 51.10 \tag{13}$$

Step 2 example: The electron's rest energy (5.11×10^5 eV) is divided by K^4. The best fit is when K=10 for the electron. The values from Step 1 and Step 2 are inserted into the column for K=10 in Table 3 and assigned to the electron particle.

$$\frac{5.11\times10^5}{(10)^4} = 51.10 \tag{14}$$

Particle Number (K)	**1**	***8***	**10**	***20***	***28***	***50***	**117**
Particle Name	Neutrino	Muon Neutrino	Electron	Tau Neutrino	Muon Electron	Tau Electron	Higgs Boson
Energy(eV) [a]	2.20	1.70×10^5	5.11×10^5	1.55×10^7	1.06×10^8	1.78×10^9	1.25×10^{11}
y=2.38925×K×∑ [b]	2.38925	39.73	51.10	108.38	154.37	280.99	666.92
En. (eV)/K^4 [c]	2.20	41.50	51.10	96.88	172.45	284.32	668.66

Table 3. Particle rest energy of leptons and the Higgs boson compared to a linear particle energy equation y=2.38925×K×∑. Particles were placed into the column with the closest fit for the value of K (particle number). 2, 8, 20, 28 and 50 are magic numbers from atomic elements.

[a] Values obtained from Particle Data Group [10].
[b] Calculated value using Eq. 12.
[c] Calculated value using Particle Data Group energy [10] divided by particle number to the fourth degree (K^4).

The same process was repeated for all of the particles in the previous section and charted against the final linear equation from Eq. (12).

