The Relation of Ohm's Law to Newton's 2nd Law

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Summary

Ohm's law relates voltage (V), current (I) and resistance (R) in electrical systems as V=IR. Newton's 2nd law relates force (F), mass (m) and acceleration (a) as F=ma. Although electrical and mechanical systems are supported by different equations to describe these relationships today, it will be shown that the laws of Ohm and Newton are related and are built from the same foundation that governs the force of particles and all of matter.

In nature, electrical and mechanical systems are indeed related. Electrical power is often used for mechanical systems such elevators or electric cars, in which power is transferred from moving electrons to moving masses of larger size. Or vice versa, that electrical power can be generated from a mechanical system. Nature relates these processes, but mankind's equations do not.

Relating Ohm's Law and Newton's Law in Units

The primary reason that electrical and mechanical systems are not related by equations is because the components of these equations do not share the same units. For example, in mechanical systems the motion of a mass is measured as velocity [m/s], but in electrical systems the motion of electrons, called current, is measured as amperes [A].

The issue is the definition of electrical charge, which is a property of particles in the atom like the electron and proton. Charge is measured in a unit called the Coulomb [C]. It is a misunderstanding of the meaning of charge that causes the separation between electrical and mechanical systems. If charge is measured as a wave amplitude, which is a distance, then all of the units align and the equations can be consolidated. Waves are known to have properties of constructive and destructive interference, allowing for charges to be combined or be neutralized based on their constructive wave interference.

Using this principle, the Coulomb units [C] for the elementary charge (e) are replaced as distance units [m]:

$$e_{units} = C = m \quad \textbf{(1.1)}$$

Since all of the components of Ohm's law depend on Coulomb units, they are replaced as follows:

$$I_{units} = A = \frac{C}{s} = \frac{m}{s} \quad \textbf{(1.2)}$$

$$V_{units} = \frac{kg\,(m^2)}{s^3\,(A)} = \frac{kg\,(m^2)}{s^3\left(\frac{m}{s}\right)} = \frac{kg\,(m)}{s^2} \tag{1.3}$$

$$R_{units} = \frac{kg\,(m^2)}{s^3\,(A^2)} = \frac{kg\,(m^2)}{s^3\left(\frac{m}{s}\right)^2} = \frac{kg}{s} \tag{1.4}$$

The power of electrical systems is measured in watts, as P=VI. The power of mechanical systems is also measured in watts (energy over time), which is why the two systems can be calculated at the power level. But now the units of the components that form these equations are also equal. The units for electrical power are:

$$P_{units} = VI = \frac{kg\,(m)}{s^2}\left(\frac{m}{s}\right) = \frac{kg\,(m^2)}{s^3} \tag{1.5}$$

The following table summarizes the corrected units for electrical systems and how they are related to mechanical systems.

	Electrical		Mechanical	New Units
Current (I)	amperes	=	velocity	$\frac{m}{s}$
Voltage (V)	volts	=	newtons	$\frac{kg\,(m)}{s^2}$
Resistance (R)	ohms	=	mass/time	$\frac{kg}{s}$
Power (P)	watts	=	watts	$\frac{kg\,(m^2)}{s^3}$

Table 1.1 – Relation of units in electrical and mechanical systems

Current is now a velocity. Voltage is now a force, correctly expressed in terms of newtons. The SI units are now the same across electrical and mechanical systems, which explains how power is consistent between the two measured in watts [kg*m^2/s^3].

For example, the force of an electron moving through a wire is measured in newtons, just the same as the force of a car accelerating down the street. While there are physical size differences between an electron and the car, the physics that explains each object is now the same.

Proving the Relation of Ohm's Law and Newton's Law

Let's start with two simple examples of Newton's second law of motion, where F=ma. In the first example, a 1 kg ball is dropped on Earth where the acceleration of gravity is 10 m/s^2. It has a force of 10 newtons. Two seconds after being dropped, it would have a velocity of 20 m/s (velocity is calculated as v=at, where t is time).

