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NC-TODIM Based MAGDM under Neutrosophic Cubic Set Environment

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Abstract: Neutrosophic cubic set is the hybridization of the concept of neutrosophic set and interval neutrosophic set. Neutrosophic cubic set has the capacity to express the hybrid information of both the interval neutrosophic set and the single valued neutrosophic set simultaneously. As newly defined, little research on the operations and applications of neutrosophic cubic sets appear in the current literature. In the present paper we propose the score, accuracy functions for neutrosophic cubic sets and prove their basic properties. We firstly develop TODIM method in neutrosophic cubic set environment, which we call NC-TODIM. We establish a new NC-TODIM method in neutrosophic cubic set environment for solving MAGDM in neutrosophic cubic set environment problems. We illustrate the proposed NC-TODIM method for solving a MAGDM problem to show applicability and effectiveness of the developed method. We also conduct sensitivity analysis to show the impact of ranking order of the alternatives for different values of attenuation factor of losses for multi-attribute group decision making problem.

Keywords: neutrosophic cubic set; single valued neutrosophic set; interval neutrosophic set; multi attribute group decision making; TODIM method; NC-TODIM

1. Introduction

While modelling multi attribute decision making (MADM) and multi attribute group decision making (MAGDM), it is often observed that the parameters of the problem are not precisely known. The parameters often involve uncertainty. To deal uncertainty, Zadeh [1] left an important mark to represent and compute with imperfect information by introducing fuzzy set. Fuzzy set fostered a broad research community, and their impact has also been clearly felt at the application level in MADM [2-4] and MAGDM [5-9].

Atanassov [10] incorporated non membership function as independent component and defined intuitionistic fuzzy set (IFS) at first to express uncertainty in more meaningful way. IFSs have been applied in many MADM problems [11-13]. Smarandache [14] proposed the notion of neutrosophic set (NS) by introducing indeterminacy as independent component. Wang et al. [15] grounded the concept of single valued neutrosophic set (SVNS), an instance of neutrosophic set to deal with incomplete, inconsistent and indeterminate information in realistic way. Wang et al. [16] proposed the interval neutrosophic sets (INS) as a subclass of neutrosophic sets in which the values of truth, indeterminacy and falsity membership degrees are interval numbers. Applications of SVNSs and INSs are found in [17-20] and [21-23] for MADM and MAGDM respectively.

Neutrosophic sets and INS are both capable of handling uncertainty and incomplete information. By fusing neutrosophic set and INS, Ali et al. [24] proposed neutrosophic cubic set and defined external and internal neutrosophic cubic sets and established some of their properties. Jun et al. [25] also defined neutrosophic cubic set by combining neutrosophic set and INS. Neutrosophic cubic set is more capable to express the hybrid information of both the INS and the SVNS simultaneously. However, there are only few studies in the literature to deal with MADM and

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MAGDM in neutrosophic cubic set environment. Banerjee et al. [26] developed grey relational analysis [27-28] based new MADM method in neutrosophic cubic set environment.

Similarity measure is an important mathematical tool in decision-making problems. Pramanik et al. [29] at first defined similarity measure for neutrosophic cubic sets and proved its basic properties. In the same study, Pramanik et al. [29] developed a new MAGDM method in neutrosophic cubic set environment. Lu and Ye [30] proposed cosine measures between neutrosophic cubic sets and proved their basic properties. In the same study, Lu and Ye [30] proposed a new cosine measures-based MADM method under a neutrosophic cubic environment.

Due to little research on operations and application of neutrosophic cubic sets, Pramanik et al. [31] proposed several operational rules on neutrosophic cubic sets and defined Euclidean distance and arithmetic average operator in neutrosophic cubic sets environment. Pramanik et al. [31] also employed information entropy scheme to calculate unknown weights of the attributes and developed a new extended TOPSIS method for MADM under neutrosophic cubic set environment. Zhan et al. [32] developed a new algorithm for multi-criteria decision making (MCDM) in neutrosophic cubic set environment based on weighted average operator and weighted geometric operator. Ye [33] established the concept of a linguistic neutrosophic cubic number (LNCN). In the same study, Ye [33] developed a new MADM method based on LNCN weighted arithmetic averaging (LNCNWAA) operator and a LNCN weighted geometric averaging (LNCNWGA) operator under a linguistic neutrosophic cubic environment.

In the literature there are only five methods [26-33] for MADM and MAGDM in neutrosophic cubic set environment. However, we say that none of them is generally superior to all others. So, new methods for MADM and MAGDM should be explored under neutrosophic cubic set environment.

TODIM (an acronym in Portuguese for Interactive Multi-criteria Decision Making) is an important MADM method, since it considers decision makers' bounded rationality. Firstly, Gomes and Lima [34] introduced TODIM method based on prospect theory [35]. Krohling and Souza [36] defined fuzzy TODIM method to solve MCDM problems. Several researchers applied fuzzy TODIM method in various fuzzy MADM or MAGDM problems [37-39]. Fan et al [40] introduced extended TODIM method to deal with the hybrid MADM problems. Krohling et al. [41] extended TODIM method from fuzzy environment to intuitionistic fuzzy environment by extending TODIM method to process the intuitionistic fuzzy information. Wang [42] introduced TODIM method to neutrosophic environment. Zhang et al. [43] proposed TODIM method for MAGDM problems under neutrosophic environment. Ji et al [44] proposed TODIM method under multi valued neutrosophic environment and applied it to personal selection. In 2017, Xu et al. [45] develop TODIM in single valued neutrosophic setting. In neutrosophic cubic set environment TODIM is yet to appear. To fill the gap, we initiate the study of TODIM in neutrosophic cubic set environment which we call as NC-TODIM.

In this paper we develop a TODIM method (for short, NC-TODIM method) for MAGDM in neutrosophic cubic set environment. We solve an illustrative numerical example of MAGDM problem in neutrosophic cubic set environment to show the applicability and effectiveness of the proposed NC-TODIM method.

Remainder of the paper is divided into five sections that are organized as follows: Section 2 presents some basic definition of neutrosophic sets, interval-valued neutrosophic sets, neutrosophic cubic sets. Section 3 is devoted to present the proposed NC-TODIM method. Section 4 presents an illustrative numerical example. Section 5 is devoted to analyse the ranking order with

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different values of attenuation factor of losses. Finally, Section 6 presents conclusion and future scope of research.

2. Preliminaries

In this section, we review some basic definitions which are important to develop the paper.

Definition 1. [14] Neutrosophic set (NS)

Let U be a space of points (objects) with a generic element in U denoted by u i.e. $u \in U$. A neutrosophic set R in U is characterized by truth-membership function t_R, indeterminacy-membership function i_R and falsity-membership function f_R, where t_R, i_R, f_R are the functions from U to $]^-0, 1^+[$ i.e. $t_R, i_R, f_R : U \rightarrow]^-0, 1^+[$ that means $t_R(u), i_R(u), f_R(u)$ are the real standard or non-standard subset of $]^-0, 1^+[$. Neutrosophic set can be expressed as $R = \{<u; (t_R(u), i_R(u), f_R(u))>: \forall u \in U\}$. Since $t_R(u), i_R(u), f_R(u)$ are the subset of $]^-0, 1^+[$, there the sum $(t_R(u) + i_R(u) + f_R(u))$ lies between $^-0$ and 3^+, where $^-0 = 0 - \varepsilon$ and $3^+ = 3 + \varepsilon$, $\varepsilon > 0$.

Example 1. Suppose that $U = \{u_1, u_2, u_3, ...\}$ be the universal set. Let R_1 be any neutrosophic set in U. Then R_1 expressed as $R_1 = \{<u_1; (.6, .3, .4)>: u_1 \in U\}$.

