Beal Conjecture as a General Form of Fermat's Last Theorem

- a mathematical essay. Stefan Bereza, Fall 2018, Philadelphia, PA

Contents:

0. Abstract.

Fermat's Last Theorem (FLT) $\mathbf{x^{\wedge p} + y^{\wedge p} = z^{\wedge p}}$ could be seen as a special case of more generalized Beal's Conjecture (BC) $\mathbf{x^{\wedge m} + y^{\wedge n} = z^{\wedge r}}$. Those equations are **impossible** when **x**, **y** and **z** are natural numbers and coprimes and {**p**, **m**, **n**, **r**}> = **3;** if m = n = r (= p), then it is FLT; if not, Beal's Conjecture.
In BC, if **x**, **y** and **z** are **integers** and have a **common factor**, they can be measured (without rest) with this factor as a common unit - making **x**, **y** and **z** in the equation rational to each other. FLT can be proved with proving irrationality of triangles inscribed into an ellipse whose sides **x**, **y** and **z** represent the Fermat's equation $\mathbf{x^{\wedge p} + y^{\wedge p} = z^{\wedge p}}$; here, for **x**, **y** and **z** a common unit cannot be found. The BC equation $\mathbf{x^{\wedge m} + y^{\wedge n} = z^{\wedge r}}$ (without a common factor) can be simplified to the Fermat's equation $\mathbf{x^{\wedge p} + y^{\wedge p} = z^{\wedge p}}$ which - at the lacking common unit - makes **x**, **y** and **z** impossible to be all rational to each other.

1. Fermat's Last Theorem (FLT)

Fermat's Last Theorem (FLT) is about the equation $\mathbf{x^{\wedge p} + y^{\wedge p} = z^{\wedge p}}$ where {**x, y, z**} should be natural numbers (positive integers) and coprimes and the power **p** is a natural number **p > = 3**.
Pierre de Fermat (1607 - 1665) knew that it is **impossible** to have all three elements of the equation {**x, y, z**} as integers at the same time; in the simplest case, when {**x, y}** are integers, **z** cannot be integer; but it cannot be a (rational) fraction either - since **(fraction)**$^{\wedge p}$ = **FRACTION** and the sum of two integers on the left side cannot result in the fraction on the right. If so, **z** can only be an irrational number of the (algebraic) type**:**
(Integer)$^{\wedge 1/p}$ = (**irrational number**) - since then **(irrational number)**$^{\wedge p}$ = **(Integer)** and the both sides of the equation $\mathbf{x^{\wedge p} + y^{\wedge p} = z^{\wedge p}}$ are equal and result in integers. The Fermat's equation $\mathbf{x^{\wedge p} + y^{\wedge p} = z^{\wedge p}}$ **(**in its simplest form as $\mathbf{Int_x{}^{\wedge p} + Int_y{}^{\wedge p}}$ **=**...) can exist only as $\mathbf{Int_x{}^{\wedge p} + Int_y{}^{\wedge p} = Irr_z{}^{\wedge p}}$.
It is generally accepted that Andrew Wiles proved FLT in 1994. However, straightforward and clear concept of Pierre de Fermat found its proof in complicated and highly abstract, almost 200 pages work. Not many can maintain that they comprehend the proof... Hence many still search for a simpler solution - see only the results of googling on the Internet; a particular approach: http://vixra.org/abs/1805.0187
Be it as it may, proven FLT makes possible to directly prove Beal Conjecture.

2. Beal Conjecture (BC)

Andrew Beal in 1993 conceived more general form of the Fermat's equation: $\mathbf{x^{\wedge m} + y^{\wedge n} = z^{\wedge r}}$ where {**x, y, z, m, n, r**} are natural numbers; condition: **3 < =** {**m, n, r**} but **NOT m = n = r**. (If **m = n = r**, then there is a Fermat's equation.) Similarly to the Fermat's Last Theorem, Beal Conjecture (BC) says that the equation cannot exist - if {**x, y, z**} are natural numbers and pairwise coprimes. However, if {**x, y, z**} have a common factor, then the equation can exist with all {**x, y, z**} as positive integers at the same time. The opposite should be also true: if the equation exists, it must have a common factor.
Example of the BC with a common factor: $\mathbf{19^{\wedge 4} + 38^{\wedge 3} = 57^{\wedge 3}}$ - the common factor is $\mathbf{19^{\wedge 3}}$.