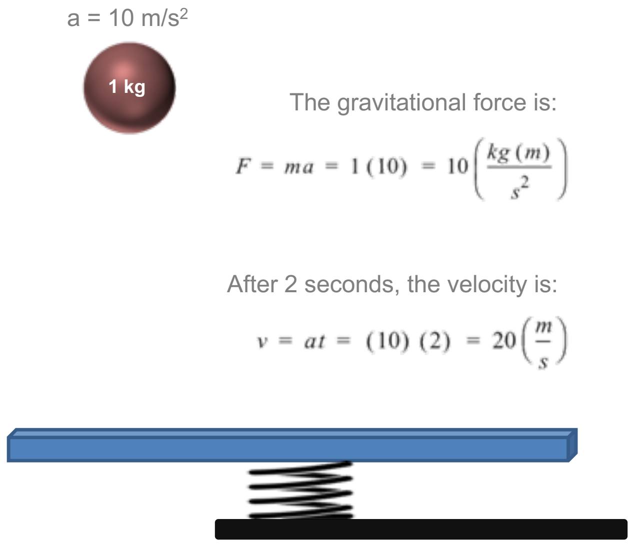

Fig 2.1 – A 1 kg ball dropped on Earth where gravitational acceleration is 10 m/s²

Summary of Example #1:

- v_1=20 m/s (velocity of ball)

In a second example, imagine a second 1 kg ball is added. It is dropped but collides with a part of the platform at 1 second, never reaching the bottom. The force of both balls are 10 newtons each. The velocity of the ball on the left is 20 m/s after two seconds, and the velocity of the ball on the right is 10 m/s after one second when it collides.

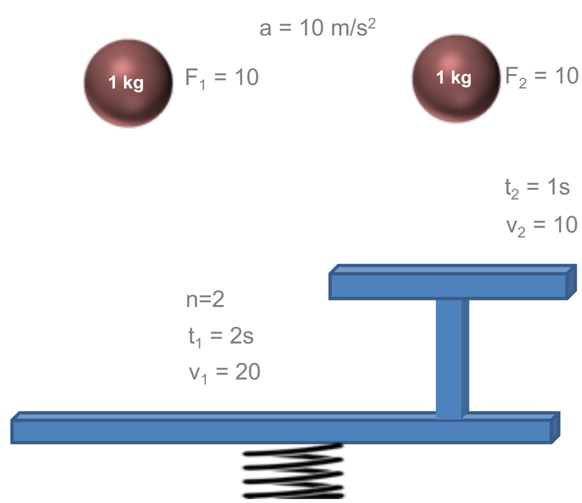

Fig 2.2 – Two 1 kg balls dropped on Earth where gravitational acceleration is 10 m/s²

The average velocity for balls (n) in this system uses the following equation:

$$v_{avg} = \frac{1}{n}\left(\sum_{i=1}^{n} v_i\right) \quad \textbf{(2.1)}$$

Since there are two balls, n=2. Thus, the average velocity is 15 m/s.

$$v_{avg} = \frac{1}{n}\left(\sum_{i=1}^{n} v_i\right) = \frac{1}{2}(20+10) = 15\left(\frac{m}{s}\right) \quad \textbf{(2.1)}$$

Summary of Example #2:

- v_1=20 m/s (velocity of left ball)
- v_2=10 m/s (velocity of right ball)
- **v_{avg}=15 m/s** (average velocity of all balls in the system)

Next, imagine a single electron traveling through a wire (conductor) with a voltage of 10V. The illustration shows the electron moving from top-down to show comparisons to gravity, although it is a voltage affecting the electron, not gravity (the same force would apply if the motion was left-right instead of top-down).

For the purpose of simple calculations, **imagine the electron has a mass of 1 kg.** This example, and the first example with the ball, could have been equivalent if the ball was set to the electron's mass, but then the example would be doing math with a mass of 9.1 x 10^{-31} kg (mass of electron). It can also be thought of as many electrons flowing together that equal the mass of 1 kg and the same math works. It's kept simple for illustration purposes.

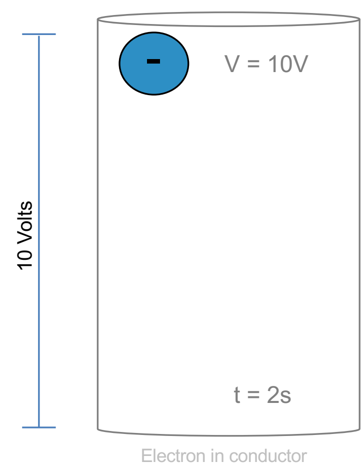

Fig 2.3 – An electron in a conductor with a voltage of 10V measured after two seconds

In Table 1.1, resistance is now measured as how much mass can travel during a specified timeframe. It has units of [kg/s]. Thus, for an electron with a mass a 1 kg traveling over 2 seconds, the resistance would be:

$$R = \frac{m}{t} = \frac{1}{2}\left(\frac{kg}{s}\right) \quad \textbf{(2.2)}$$

Since voltage is known (10 volts), current can be calculated using Ohm's law (V=IR, or I=V/R) as follows. Note that it is the same as Example #1 for the ball being dropped on Earth (20 m/s).

$$I = \frac{V}{R} = \frac{10}{\left(\frac{1}{2}\right)} = 20\left(\frac{m}{s}\right) \quad (2.3)$$

Summary of Example #3:

- **I_1=20 m/s** (velocity/current of electron)

In a fourth example, imagine two electrons in a wire at 10 volts where one electron collides with an atom after one second (electron on the right). The electron on the left continues in motion undisturbed and is measured after two seconds.