Definition 2. [16] Interval neutrosophic set (INS)

Let G be a non-empty set. An interval neutrosophic set $\tilde{G}$ in G is characterized by truth-membership function $t_{\tilde{G}}$, the indeterminacy membership function $i_{\tilde{G}}$ and falsity membership function $f_{\tilde{G}}$. For each $g \in G$, $t_{\tilde{G}}(g)$, $i_{\tilde{G}}(g)$, $f_{\tilde{G}}(g) \subseteq [0, 1]$ and $\tilde{G}$ defined as
$\tilde{G} = \{<g; [t^-_{\tilde{G}}(g), t^+_{\tilde{G}}(g)], [i^-_{\tilde{G}}(g), i^+_{\tilde{G}}(g)], [f^-_{\tilde{G}}(g), f^+_{\tilde{G}}(g)]: \forall g \in G\}$. Here, $t^-_{\tilde{G}}(g)$, $t^+_{\tilde{G}}(g)$, $i^-_{\tilde{G}}(g)$, $i^+_{\tilde{G}}(g)$, $f^-_{\tilde{G}}(g)$, $f^+_{\tilde{G}}(g) : G \rightarrow]^-0, 1^+[$ and
$^-0 \le \sup t^+_{\tilde{G}}(g) + \sup i^+_{\tilde{G}}(g) + \sup f^+_{\tilde{G}}(g) \le 3^+$,
In real problems it is difficult to express the truth-memberships function, indeterminacy-membership function and falsity-membership function in the form of $t^-_{\tilde{G}}(g)$, $t^+_{\tilde{G}}(g)$, $i^-_{\tilde{G}}(g)$, $i^+_{\tilde{G}}(g)$, $f^-_{\tilde{G}}(g)$, $f^+_{\tilde{G}}(g) : G \rightarrow]^-0, 1^+[$.
Here, $t^-_{\tilde{G}}(g)$, $t^+_{\tilde{G}}(g)$, $i^-_{\tilde{G}}(g)$, $i^+_{\tilde{G}}(g)$, $f^-_{\tilde{G}}(g)$, $f^+_{\tilde{G}}(g): G \rightarrow [0, 1]$.

Example 2.

Suppose that $G = \{g_1, g_2, g_3, ..., g_n\}$ be a non-empty set. Let $\tilde{G}_1$ be any interval neutrosophic set. Then $\tilde{G}_1$ expressed as $\tilde{G}_1 = \{<g_1; [.39, .47], [.17, .43], [.18, .36]: g_1 \in G\}$.

Definition 3. [24] Neutrosophic cubic set (NCS)

A neutrosophic cubic set in a non-empty set G is defined as © = $\{<g; \tilde{G}(g), R(g)>: \forall g \in G\}$, where $\tilde{G}$ and R are the interval neutrosophic set and neutrosophic set in G respectively. Neutrosophic cubic set can be presented as an order pair © $= <\tilde{G}, R>$, then we call it as neutrosophic cubic number (NC-number).

Example 3.

Suppose that $G = \{g_1, g_2, g_3, ..., g_n\}$ be a non-empty set. Let $©_1$ be any NC-number. Then $©_1$ can be express as $©_1 = \{<g_1; [.39, .47], [.17, .43], [.18, .36], (.6, .3, .4)>: g_1 \in G\}$

Some operations of NC-numbers:

i. Union of any two NC-numbers

Let $©_1 = <\tilde{G}_1, R_1>$ and $©_2 = <\tilde{G}_2, R_2>$ be any two NC-numbers in a non-empty set G. Then the union of $©_1$ and $©_2$ denoted by $©_1 \cup ©_2$ and defined as
$©_1 \cup ©_2 = <\tilde{G}_1(g) \cup \tilde{G}_1(g), R_1(g) \cup R_2(g) \forall g \in G>$, where
$\tilde{G}_1(g) \cup \tilde{G}_1(g) = \{<g, [\max\{t^-_{\tilde{G}_1}(g), t^-_{\tilde{G}_2}(g)\}, \max\{t^+_{\tilde{G}_1}(g), t^+_{\tilde{G}_2}(g)\}], [\max\{i^-_{\tilde{G}_1}(g), i^-_{\tilde{G}_2}(g)\}, \max\{i^+_{\tilde{G}_1}(g), i^+_{\tilde{G}_2}(g)\}], [\min\{f^-_{\tilde{G}_1}(g), f^-_{\tilde{G}_2}(g)\}, \min\{f^+_{\tilde{G}_1}(g), f^+_{\tilde{G}_2}(g)\}]>: g \in G\}$ and $R_1(g) \cup R_2(g) = \{<g, \max\{t_{R_1}(g), t_{R_2}(g)\}, \max\{i_{R_1}(g), i_{R_2}(g)\}, \min\{f_{R_1}(g), f_{R_2}(g)\}>: \forall g \in U\}$.

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Example 4.

Let $©_1$ and $©_2$ be two NC-numbers in G presented as follows:

$©_1$ = < [.39, .47], [.17, .43], [.18, .36], (.6, .3, .4)> and $©_2$ = < [.56, .70], [.27, .42], [.15, .26], (.7, .3, .6)>.

Then $©_1 \cup ©_2$ = < [.56, .7], [.27, .43], [.15, .26], (.7, .3, .4)>.

ii. Intersection of any two NC-numbers

Intersection of two NC-numbers denoted and defined as follows:

$©_1 \cap ©_2 = <\widetilde{G}_1(g) \cap \widetilde{G}_1(g), R_1(g) \cap R_2(g) \forall g \in G>$, where $\widetilde{G}_1(g) \cap \widetilde{G}_1(g)$ = {<g, [min { $t^-_{\widetilde{G}_1}(g)$, $t^-_{\widetilde{G}_2}(g)$},min { $t^+_{\widetilde{G}1}(g)$, $t^+_{\widetilde{G}_2}(g)$}], [min { $i^-_{\widetilde{G}_1}(g)$, $i^-_{\widetilde{G}_2}(g)$}, min { $i^+_{\widetilde{G}_1}(g)$, $i^+_{\widetilde{G}_2}(g)$}], [max { $f^-_{\widetilde{G}_1}(g)$, $f^-_{\widetilde{G}_2}(g)$}, max { $f^+_{\widetilde{G}_1}(g)$, $f^+_{\widetilde{G}_2}(g)$}]>: g∈ G} and $R_1(g) \cap R_2(g)$ = {<g, min { $t_{R_1}(g)$, $t_{R_2}(g)$}, min { $i_{R_1}(g)$, $i_{R_2}(g)$}, max { $f_{R_1}(g)$, $f_{R_2}(g)$}>: ∀ g∈ U}.

Example 5.

Let $©_1$ and $©_2$ be any two NC-numbers in G presented as follows:

$©_1$ = < [.45, .57], [.27, .33], [.18, .46], (.7, .3, .5)> and $©_2$ = < [.67, .75], [.22, .44], [.17, .21], (.8, .4, .4)>.

Then $©_1 \cap ©_2$ = < [.45, .57], [.22, .33], [.18, .46], (.7, .3, .4)>.

iii. Compliment of a NC-number

Let $©_1 = <\widetilde{G}_1, R_1>$ be any neutrosophic cubic set in G. Then compliment of $©_1 = <\widetilde{G}_1, R_1>$ denoted by $©^c_1$ = {<g, $\widetilde{G}^c_1(g)$, $R^c_1(g)$>: ∀ g∈ G}.