3. FLT as a basis for proving Beal Conjecture

The cardinal feature of the Fermat's equation $x^{\wedge p} + y^{\wedge p} = z^{\wedge p}$ (solved as $Int_x^{\wedge p} + Int_y^{\wedge p} = Irr_z^{\wedge p}$) is that the three bases Int_x , Int_y and Irr_z cannot be gauged with a common unit... If divided by a rational unit, the Irr_z stays with a rest behind; using a unit derived from Irr_z will measure fully only Irr_z [1]) and not **Int**.

If the power **p** is **2**, the Fermat's equation morphs into Pythagorean triangle equation.
The equation $a^{\wedge 2} + b^{\wedge 2} = c^{\wedge 2}$ can be modeled by a (rectangular) triangle where variable **a** and **b** represent the sides **a** and **b** and **c** represents the hypotenuse. At constant **c**, changing **a** & **b** cause the corner (opposite the hypotenuse **c**) draw a semicircle; all emerging triangles **abc** are inscribed into the semi-circle. Some of the triangles can form **rational/integral triangles** called Pythagorean triangles.

If the power **p** is $1 < p < 2$, **c** is constant and **a** & **b** change - in the equation $a^{\wedge p} + b^{\wedge p} = c^{\wedge p}$ - the variable **a** & **b** draw with their corner a (horizontal) semi-ellipse and the emerging (obtuse) **triangles abc** with their sides representing the equation $a^{\wedge p} + b^{\wedge p} = c^{\wedge p}$ are **never rational**.

If the power (natural number) **p** is $p > 2$, at **c** constant and changing **a** & **b** - in the equation $a^{\wedge p} + b^{\wedge p} = c^{\wedge p}$ - the variable **a** & **b** draw with their corner a (vertical) semi-ellipse and the emerging (acute angle) **triangles abc** with their sides representing the equation $a^{\wedge p} + b^{\wedge p} = c^{\wedge p}$ are **never rational**. (FLT).
For detailed, rather elementary explanations see http://vixra.org/abs/1805.0187

Having assumed FLT as proved, there is a **short way** to show that BC is also true.
The **z** of the right side of the equation $x^{\wedge m} + y^{\wedge n} = z^{\wedge r}$ will be the reference point. Bring now the left side of the Beal's equation to the same power **r** as the right side:

$$x^{\wedge m} + y^{\wedge n} = z^{\wedge r}$$
$$[x^{\wedge m/r}]^{\wedge r} + [y^{\wedge n/r}]^{\wedge r} = z^{\wedge r} \qquad [x^{\wedge m/r}]^{\wedge r} = [a]^{\wedge r}\ ; \quad [y^{\wedge n/r}]^{\wedge r} = [b]^{\wedge r}$$
$$[a]^{\wedge r} + [b]^{\wedge r} = z^{\wedge r}$$

Look through all combinations for **a** & **b** of being rational (**rat**) or irrational (**irr**) to **z**:
$a = a_{rat}$ or $a = a_{irr}$ $\qquad$ $b = b_{rat}$ or $b = b_{irr}$

If **a** & **b** are rational/integers, then **z** must be necessarily irrational (FLT).
Then, there are: $[a_{irr}]^{\wedge r} + [b_{rat}]^{\wedge r} = z_{rat}^{\wedge r}$ or $[a_{rat}]^{\wedge r} + [b_{irr}]^{\wedge r} = z_{rat}^{\wedge r}$ and finally $[a_{irr}]^{\wedge r} + [b_{irr}]^{\wedge r} = z_{rat}^{\wedge r}$; those configurations stay also in direct contradiction to the assumption that all three elements **a**, **b**, **z** are rational/integral to each other.

That constitutes the proof of the first part of BC: in the equation $x^{\wedge m} + y^{\wedge n} = z^{\wedge r}$ the values **x**, **y**, **z** (being coprimes) cannot be all rational/integral at the same time.

If, in the $x^{\wedge m} + y^{\wedge n} = z^{\wedge r}$, there is a common factor **F**, it can be employed as a common unit; then the equation could be: $F{*}value_1 + F{*}value_2 = F{*}value_3$. Since $values_{1,2,3}$ should be integers/rationals and there is a common (integer) unit **F**, both sides of the equation with all their elements can be measured (gauged) with **F** completely without leaving a rest; **x**, **y**, **z** are rational to each other. There is nothing which may contradict that. That proves the second part of the BC.