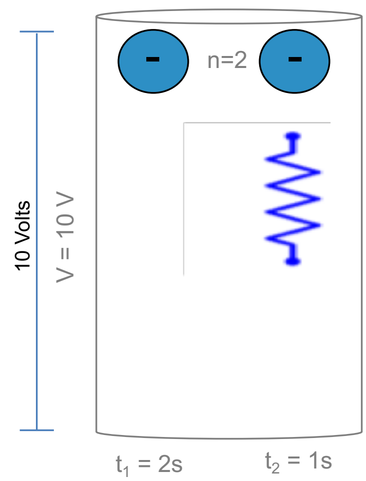

Fig 2.4 – Two electrons in a conductor with 10 volts, where one electron collides with an atom (resistance) after 1 second

The total resistance of this system requires knowing the time, if any, when an electron collides with an atom. It is a measurement of mass over time. The summation of resistance is:

$$R_{tot} = \frac{\sum_{i=1}^{n} m_i}{\sum_{i=1}^{n} t_i} \quad (2.4)$$

In this simple example, it is a total of 2 kg of mass traveling for a total time of 3 seconds (1 electron travels for 2 seconds, and 1 electron travels for 1 second). The total resistance is therefore 2/3 [kg/s]

$$R_{tot} = \frac{\sum_{i=1}^{n} m_i}{\sum_{i=1}^{n} t_i} = \frac{1+1}{2+1} = \frac{2}{3}\left(\frac{kg}{s}\right) \quad (2.5)$$

Current is the average velocity of each electron. To get to the average current across all electrons, each one is calculated using Ohm's law (I=V/R):

$$I_1 = \frac{V_1}{R_1} = \frac{10}{\left(\frac{1}{2}\right)} = 20\left(\frac{m}{s}\right) \quad \textbf{(2.6)}$$

$$I_2 = \frac{V_2}{R_2} = \frac{10}{\left(\frac{1}{1}\right)} = 10\left(\frac{m}{s}\right) \quad \textbf{(2.7)}$$

With the velocities of each electron known, the average current can be calculated. I_{avg} is calculated to be 15 m/s. **Once again, it's the same as Newton's law for two balls being dropped based on gravity (Example #2).**

$$I_{avg} = \frac{1}{n}\left(\sum_{i=1}^{n} I_i\right) = \frac{1}{2}(20 + 10) = 15\left(\frac{m}{s}\right) \quad \textbf{(2.8)}$$

Think these examples were too simple? Try the values with any number of combination of balls and resistance points and the calculations will always be the same between Newton's law of Ohm's law. It's because the physics for a single electron is the same as the physics for a larger object with mass.

Why do we use properties like voltage, current and resistance? Because it is impossible to know and measure the position and velocity of every electron in a wire. Current will continue to be the average velocity of all electrons in the system, as we cannot measure each one. Resistance will continue to be the probability of electrons colliding with atoms in a material that has demonstrated that level of collision activity in the past. It is simpler to manage one number for current and one number for resistance than billions of numbers calculating each and every electron. Therefore, Ohm's law will continue to be used. However, we should recognize that current is truly the average velocity of electrons and that resistance is the total mass traveling in a specified timeframe. Completing this example, we find that voltage to be correct using Ohm's law where V=IR. It is 10 volts. It is 10 newtons of force. They are one in the same.

$$V = I_{avg} R_{tot} = (15)\left(\frac{2}{3}\right) = 10\left(\frac{kg\,(m)}{s^2}\right)(volts) \quad \textbf{(2.9)}$$

Summary of Example #4:

- I_1=20 m/s (velocity of electron on left)
- I_2=10 m/s (velocity of electron on right)
- **I_{avg}=15 m/s** (average velocity of all electrons in the system)
- **R_{tot}=2/3 kg/s** (total resistance for all electrons in the system)
- **V=10 newtons** (voltage)

Despite the fact that we won't be able to measure each and every electron, Ohm's law and Newton's law are indeed related.