Here, $\widetilde{G}_1^{\,c}$ = {<g, [$t^-_{\widetilde{G}_1^c}(g)$, $t^+_{\widetilde{G}_1^c}(g)$], [$i^-_{\widetilde{G}_1^c}(g)$, $i^+_{\widetilde{G}_1^c}(g)$], [$f^-_{\widetilde{G}_1^c}(g)$, $f^+_{\widetilde{G}_1^c}(g)$]>: ∀ g∈ G}, where, $t^-_{\widetilde{G}_1^c}(g) = f^-_{\widetilde{G}_1}(g)$, $t^+_{\widetilde{G}_1^c}(g) = f^+_{\widetilde{G}_1}(g)$, $i^-_{\widetilde{G}_1^c}(g) = \{1\} - i^-_{\widetilde{G}_1}(g)$, $i^+_{\widetilde{G}_1^c}(g) = \{1\} - i^+_{\widetilde{G}_1}(g)$, $f^-_{\widetilde{G}_1^c}(g) = t^-_{\widetilde{G}_1}(g)$, $f^+_{\widetilde{G}_1^c}(g) = f^+_{\widetilde{G}_1}(g)$ and $t_{R^c_1}(g) = f_{R_1}(g)$, $i_{R?1^c}(g) = \{1^+\} - i_{R_1}(g)$, $f_{R^c_1}(g) = t_{R_1}(g)$.

Example 6.

Assume that $©_1$ be any NC-number in G in the form:

$©_1$ = < [.45, .57], [.27, .33], [.18, .46], (.7, .3, .5)>. Then compliment of $©_1$ is obtained as $©^c_1$ = < [.18, .46], [.73, .67], [.45, .57], (.5, .7, .7) >.

Definition 4. Score function

Let $©_1$ be a NC-number in a non-empty set G. Then, a score function of $©_1$, $Sc(©_1)$ is defined as:

$$Sc(©_1) = \frac{1}{2}\left[\left(\frac{2+a_1+a_2-2b_1-2b_2-c_1-c_2}{4}\right)+\left(\frac{1+a-2b-c}{2}\right)\right] \qquad (2.1)$$

where, $©_1$ = < [a_1, a_2], [b_1, b_2], [c_1, c_2], (a, b, c) > and $Sc(©_1) \in [-1, 1]$.

Proposition 1. Score function of two NC-numbers lies between -1 to 1.

Proof.

Using the definition of interval neutrosophic set and neutrosophic set, we have all a_1, a_2, b_1, b_2, c_1, c_2, a, b, and c $\in [0,1]$.

Since, $0 \le a_1 \le 1$, $0 \le a_2 \le 1$

$$\Rightarrow 0 \le a_1 + a_2 \le 2,$$

$$\Rightarrow 2 \le 2 + a_1 + a_2 \le 4 \qquad (2.2)$$

$0 \le b_1 \le 1 \Rightarrow 0 \le 2b_1 \le 2$, and $0 \le b_2 \le 1 \Rightarrow 0 \le 2b_2 \le 2$

$$\Rightarrow -2 \le -2b_1 \le 0$$

$$\Rightarrow -2 \le -2b_2 \le 0$$

$$\Rightarrow -4 \le -2b_1 - 2b_2 \le 0 \qquad (2.3)$$

$$0 \le c_1 \le 1 \quad \Rightarrow -1 \le -c_1 \le 0$$

$$0 \le c_2 \le 1 \qquad \Rightarrow -1 \le -c_2 \le 0$$

$$\Rightarrow -2 \le -c_1 - c_2 \le 0 \qquad (2.4)$$

Adding (2.2), (2.3) and (2.4), we obtain

$$\Rightarrow -4 \le 2 + a_1 + a_2 - 2b_1 - 2b_2 - c_1 - c_2 \le 4,$$

$$\Rightarrow -1 \le \frac{2 + a_1 + a_2 - 2b_1 - 2b_2 - c_1 - c_2}{4} \le 1 \qquad (2.5)$$

Again,

$$0 \le a \le 1 \Rightarrow 1 \le 1 + a \le 2, \qquad (2.6)$$

$$0 \le b \le 1 \Rightarrow 0 \le 2b \le 2,$$

$$0 \le c \le 1,$$

$$\Rightarrow 0 \le 2b + c \le 3,$$

$$\Rightarrow -3 \le -2b - c \le 0 \qquad (2.7)$$

Adding (2.6) and (2.7), we obtain

$$-2 \le 1 + a - 2b - c \le 2,$$

$$\Rightarrow -1 \le \frac{1 + a - 2b - c}{2} \le 1 \qquad (2.8)$$

Adding (2.5) and (2.8) and dividing by 2, we obtain

$$-1 \le \frac{1}{2}\left[\left(\frac{2 + a_1 + a_2 - 2b_1 - 2b_2 - c_1 - c_2}{4}\right) + \left(\frac{1 + a - 2b - c}{2}\right)\right] \le 1$$

$Sc(©_1) \in [-1, 1]$,

Hence complete the proof.

Example 7.

Let $©_1$ and $©_2$ be two NC-numbers in G presented as follows:

$©_1$ = < [.39, .47], [.17, .43], [.18, .36], (.6, .3, .4)> and $©_2$ = < [.56, .70], [.27, .42], [.15, .26], (.7, .3, .6)>.

Then, by applying Definition 4, we obtain $Sc(©_1)$ = -.01 and $Sc(©_2)$ = .07, In this case, we can say that $©_2 > ©_1$.

Definition 5. Accuracy function

Let $©_1$ be a NC-number in a non-empty set G, an accuracy function of $©_1$ is defined as:

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$$Ac(©_1)=\frac{1}{2}[\frac{1}{2}(a_1+a_2-b_2(1-a_2)-b_1(1-a_1)-c_2(1-b_1)-c_1(1-b_2))+a-b(1-a)-c(1-b)] \qquad (2.9)$$

Here, $Ac(©_1) \in [-1, 1]$.

When the value of $Ac(©_1)$ increases, we say that the degree of accuracy of the NC-number $©_1$ increases.

Proposition 2. Accuracy function of two NC-numbers lies between -1 to 1.

Proof.

The values of accuracy function depend upon $\{\frac{1}{2}(a_1+a_2-b_2(1-a_2)-b_1(1-a_1)-c_2(1-b_1)-c_1(1-b_2))$ and $\{a-b(1-a)-c(1-b)\}$ The values of

$\{\frac{1}{2}(a_1+a_2-b_2(1-a_2)-b_1(1-a_1)-c_2(1-b_1)-c_1(1-b_2))\}$ and $\{a-b(1-a)-c(1-b)\}$ lies between -1 to 1 from [18].

Thus, $-1 \le Ac(©_1) \le 1$.

Hence complete the proof.

Example 8.

Let $©_1$ and $©_2$ be two NC-numbers in G presented as follows: $©_1 = <[.41,.52],[.10,.18],[.06,.17],(.48,.11,.11)>$ and $©_2 = <[.40,.51],[.10,.20],[.10,.19],(.50,.11,.11)$. Then, by applying Definition 5, we obtain $Ac(©_1) = .14$ and $Ac(©_2) = .30$. In this case, we can say that alternative $©_2$ is better than $©_1$.

With respect to the score function Sc and the accuracy function Ac, a method for comparing NC-numbers can be defined as follows:

Comparison procedure of two NC-numbers

Let $©_1$ and $©_2$ be any two NC-numbers. Then we define comparison method as follows:

i. If $Sc(©_1) > Sc(©_2)$, then $©_1 > ©_2$. (2.10)

ii. If $Sc(©_1) = Sc(©_2)$ and $Ac(©_1) > Ac(©_2)$, then $©_1 > ©_2$. (2.11)

iii. If $Sc(©_1) = Sc(©_2)$ and $Ac(©_1) = Ac(©_2)$, then $©_1 = ©_2$. (2.12)

Example 9.