---

[1]) It's easier when Int_x , Int_y and Irr_z are imagined as **sections** of the line and a **unit** as a tiny portion derived from one of them by dividing the section through an integer. A common unit measuring all three sections cannot exist here.

There is also a **longer way** of proving BC: through demonstrating once again that FLT is correct and then simplifying BC to the FLT i.e. reducing Beal's equation to a common power like above.

Let the Fermat equation $\mathbf{x^{\wedge p} + y^{\wedge p} = z^{\wedge p}}$ **-** where {**x, y, z**} should be natural numbers (positive integers) and coprimes and the power **p** is a natural number **p > = 3** - be expressed by a triangle with the sides **x = a** and **y = b** and the basis **z**; so there is now an equation $\mathbf{a^{\wedge p} + b^{\wedge p} = z^{\wedge p}}$ with the representing it triangle **abz**.

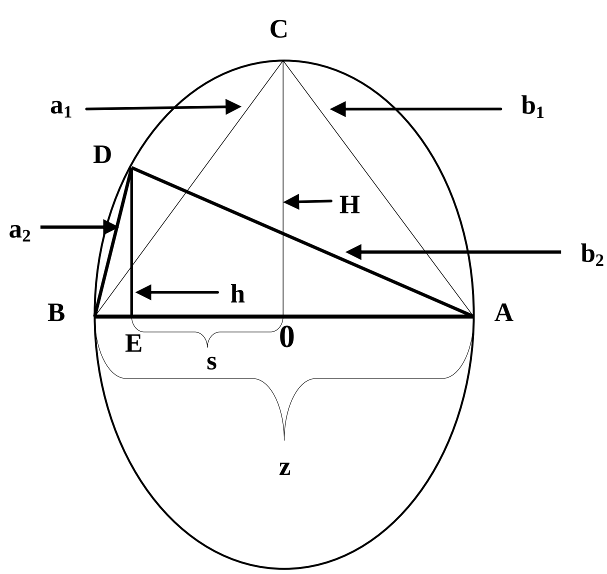
Let $\mathbf{z^{\wedge p} = [z^{\wedge p}]/2 + [z^{\wedge p}]/2}$

$\mathbf{z^{\wedge p} = [z * 2^{-1/p}]^{\wedge p} + [z * 2^{-1/p}]^{\wedge p}}$

- Now let $\mathbf{[z * 2^{-1/p}]^{\wedge p} = [a_1]^{\wedge p} = [b_1]^{\wedge p}}$
- Build an isosceles triangle with the sides $\mathbf{a_1}$, $\mathbf{b_1}$ and **z**.
- Keeping the equation $\mathbf{[a]^{\wedge p} + [b]^{\wedge p} = z^{\wedge p}}$ valid, at **constant z**, successively change **a**; **b** will be changed respectfully. Thus, **a** should be changing from $\mathbf{a_1 = b_1}$ to **a =** approaching **0** while **b** grows at that time from $\mathbf{b_1 = a_1}$ to **b =** approaching **z**. Then, repeat the procedure on the right side - this time with the growing **a** and diminishing **b**.

At **p > = 3**, the (top) corner between **a** & **b** (of all triangles with the sides **a, b,** and **z**) will draw an vertical **semi-ellipse**; all the time the sum $\mathbf{[a]^{\wedge p} + [b]^{\wedge p}}$ will be equal to $\mathbf{z^{\wedge p}}$.

- In the course of **a & b** changes, at one moment, these sides (call them now $\mathbf{a_2}$ & $\mathbf{b_2}$) shall have the values of the **x** & **y** from the equation $\mathbf{[x]^{\wedge p} + [y]^{\wedge p} = z^{\wedge p}}$. Again, the triangle sides $\mathbf{a_2}$, $\mathbf{b_2}$ and **z** represent now the **x**, **y** and **z** from the equation $\mathbf{x^{\wedge p} + y^{\wedge p} = z^{\wedge p}}$.
- Test the triangle $\mathbf{a_2b_2z}$ whether it could be **rational** - i.e. if all the sides $\mathbf{a_2}$ $\mathbf{b_2}$ **z** are measurable (without a rest) with a common unit. In the rational triangles the height **h** (or $\mathbf{h}^{\wedge 2}$) must be rational to their sides. Here the reference point of rationality is **z**; the height **h** (or $\mathbf{h}^{\wedge 2}$) must be thus rational with **z** (as well as with other sides if the triangle is rational). The act of measuring can be done by dividing **h** (or $\mathbf{h}^{\wedge 2}$) through **z**. Only if the division renders integer/rational values, the triangle is rational, and, consequently, its sides are.