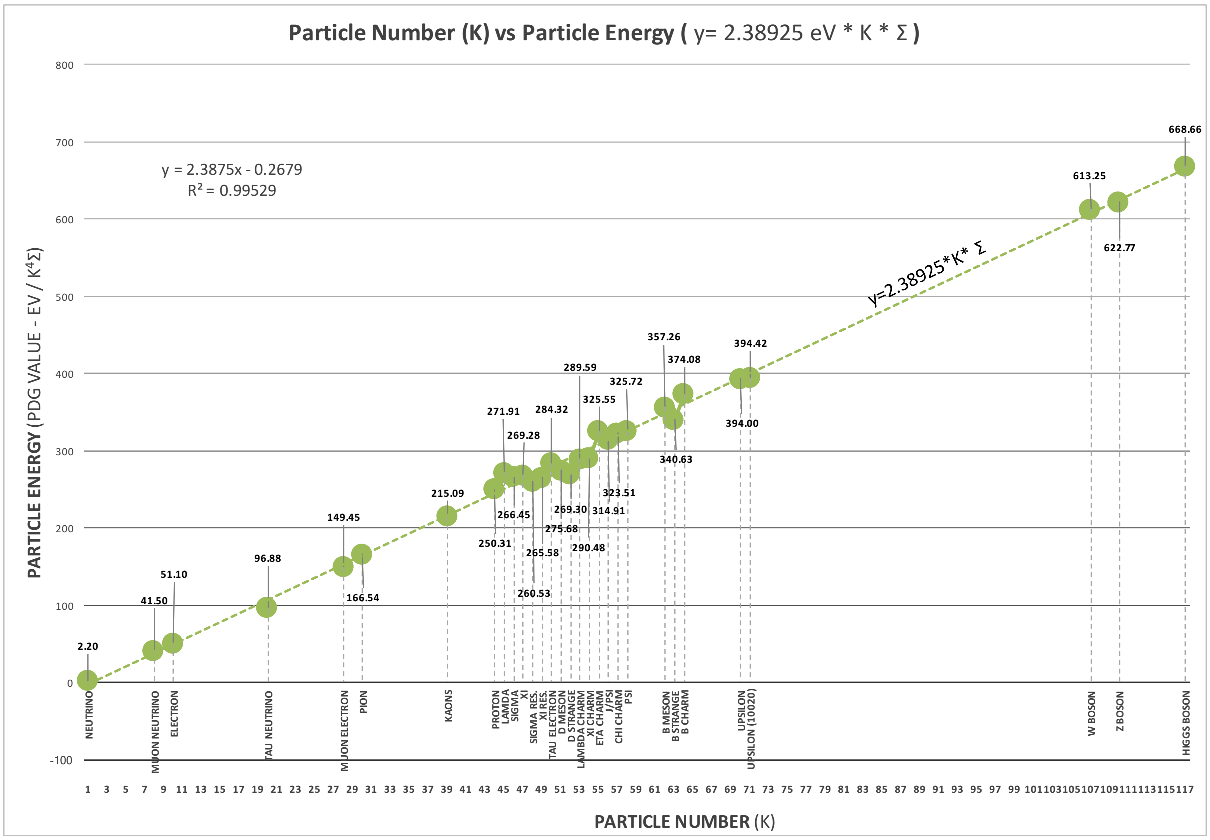

Fig. 4. Relation of particle rest energy to particle number (K) based on linear equation y=2.38925×K×∑. The Y axis is the Particle Data Group (PDG) rest energy value of a particle in electron-volts (eV) divided by a value that is based on the particle number to the fourth power (K^4). The lepton sequence now matches the magic number sequence of atomic elements and the Higgs boson is within the range of the Periodic Table of Elements (K=117).

Observations:

- In Table 3, the italicized particle numbers (K) are the magic numbers from atomic elements: 2, 8, 20, 28 and 50. The leptons are now found at particle numbers that match stable atomic elements with the exception of K=2. This leaves the possibility of finding a neutrino particle at K=2 since this energy value does not match a known particle. The rest energy of this missing particle would be 110 eV.

- In Table 3, the predicted electron rest energy and measured electron rest energy (divided by K^4) are now identical at 51.10.

- The Higgs boson has a best fit at particle number K=117, within the Periodic Table of Elements which has a range from Z=1 to Z=118. However, it is still likely that particles could be found with higher energy levels beyond K=118. When including neutrons, in addition to protons, atomic elements have nucleon counts that exceed 118.

- The slope of the line in Fig. 4 for particles is y=2.39x. The slope of the line for atomic elements in Fig. 1 is y=2.58x.

3. CONCLUSIONS

Using a linear equation to simplify the results of particle energy experiments, we find that there is an equation that can be used to predict new particles and their energy values. It also reveals a slope that may be proven when neutrino experiments narrow the correct value of the neutrino's rest energy, expected to be around 2.38925 eV in the calculations of this paper. The final equation that can be used to predict particle rest energies in electron-volts (eV) is found in Eq. (15).

$$eV = 2.38925K^5 \sum_{n=1}^{K} \frac{n^3 - (n-1)^3}{n^4} \tag{15}$$

The function that relates particle numbers to energy also shows similarities to atomic elements in a few ways: 1) the slopes are nearly the same (y=2.39x for particles; y=2.58x for atomic elements), 2) they share roughly the same number of known particle numbers and atomic numbers (117 for the Higgs boson and 118 for ununoctium) and 3) they share a commonality that particles and atomic elements tend to be more stable, relative to others, at certain numbers (2, 8, 20, 28, 50).

This relation of subatomic particles to atomic elements brings hope that the equivalent of the proton will be discovered for particles, unifying various particles that have been found or continue to be discovered into a simpler definition of their creation. Mathematically, K=1 in the equation is a particle at 2.38925 eV, very close to the estimated energy of the neutrino (2.2 eV). Therefore, it is possible in the context of this equation that the neutrino is the fundamental particle (K), similar to the role of the proton (Z) in the atom. In the decay of some particles, such as neutron beta decay, neutrinos are produced [16]. Hence, it is within the realm of possibility that the neutrino is the fundamental particle creating other particles.

The mapping of subatomic particles to particle numbers yields an extraordinary similarity to atomic elements as it becomes linear, now producing a function that can predict not only the energy values of remaining particles, but also how many particles may be waiting to be discovered. This is the process that Mendeleev used in the creation of the Periodic Table of Elements that eventually simplified our understanding of elements and the atom. Next, we may find that the same structure is true for subatomic particles.

ACKNOWLEDGEMENTS

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