Let $©_1$ and $©_2$ be two NC-numbers in G presented as follows:

$©_1 = <[.23,.29],[.37,.46],[.34,.42],(.26,.26,.26)>$ and

$©_2 = <[.25,.31],[.35,.44],[.35,.44],(.28,.28,.28)>$. Then, applying Definition 4, we obtain $Sc(©_1) = .13$ and $Sc(©_2) = .13$. Applying Definition 5, we obtain $Ac(©_1) = -.20$ and $Ac(©_2) = -.18$. In this case, we say that alternative $©_2 > ©_1$. (Score values and Accuracy values taking correct up to two decimal places)

Definition 6.

Let $©_1$ and $©_2$ be any two NC-numbers, then distance between them is defined by

$$\partial(©_1, ©_2) = \frac{1}{9}[|a_1-d_1|+|a_2-d_2|+|b_1-e_1|+|b_2-e_2|+|c_1-f_1|+|c_2-f_2|+|a-d|+|b-e|+|c-f|] \qquad (2.13)$$

where, $©_1 = <[a_1, a_2], [b_1, b_2], [c_1, c_2], (a, b, c)>$ and $©_2 = <[d_1, d_2], [e_1, e_2], [f_1, f_2], (d, e, f)>$.

Example 10.

Let $©_1$ and $©_2$ be two NC-numbers in G presented as follows:

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$©_1$ = < [.66, .75], [.25, .32], [.17, .34], (.53, .17, .22)> and $©_2$ = < [.35, .55], [.12, .25], [.12, .20], (.60, .23, .43)>. Then, applying Definition 6, we obtain ∂ ($©_1$, $©_2$) = .12.

Definition 7.

Let $©_{ij} = \{<[t_{ij}^{-}, t_{ij}^{+}], [i_{ij}^{-}, i_{ij}^{+}], [f_{ij}^{-}, f_{ij}^{+}], (t, i, f)>\}$ be any neutrosophic cubic value. $©_{ij}$ used to evaluate i-th alternative with respect to j-th criterion. The normalized form of $©_{ij}$ is defined as follows:

$$©_{ij}^{\otimes} = \{<[\frac{t_{ij}^{-}}{(\sum_{i=1}^{m}(t_{ij}^{-})^2 + (t_{ij}^{+})^2)^{\frac{1}{2}}}, \frac{t_{ij}^{+}}{(\sum_{i=1}^{m}(t_{ij}^{-})^2 + (t_{ij}^{+})^2)^{\frac{1}{2}}}],\ [\frac{i_{ij}^{-}}{(\sum_{i=1}^{m}(i_{ij}^{-})^2 + (i_{ij}^{+})^2)^{\frac{1}{2}}}, \frac{i_{ij}^{+}}{(\sum_{i=1}^{m}(i_{ij}^{-})^2 + (i_{ij}^{+})^2)^{\frac{1}{2}}}],$$

$$[\frac{f_{ij}^{-}}{(\sum_{i=1}^{m}(f_{ij}^{-})^2 + (f_{ij}^{+})^2)^{\frac{1}{2}}}, \frac{f_{ij}^{+}}{(\sum_{i=1}^{m}(f_{ij}^{-})^2 + (f_{ij}^{+})^2)^{\frac{1}{2}}}]$$

$$[\frac{t_{ij}}{(\sum_{i=1}^{m}(t_{ij})^2 + (i_{ij})^2 + (f_{ij})^2)^{\frac{1}{2}}}, \frac{i_{ij}}{(\sum_{i=1}^{m}(t_{ij})^2 + (i_{ij})^2 + (f_{ij})^2)^{\frac{1}{2}}}, \frac{f_{ij}}{(\sum_{i=1}^{m}(t_{ij})^2 + (i_{ij})^2 + (f_{ij})^2)^{\frac{1}{2}}}]>\}. \quad (2.14)$$

2.1. A conceptual model of evolution of neutrosophic cubic set is shown in Figure 1.

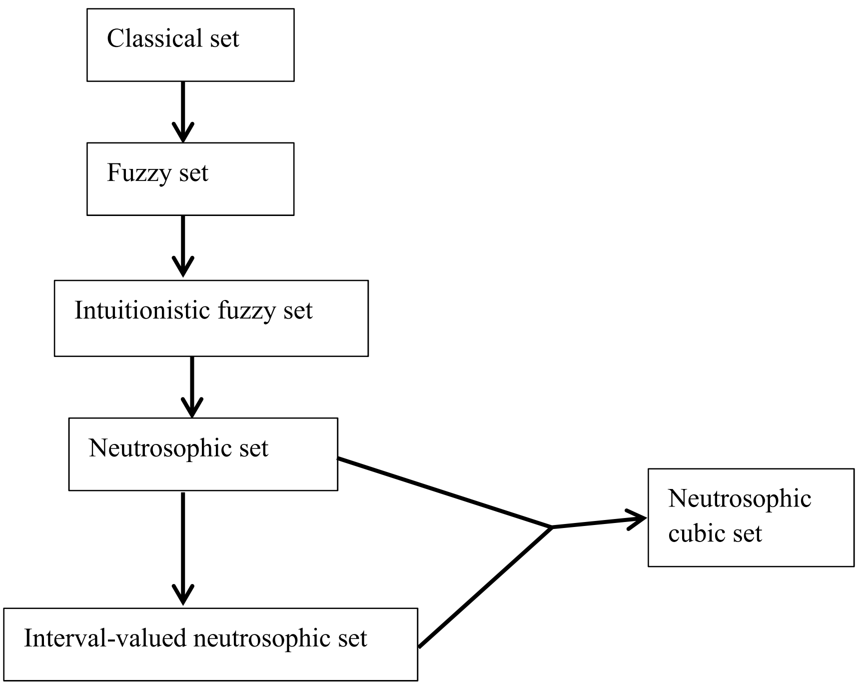

Figure.1. Evolution of neutrosophic cubic set

3. NC-TODIM method for solving MAGDM problem under neutrosophic cubic set environment

Classical TODIM is not enough to deal neutrosophic MAGDM problems due to presence of indeterminacy and complexity of decision environment. However, NC-numbers can express the indeterminate information. In this study we extend the TODIM method to NC-TODIM to solve the MAGDM problems under neutrosophic cubic set environment.

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3.1. Description about MAGDM problems

Assume that $A = \{A_1, A_2, \ldots, A_m\}$ $(m \geq 2)$, $C = \{C_1, C_2, \ldots, C_n\}$ $(n \geq 2)$ be the discrete set of alternatives and attributes respectively. $W = \{W_1, W_2, \ldots, W_n\}$ is the weight vector of attribute C_j $(j = 1, 2, \ldots, n)$, where $W_j > 0$ and $\sum_{j=1}^{n} W_j = 1$. Let $E = \{E_1, E_2, \ldots, E_r\}$ be the set of decision makers and $\gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_r\}$ be the weight vector of decision makers, where $\gamma_k > 0$ and $\sum_{k=1}^{r} \gamma_k = 1$

3.2. NC-TODIM method

Now, we describe the procedure of NC-TODIM method to solve the MAGDM problems with NC-numbers. The method consists of following steps:

Step1. Formulate the decision matrix

Assume that $M^k = \left(©_{ij}^k\right)_{m \times n}$ be the decision matrix, where $©_{ij}^k = <\tilde{G}_{ij}^k, R_{ij}^k>$ is the rating value provided by the decision maker E_k for alternative A_i with respect to attribute C_j. The matrix form of M^k is presented below

$$M^k = \begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ . \\ A_m \end{array} & \begin{pmatrix} ©_{11}^k & ©_{12}^k & \ldots & ©_{1n}^k \\ ©_{21}^k & ©_{22}^k & & ©_{2n}^k \\ . & & \ldots & . \\ ©_{m1}^k & ©_{m2}^k & \ldots & ©_{mnj}^k \end{pmatrix} \end{array} \qquad (3.1)$$

Step 2. Normalize the decision matrix

MAGDM problem generally consists of cost criteria and benefit criteria. So, the decision matrix needs to be normalized. For cost criterion C_j we use the Equation (7) to normalize the decision matrix (Equation (3.1)) provided by the decision makers. For benefit criterion C_j we don't need to normalize the decision matrix. When C_j is a cost criterion, the normalized form of decision matrix (see Equation (3.1)) is presented below.

$$M^{\otimes k} = \begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ . \\ A_m \end{array} & \begin{pmatrix} ©_{11}^{\otimes k} & ©_{12}^{\otimes k} & \ldots & ©_{1n}^{\otimes k} \\ ©_{21}^{\otimes k} & ©_{22}^{\otimes k} & & ©_{2n}^{\otimes k} \\ . & & \ldots & . \\ ©_{m1}^{\otimes k} & ©_{m2}^{\otimes k} & \ldots & ©_{mnj}^{\otimes k} \end{pmatrix} \end{array} \qquad (3.2)$$

Here $©_{ij}^{\otimes k}$ is the normalized form of NC-number.

Step 3. Determine the relative weight of each criterion

Relative weight W_{ch} of each criterion is obtained by the following equation.

$$W_{ch} = \frac{W_C}{W_h} \qquad (3.3)$$

where, $W_h = \max\{W_1, W_2, \ldots, W_n\}$.

Step 4. Calculate score values

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Using Equation (2.1), calculate score value $Sc(©_{ij}^{\otimes k})$ (i = 1, 2, …, m; j = 1, 2, …, n) of $©_{ij}^{\otimes k}$ if C_j is a cost criterion. Using Equation (2.1), calculate score value $Sc(?_{ij}^{k})$ (i = 1, 2, …, m; j = 1, 2, …, n) of $©_{ij}^{k}$ if C_j is a benefit criterion.

Step 5: Calculate accuracy values

Using Equation (2.9), calculate accuracy value $Ac(©_{ij}^{\otimes k})$ (i= 1, 2, …, m; j= 1, 2, …, n) of $©_{ij}^{\otimes k}$ if C_j is a cost criterion. Using Equation (2.9), calculate accuracy value $Ac(©_{ij}^{k})$ (i= 1, 2, …, m; j= 1, 2, …, n) of $©_{ij}^{k}$ if C_j is a benefit criterion.

Step 6. Formulate the dominance matrix

Calculate the dominance of each alternative A_i over each alternative A_j with respect to the criteria C ($C_1, C_2, \ldots, C_n$), of the k-th decision maker E_k by the following Equation (3.4) and Equation (3.5).

(For cost criteria)

$$\Psi_c^k(A_i, A_j) = \begin{cases} \sqrt{\left(\dfrac{W_{Ch}}{\sum_{c=1}^{n} W_{ch}}\partial(©_{ic}^{\otimes k}, ©_{jc}^{\otimes k}\right)}, & \text{if } ©_{ic}^{\otimes k} > ©_{jc}^{\otimes k} \\ = 0 & , \text{if } ©_{ic}^{\otimes k} = ©_{jc}^{\otimes k} \\ = -\dfrac{1}{\alpha}\sqrt{\left(\dfrac{\sum_{c=1}^{n} W_{ch}}{W_{Ch}}\partial(©_{ic}^{\otimes k}, ©_{jc}^{\otimes k}\right)}, & \text{if } ©_{ic}^{\otimes k} < ©_{jc}^{\otimes k} \end{cases} \quad (3.4)$$

(For benefit criteria)

$$\Psi_c^k(A_i, A_j) = \begin{cases} \sqrt{\left(\dfrac{W_{Ch}}{\sum_{c=1}^{n} W_{ch}}\partial(©_{ic}^{k}, ©_{jc}^{k}\right)}, & \text{if } ©_{ic}^{k} > ©_{jc}^{k} \\ = 0 & , \text{if } ©_{ic}^{k} = ©_{jc}^{k} \\ = -\dfrac{1}{\alpha}\sqrt{\left(\dfrac{\sum_{c=1}^{n} W_{ch}}{W_{Ch}}\partial(©_{ic}^{k},, ©_{jc}^{k}\right)}, & \text{if } ©_{ic}^{k} < ©_{jc}^{k} \end{cases} \quad (3.5)$$

Where, parameter 'α' represents the attenuation factor of losses and α must be positive.

Step 7. Formulate the individual total dominance matrix

Using Equation (3.6), calculate the individual total dominance matrix of each alternative A_i over each alternative A_j.

$$\lambda^k = (A_i, A_j) = \sum_{c=1}^{n} \Psi_c^k(A_i, A_j) \quad (3.6)$$

Step 8. Aggregate the dominance matrix

Using Equation (3.7), calculate the collective overall dominance of alternative A_i over each alternative A_j.

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$$\lambda(A_i, A_j) = \sum_{k=1}^{m} \gamma_k \lambda^k (A_i, A_j) \qquad (3.7)$$

Step 9. Calculate global values

Using the Equation (3.8), we calculate global value of each alternative

$$\Omega_i = \frac{\sum_{j=1}^{n} \lambda(A_i, A_j) - \min_{1 \le i \le m} \left(\sum_{j=1}^{n} \lambda(A_i, A_j)\right)}{\max_{1 \le i \le m} \left(\sum_{j=1}^{n} \lambda(A_i, A_j)\right) - \min_{1 \le i \le m} \left(\sum_{j=1}^{n} \lambda(A_i, A_j)\right)} \qquad (3.8)$$

Step 10. Rank the priority

Sorting the values of Ω_i provides the rank of each alternative. A set of alternatives can be preference ranked according to the descending order of Ω_i. Highest global value corresponds to the best alternative.