Calculations:
BC = a_1 ; **BA = z** ; **B0 = z/2** ; **E0 = s** ; **DE = h** ; **C0 = H** ; **s, h** (or **$h^{\wedge 2}$**) must be rational in the rational triangles; **z** is the reference of rationality, so **z/2** is also rational.

$z^{\wedge p} = 2(a_1)^{\wedge p}$ $(a_1)^{\wedge p} = z^{\wedge p}/2$ $a_1 = z/2^{\wedge 1/p}$ $a_1 = 2^{\wedge -1/p} * z$ $(a_1)^{\wedge 2} = 4^{\wedge -1/p} * z^{\wedge 2}$

$H^{\wedge 2} = (a_1)^{\wedge 2} - (z/2)^{\wedge 2}$ $H^{\wedge 2} = z^{\wedge 2} * 4^{\wedge -1/p} - z^{\wedge 2} * 4^{-1}$ $H^{\wedge 2} = z^{\wedge 2}(4^{\wedge -1/p} - 4^{-1})$ $H = z*(4^{\wedge -1/p} - 4^{-1})^{1/2}$

h is calculated from the equation of the ellipse: $[h/H]^{\wedge 2} + [s/(z/2)]^{\wedge 2} = 1$
$h^{\wedge 2} = H^{\wedge 2} - (4s^2 * H^{\wedge 2})/z^{\wedge 2}$ $h^{\wedge 2} = (H^{\wedge 2}/z^2)*(z^{\wedge 2} - 4s^{\wedge 2})$ $h = (H/z)*(z^{\wedge 2} - 4s^{\wedge 2})^{\wedge 1/2}$

$h^{\wedge 2}/z = (H^{\wedge 2}/z^2)*(z^{\wedge 2} - 4s^{\wedge 2})/z = z^{\wedge 2}(4^{\wedge -1/p} - 4^{-1})*(z^{\wedge 2} - 4s^{\wedge 2})/z^{\wedge 3} = (4^{\wedge -1/p} - 4^{-1})*(z^{\wedge 2} - 4s^{\wedge 2})/z$
$h^{\wedge 2}/z = (4^{\wedge -1/p} - 4^{-1})*(z^{\wedge 2} - 4s^{\wedge 2})/z$; $(4^{\wedge -1/p} - 4^{-1}) = irr$; $(z^{\wedge 2} - 4s^{\wedge 2})/z = rat$
$h^{\wedge 2}/z = (irr)*(rat) = irr$
$h^{\wedge 2}/z = irr$

$h/z = [(4^{\wedge -1/p} - 4^{-1})*(z^{\wedge 2} - 4s^{\wedge 2})]^{\wedge 1/2}/z = [(irr)*(rat)]^{\wedge 1/2}/rat = [irr]^{1/2}/rat = irr/rat$
$h/z = irr/rat = irr$
$h/z = irr$

The **h** and **$h^{\wedge 2}$** measured with **z** show irrational values; that means that the triangle **$a_2 b_2 z$** representing the **x, y** and **z** in the equation $x^{\wedge p} + y^{\wedge p} = z^{\wedge p}$ **is not a rational triangle**; thus, **x**, **y**, **z** cannot be all rational at the same time... So, it proves FLT. If FLT is true, so must be BC like has been shown above.

4. Literature

About Fermat's Last Theory:
[1] Recreations in the Theory of Numbers, Albert H. Beiler, Dover Publications, 1966
[2] Fermat's enigma, Simon Singh, Anchor Books, 1998
[3] http://vixra.org/abs/1805.0187

About Beal Conjecture:
[4] R. Daniel Mauldin**,** Dec 1997**;** NOTICES OF THE AMS**,** A Generalization of Fermat's Last Theorem: The Beal Conjecture and Prize Problem**,** https://www.ams.org/notices/199711/beal.pdf
[5] Beal conjecture, https://en.wikipedia.org/wiki/Beal_conjecture
[6] http://earthmatrix.com/beal/proof-and-counterexamples.html

For triangles, ellipse, (ir)rationality, powers etc. look in ordinary textbooks or google the Internet.