3.3. A conceptual model of the proposed approach is shown in Figure 2.

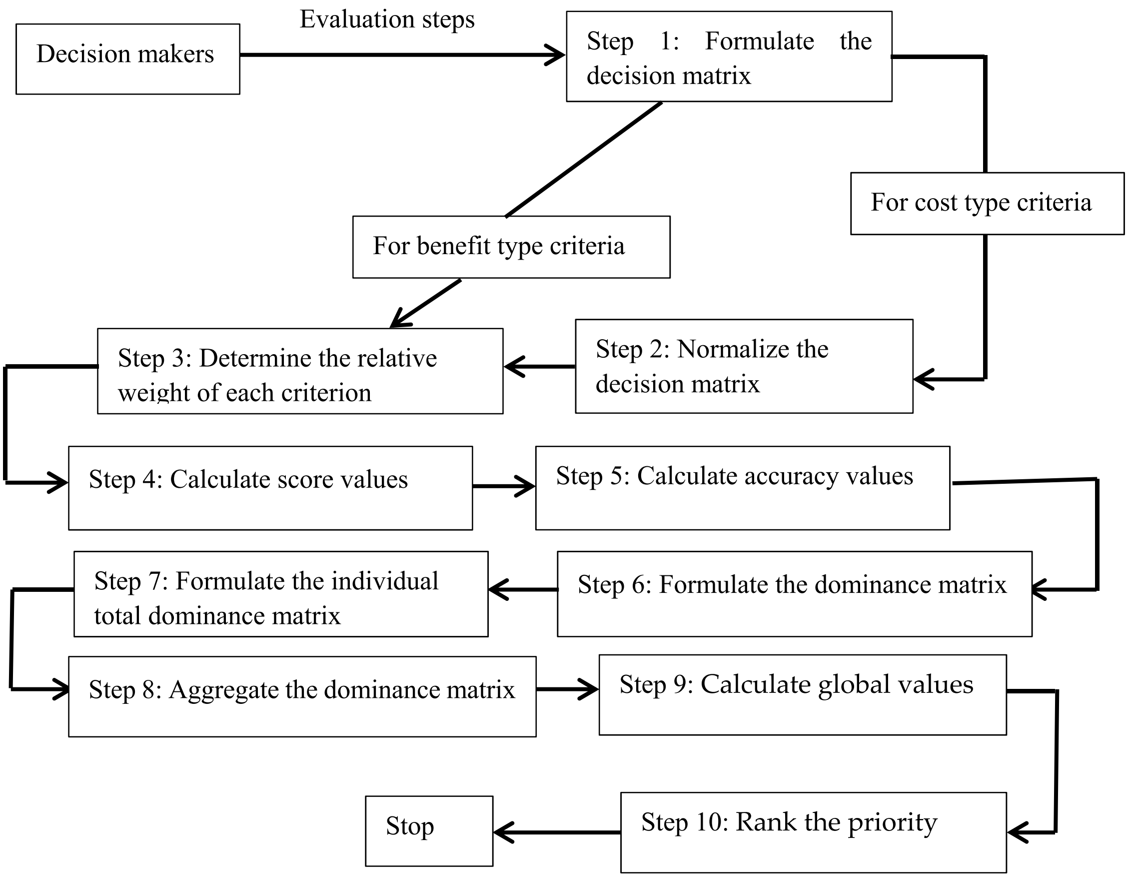

Figure 2: A flow chart of the proposed method.

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4. Illustrative example

In this section, a MAGDM problem is adapted from the study [18] under neutrosophic cubic set environment. An investment company wants to select a best alternative among the set of feasible alternatives. The feasible alternatives are

1. Car company (A_1)
2. Food company (A_2)
3. Computer company (A_3)
4. Arms company (A_4)

The best alternative is selected based on the following criteria:

1. Risk analysis (C_1)
2. Growth analysis (C_2)
3. Environmental impact analysis (C_3)

An investment company forms a panel of three decision makers {E_1, E_2, E_3} who evaluate four alternatives in decision making process. The weight vector of attributes and decision makers are considered as $W = (.4, .35, .25)^T$ $\gamma = (.32, .33, .35)^T$ respectively.

The proposed method is presented using the following steps:

Step 1. Formulate the decision matrix

Formulate the decision matrices $M^k (k = 1, 2, 3)$ using the rating values of alternatives with respect to three criteria provided by the three decision makers in terms of neutrosophic cubic numbers. Assume that the NC-numbers $©^k_{ij} = \langle \tilde{G}^k_{ij}, R^k_{ij} \rangle$ presents rating value provided by the decision maker E_k for alternative A_i with respect to attribute C_j. Using these rating values $©^k_{ij}$ (k = 1, 2, 3; i = 1, 2, 3, 4; ; j = 1, 2, 3) , three decision matrices $M^k = (©^k_{ij})_{4\times3}$ (k = 1, 2, 3) are constructed (see Equations (4.1), (4.2) and (4.3)).

Decision matrix for E_1

$$M^1 = \begin{array}{cc} & \begin{array}{ccc} \quad C_1 \quad & \quad C_2 \quad & \quad C_3 \quad \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} \langle[.41,.52],[.10,.18],[.06,.17],(.48,.11,.11)\rangle & \langle[.40,.51],[.10,.20],[.10,.19],(.50,.11,.11)\rangle & \langle[.22,.27],[.41,.52],[.41,.52],(.31,.31,.31)\rangle \\ \langle[.35,.46],[.18,.27],[.17,.34],(.43,.16,.21)\rangle & \langle[.22,.28],[.40,.50],[.39,.48],(.28,.28,.28)\rangle & \langle[.38,.49],[.10,.21],[.10,.21],(.57,.12,.12)\rangle \\ \langle[.23,.29],[.36,.45],[.34,.42],(.26,.26,.26)\rangle & \langle[.34,.45],[.20,.30],[.19,.39],(.44,.16,.22)\rangle & \langle[.22,.27],[.41,.52],[.41,.52],(.31,.31,.31)\rangle \\ \langle[.17,.23],[.45,.55],[.42,.59],(.21,.32,.37)\rangle & \langle[.22,.28],[.40,.50],[.39,.48],(.28,.28,.28)\rangle & \langle[.38,.49],[.10,.21],[.10,.21],(.57,.12,.12)\rangle \end{pmatrix} \end{array} \quad (4.1)$$

Decision matrix for E_2

$$M^2 = \begin{array}{cc} & \begin{array}{ccc} \quad C_1 \quad & \quad C_2 \quad & \quad C_3 \quad \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} \langle[.17,.23],[.46,.55],[.42,.59],(.21,.32,.37)\rangle & \langle[.25,.31],[.35,.44],[.35,.44],(.28,.28,.28)\rangle & \langle[.34,.43],[.13,.27],[.13,.27],(.49,.11,.11)\rangle \\ \langle[.23,.29],[.37,.46],[.34,.42],(.26,.26,.26)\rangle & \langle[.25,.31],[.35,.44],[.35,.44],(.28,.28,.28)\rangle & \langle[.34,.43],[.13,.27],[.13,.27],(.49,.11,.11)\rangle \\ \langle[.41,.52],[.10,.18],[.10,.17],(.48,.11,.11)\rangle & \langle[.44,.57],[.10,.17],[.10,.17],(.51,.11,.11)\rangle & \langle[.19,.24],[.53,.67],[.53,.67],(.27,.27,.27)\rangle \\ \langle[.35,.46],[.20,.28],[.17,.34],(.42,.16,.21)\rangle & \langle[.25,.31],[.35,.44],[.35,.44],(.28,.28,.28)\rangle & \langle[.34,.43],[.13,.27],[.13,.27],(.49,.11,.11)\rangle \end{pmatrix} \end{array} \quad (4.2)$$

Decision matrix for E_3

$$M^3 = \begin{array}{cc} & \begin{array}{ccc} \quad C_1 \quad & \quad C_2 \quad & \quad C_3 \quad \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} \langle[.22,.27],[.42,.52],[.42,.52],(.28,.28,.28)\rangle & \langle[.22,.28],[.40,.50],[.39,.48],(.28,.28,.28)\rangle & \langle[.41,.52],[.10,.18],[.10,.17],(.48,.11,.11)\rangle \\ \langle[.22,.27],[.42,.52],[.42,.52],(.28,.28,.28)\rangle & \langle[.40,.51],[.10,.20],[.10,.19],(.50,.11,.11)\rangle & \langle[.23,.29],[.36,.45],[.34,.42],(.26,.26,.26)\rangle \\ \langle[.38,.49],[.10,.21],[.10,.21],(.50,.11,.11)\rangle & \langle[.34,.45],[.20,.30],[.19,.39],(.44,.16,.22)\rangle & \langle[.38,.49],[.10,.21],[.10,.21],(.50,.11,.11)\rangle \\ \langle[.38,.49],[.10,.21],[.10,.21],(.50,.11,.11)\rangle & \langle[.22,.28],[.40,.50],[.39,.48],(.28,.28,.28)\rangle & \langle[.17,.23],[.45,.54],[.42,.59],(.21,.32,.37)\rangle \end{pmatrix} \end{array} \quad (4.3)$$

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Step 2. Normalize the decision matrix

Since all the criteria are benefit type, we do not need to normalize the decision matrix.

Step 3. Determine the relative weight of each criterion

Using Equation (3.3), we obtain the relative weight of criteria W_{ch} as follows:

$W_{ch} = (1, .875, .625)^T$.

Step 4. Calculate score values

The score values of each alternative relative to each criterion obtained by Equation (2.1) are presented in the Tables 1, 2 and 3.

Table 1. Score values for M^1.

	C_1	C_2	C_3
A_1	.56	.54	.06
A_2	.40	.09	.54
A_3	.50	.38	.06
A_4	-.03	.09	.54

Table 2. Score values for M^2

	C_1	C_2	C_3
A_1	-.03	.13	.49
A_2	.13	.13	.49
A_3	.56	.60	-.04
A_4	.39	.13	.49

Table 3. Score values for M^3

	C_1	C_2	C_3
A_1	.07	.09	.56
A_2	.07	.52	.13
A_3	.51	.37	.39
A_4	.51	.09	-.03

Step 5. Calculate accuracy values

The accuracy values of each alternative relative to each criterion obtained by Equation (2.9). are presented in Tables 4, 5 and 6.

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Table 4. Accuracy values for M^1.

	C_1	C_2	C_3
A_1	.14	.30	-.24
A_2	.12	-.23	.32
A_3	-.20	.09	-.24
A_4	-.38	-.23	.32

Table 5. Accuracy values for M^2

	C_1	C_2	C_3
A_1	-.38	-.18	.21
A_2	-.20	-.18	.21
A_3	.14	.36	-.21
A_4	.12	-.18	.21

Table 6. Accuracy values for M^3

	C_1	C_2	C_3
A_1	-.24	-.23	.41
A_2	-.24	.30	-.20
A_3	.26	.09	.12
A_4	.26	-.23	-.38

Step 6. Formulate the dominance matrix

Using Equation (3.5), we construct dominance matrix for $\alpha = 1$ The dominance matrixes are represented in matrix form (See Equations (4.4), (4.5), (4.6), (4.7), (4.8), (4.9), (4.10), (4.11), and (4.12)).

The dominance matrix Ψ_1^1

$$\Psi_1^1 = \begin{array}{cc} & \begin{matrix} A_1 & A_2 & A_3 & A_4 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} & \begin{pmatrix} 0 & .18 & .30 & .35 \\ -.46 & 0 & -.58 & .30 \\ -.74 & .23 & 0 & .19 \\ -.88 & -.74 & -.47 & 0 \end{pmatrix} \end{array} \quad (4.4)$$

The dominance matrix Ψ_2^1

$$\Psi_2^1 = \begin{array}{cc} & \begin{matrix} A_1 & A_2 & A_3 & A_4 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} & \begin{pmatrix} 0 & .29 & .18 & .28 \\ -.82 & 0 & -.69 & 0 \\ -.51 & .24 & 0 & .29 \\ -.81 & 0 & -.65 & 0 \end{pmatrix} \end{array} \quad (4.5)$$

The dominance matrix Ψ_3^1

$$\Psi_3^1 = \begin{array}{cc} & \begin{matrix} A_1 & A_2 & A_3 & A_4 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} & \begin{pmatrix} 0 & -1 & 0 & -1 \\ .25 & 0 & .26 & 0 \\ 0 & -1 & 0 & -1 \\ .25 & 0 & .26 & 0 \end{pmatrix} \end{array} \quad (4.6)$$

The dominance matrix Ψ_1^2

$$\Psi_1^2 = \begin{array}{cc} & \begin{matrix} A_1 & A_2 & A_3 & A_4 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} & \begin{pmatrix} 0 & -.46 & -.88 & -.74 \\ .18 & 0 & -.75 & -.58 \\ .35 & .09 & 0 & .04 \\ .30 & .23 & .19 & 0 \end{pmatrix} \end{array} \quad (4.7)$$

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The dominance matrix Ψ_2^2

$$\Psi_2^2 = \begin{array}{cc} & \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} 0 & 0 & -.84 & 0 \\ 0 & 0 & -.84 & 0 \\ .29 & .29 & 0 & .29 \\ 0 & 0 & -.84 & 0 \end{pmatrix} \end{array} \quad (4.8)$$

The dominance matrix Ψ_3^2

$$\Psi_3^2 = \begin{array}{cc} & \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} 0 & 0 & .26 & 0 \\ 0 & 0 & .26 & 0 \\ -1 & -1 & 0 & -1 \\ 0 & 0 & .26 & 0 \end{pmatrix} \end{array} \quad (4.9)$$

The dominance matrix Ψ_1^3

$$\Psi_1^3 = \begin{array}{cc} & \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} 0 & 0 & -.78 & -.78 \\ 0 & 0 & -.78 & -.78 \\ .31 & .31 & 0 & 0 \\ .31 & .31 & 0 & 0 \end{pmatrix} \end{array} \quad (4.10)$$

The dominance matrix Ψ_2^3

$$\Psi_2^3 = \begin{array}{cc} & \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} 0 & -.83 & -.65 & 0 \\ .29 & 0 & .18 & .29 \\ .23 & -.51 & 0 & .23 \\ 0 & -.83 & -.65 & 0 \end{pmatrix} \end{array} \quad (4.11)$$

The dominance matrix Ψ_3^3

$$\Psi_3^3 = \begin{array}{cc} & \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} 0 & -.94 & -.59 & -1.1 \\ .23 & 0 & -.73 & .15 \\ -.59 & .18 & 0 & .23 \\ -1.1 & -.58 & -.94 & 0 \end{pmatrix} \end{array} \quad (4.12)$$

Step 7. Formulate the individual overall dominance matrix

The individual overall dominance matrix is calculated by the Equation (3.6) and The dominance matrixes are represented in matrix form (see Equations (4.13), (4.14), and (4.15)).

First decision maker's overall dominance matrix λ^1

$$\lambda^1 = \begin{array}{cc} & \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} 0 & -.53 & .47 & -.37 \\ -1 & 0 & -1 & .30 \\ -1.3 & -.53 & 0 & -.52 \\ -1.5 & -.74 & -.86 & 0 \end{pmatrix} \end{array} \quad (4.13)$$

Second decision maker's overall dominance matrix λ^2

$$\lambda^2 = \begin{array}{cc} & \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} 0 & -.46 & -1.5 & -.74 \\ .18 & 0 & -1.3 & -.58 \\ -.36 & -.62 & 0 & -.67 \\ .30 & .23 & -.39 & 0 \end{pmatrix} \end{array} \quad (4.14)$$

Third decision maker's overall dominance matrix λ^3

$$\lambda^3 = \begin{array}{cc} & \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} 0 & -1.8 & -2 & -1.9 \\ .52 & 0 & -1.3 & -.34 \\ -.05 & -.02 & 0 & .46 \\ -.79 & .-1.1 & -1.6 & 0 \end{pmatrix} \end{array} \quad (4.15)$$

Step 8. Aggregate the dominance matrix

Using Equation (3.7), the aggregate dominance matrix is constructed (see Equation 4.16).

Aggregate the dominance matrix λ

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$$\lambda = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{pmatrix} A_1 & A_2 & A_3 & A_4 \\ 0 & -.94 & -1.1 & -.53 \\ -.10 & 0 & -1.23 & -.22 \\ -.54 & -.38 & 0 & -.23 \\ -.64\,. & -.55 & -.96 & 0 \end{pmatrix} \qquad (4.16)$$

Step 9. Calculate global values

Using Equation (3.8) we calculate the values of Ω_i (i = 1, 2, 3, 4) and represented in Table 7.

Table 7. Global values of alternatives

A_i	A_1	A_2	A_3	A_4
Ω_i	.49	.61	1	0

Step 10. Rank the priority

Since $\Omega_3 > \Omega_2 > \Omega_1 > \Omega_4$, alternatives are then preference ranked as follows:
$A_3 > A_2 > A_1 > A_4$.
Hence A_3 is the best alternative.

From the illustrative example, we see that the proposed NC-TODIM method is more suitable for real scientific and engineering applications because it can handle hybrid information consisting of INS and SVNS information simultaneously to cope indeterminate and inconsistent information. Thus, NC-TODIM extends the existing decision-making methods and provides a sophisticated mathematical tool for decision makers.

5. Rank of alternatives with different values of α

Table 8 shows that the ranking order of alternatives depends on values of attenuation factor, which reflects the importance of attenuation factor in NC-TODIM method.

Table 8. Global values and ranking of alternatives for different values of α

Values of α	Global values of alternative (Ω_i)	Rank order of A_i
0.5	$\Omega_1 = 0$, $\Omega_2 = .89$, $\Omega_3 = 1$, $\Omega_4 = .46$ $\Omega_3 > \Omega_2 > \Omega_4 > \Omega_1$	$A_3 > A_2 > A_4 > A_1$
1	$\Omega_1 = .49$, $\Omega_2 = .61$, $\Omega_3 = 1$, $\Omega_4 = 0$ $\Omega_3 > \Omega_2 > \Omega_1 > \Omega_4$	$A_3 > A_2 > A_1 > A_4$
1.5	$\Omega_1 = 0$, $\Omega_2 = .72$, $\Omega_3 = 1$, $\Omega_4 = .44$ $\Omega_3 > \Omega_2 > \Omega_4 > \Omega_1$	$A_3 > A_2 > A_4 > A_1$
2	$\Omega_1 = 0$, $\Omega_2 = 1$, $\Omega_3 = .81$, $\Omega_4 = .38$ $\Omega_2 > \Omega_3 > \Omega_4 > \Omega_1$	$A_2 > A_3 > A_4 > A_1$

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3	$\Omega_1 = 0,\ \Omega_2 = .56,\ \Omega_3 = 1,\ \Omega_4 = .45$ $\Omega_3 > \Omega_2 > \Omega_4 > \Omega_1$	$A_3 > A_2 > A_4 > A_1$

5.1. Analysis on influence of the parameter α to ranking order

The impact of parameter α on ranking order is examined by comparing the ranking orders taken with varying the different values of α. When α = .5, 1, 1.5, 2, 3, ranking order are presented in Table 8. We draw Figure 3 and Figure 4 to compare the ranking order for different values of α. When α=.5, α = 1.5 and α=3 the ranking order is unchanged and A_3 is the best alternative, A_1 is the worst alternative. When α = 1, the ranking order is changed and A_3 is the best alternative and A_4 is the worst alternative. For α = 2, the ranking order is changed and A_2 is the best alternative and A_1 is the worst alternative. From Table 8 we see that A_3 is the best alternative in four cases and A_1 is the worst. We can say that ranking order depends on parameter α and A_3 is the best alternative and A_1 is the worst alternative.

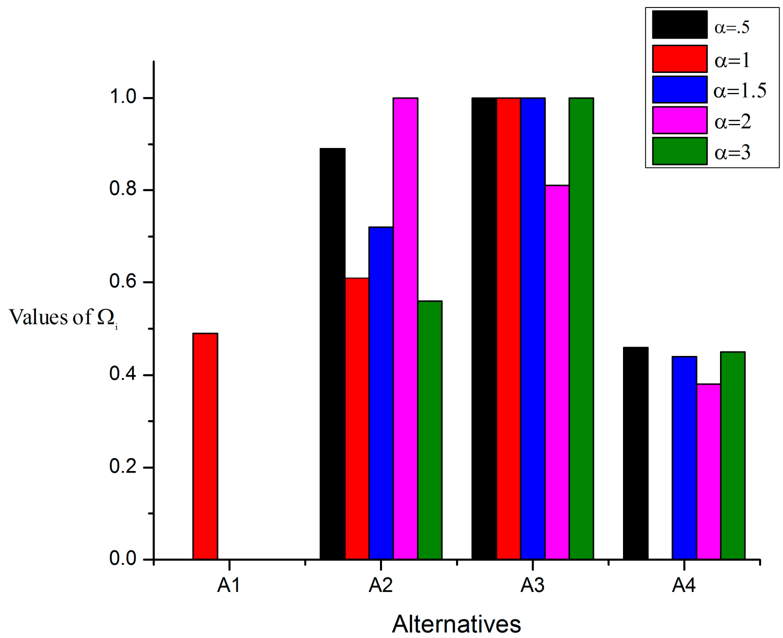

Figure.3. Global values of the alternatives for different values of attenuation factor α = .5, 1, 1.5, 2, 3.

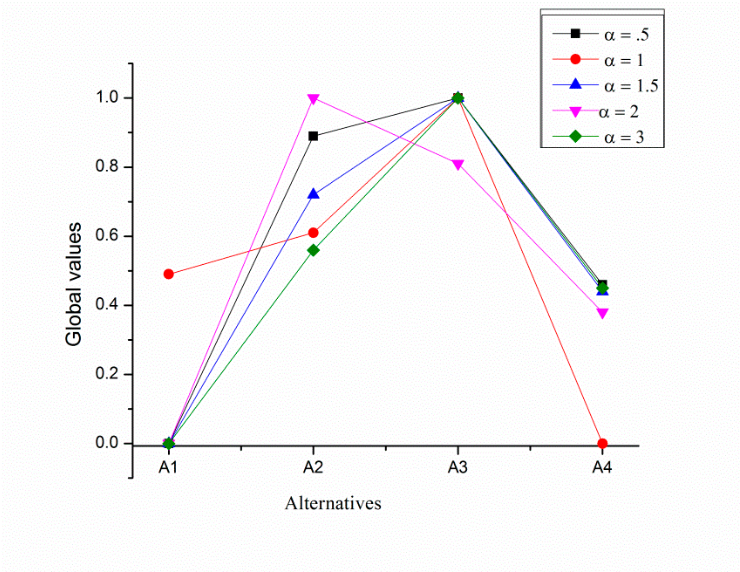

Figure.4. Ranking of the alternatives for α = .5, 1, 1.5, 2, 3.

6. Conclusion

In many real world decision-making problems, decision makers encounter uncertain decision parameters that are incomplete, indeterminate and inconsistent in nature. As a result, the decision makers cannot easily reflect their judgments on the alternatives with exact and crisp values. To tackle the situation, we propose the NC-TODIM for MAGDM problems under neutrosophic cubic information, where the preference values of alternatives over the attributes and the importance of attributes are expressed in terms of neutrosophic cubic numbers. In this study, we propose score function, accuracy functions and established some of their properties. We develop NC-TODIM method, which is capable to tackle MAGDM problems affected by uncertainty and indeterminacy represented by neutrosophic cubic numbers. The standard TODIM, in its original formulation, is only applicable to a crisp environment. Existing neutrosophic TODIM methods deal with single valued neutrosophic information only. Therefore, NC-TODIM provides more flexibility to deal with real world problems. We solve a numerical example to show the applicability and effectiveness of the proposed NC-TODIM. We investigate the influence of attenuation factor of losses α on ranking order of alternatives. The proposed NC-TODIM method can be applied to other MAGDM problems characterized by neutrosophic hybrid environments.

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