

Model of Quarks and Leptons Based on Spacetime Symmetries

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Abstract

The experimental search of standard model superpartners, and the derivation of the standard model from higher dimensional theories have been challenging for some time now. In this article these technologies are kept but they are applied to a simpler environment. A coherent scenario of particles based on Kaluza-Klein theory and unbroken supersymmetry is proposed. It offers an economic basis for constructing the standard model particles without the superpartner problem of the minimal supersymmetric standard model. With local supersymmetry one arrives at supergravity without Yang-Mills fields. A number of results in the literature would have to be reconsidered according to this model.

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1 Introduction

It is commonly stated that the CERN LHC has 'failed' to discover supersymmetry (SUSY). On the other hand, LHC has given strong support for the standard model (SM) of particles. Accordingly, I have proposed model that is supersymmetric and the constituents of which build the standard model. Supersymmetry functions on the constituent, or preon, level and it is unbroken, or mildly spontaneously broken. The SUSY preon model contains all the fields and their superpartner fields in its supermultiplets, out of which the standard model fermions are constructed. This has the consequence that, within this model, no standard model superpartners (like squarks, sleptons, gluinos etc.) exist in nature. This is the crucial test of the SUSY preon scheme.

The second motivation, in the chain of events for the present article, is the question of deriving the standard model from string theory. Kaluza-Klein theory is the early attempt to derive four dimensional physics from a higher dimensional model without matter. The derivation the standard model with $SU(3) \times SU(2) \times U(1)$ symmetry from higher dimensional models or theories has been challenging. In this article I hope to apply the Kaluza unbroken-Klein theory by defining the basic fields with fewer properties than in the SM, and limiting oneself within the abelian $U(1)$ symmetry. I keep the technologies of supersymmetry and higher dimensions but use them in an unbroken, minimal environment of preons. This article assumes that spacetime symmetries cannot be compromised, therefore supersymmetry should be valid unbroken.¹

In the preon model, quarks and leptons are represented as three preon bound states. The number of elementary superfield fermion fields is $N_F = 2$, whereas in the supersymmetric standard model $N_F = 16$ (2 quarks in 3 colors, 2 leptons and their superpartners, for the first generation in both models). On the other hand, the physics on preon level is largely open. It is contemplated that quantum gravity, when available, would provide the interaction which keeps the preons

¹Pauli had a strong case to introduce the neutrino to have spacetime conservation laws valid.

together and explains the heavier two generations as excitations of preon bound states. Such theories work in atomic and nuclear systems.

At present, an indirect case of comparing supersymmetric models with available experimental data are the CMB data of Planck 2018. The CMB measurements open a window to energies well above any accelerator energy and only a few decades below Planck scale. The agreement between the gravity driven inflationary model and data is good. The connection of the leading inflationary model to supersymmetry is elucidated. Composite models of quarks and leptons have no direct experimental support, except the peculiar spectrum of these fermions.

The article is organized as follows. In section 2 the Kaluza-Klein theory is introduced, as providing electromagnetism from the fifth dimension to four dimensions. The basic supermultiplets of supersymmetric fields are presented in section 3. With these mathematical preliminaries the preon model is defined and the standard model is heuristically constructed in section 4. In section 5 a proper case of comparing two supersymmetric models with available experimental data are at present the CMB data of Planck 2018. The CMB measurements open a window to energies well above any accelerator energy and only two or three decades below Planck scale. The agreement between the gravity driven models and data is good. The connection of the leading inflationary model to supersymmetry is elucidated. Conclusions are given in section 6. – The article is intended to be pedagogical and self-contained.

2 Kaluza-Klein Theory

The idea of unifying gravity with electromagnetism was born about one hundred years ago. Nordström [1] in 1914 and Kaluza [2] in 1921 were the first physicists to make this unification (for a careful reviews, see [3, 4]). They proposed a theory in five dimensions with variables (x^0, x^1, \dots, x^4) . An immediate question was why we do not see any fifth dimension in nature? Both men avoided this question by assuming that all derivatives with respect to the fifth dimension variable x^4 vanish. The two men obtained successfully the field equations of both gravity and electromagnetism from a five dimensional theory. This success is due to $U(1)$ gauge invariance was added onto Einstein's equations in the guise of invariance with respect of coordinate transformations in the x^4 direction. Gauge symmetry is interpreted as geometrical symmetry of spacetime in extra dimensions. Klein [5] showed that the fifth dimension should be handled by the method of compactification. It means that x^4 has circular topology and its scale is very small, like of the order of Planck scale. Compactification of extra dimensions has been studied actively beyond 5D, up to 10D superstring theory and 11D supergravity. Eleven has been shown to be (i) the maximum number of dimensions with a single graviton and (ii) the minimum number required of a KK theory to contain the standard model gauge group $SU(3) \times SU(2) \times U(1)$. But unfortunately, both these two higher dimensional theories have not been

solved satisfactorily for the present [6]. Therefore, the approach in this article is based on the original 5D Kaluza-Klein theory.

Let us briefly recap the Kaluza-Klein theory. The Einstein equations in an empty 5D space, i.e. without any 5D energy-momentum tensor of matter, read

$$\hat{R}_{AB} - \frac{1}{2}\hat{R}g_{AB} = 0 \quad (2.1)$$

where \hat{R}_{AB} is the Ricci tensor. The capital Latin indices A, B, ... have values 0, 1, 2, 3, 4. Five dimensional quantities are denoted by a hat on top of them. The corresponding 5D action is

$$S = -\frac{1}{16\pi\hat{G}} \int \sqrt{-\hat{g}} d^4x dy \hat{R} \quad (2.2)$$

where $y = x^4$ is the fifth coordinate and \hat{G} is the 5D gravitational constant.

The missing matter source terms in (2.1) and (2.2) indicates Kaluza's key point that the universe in dimensions $D > 4$ is empty. Matter in 4D would be a manifestation of geometry in higher dimensions. If matter has to be introduced by hand in higher dimensional fields, the ideal would be lost.² Meanwhile the more ambitious theories of everything are under construction, I take the humble attitude of limiting to five dimensions and organizing matter, the strong and weak interactions in a different way.³

The five dimensional Ricci tensor and Christoffel symbols are defined in terms of the metric as in 4D

$$\begin{aligned} \hat{R}_{AB} &= \partial_C \hat{\Gamma}_{AB}^C - \partial_B \hat{\Gamma}_{AC}^C + \hat{\Gamma}_{AB}^C \hat{\Gamma}_{CD}^D - \hat{\Gamma}_{AD}^C \hat{\Gamma}_{BC}^D \\ \hat{\Gamma}_{AB}^C &= \frac{1}{2} \hat{g}^{CD} (\partial_A \hat{g}_{DB} + \partial_B \hat{g}_{DA} - \partial_D \hat{g}_{AB}) \end{aligned} \quad (2.3)$$

Everything in (2.3) is like in general relativity, except indices running up to 4, not 3.

Now a form for the five dimensional metric has to be chosen. The four dimensional part $\alpha\beta$ is as before. The lower right corner contains the scalar field ϕ and the four potential takes the remaining two vacant corners. A useful realization is the following

$$\hat{g}_{AB} = \begin{pmatrix} g_{\alpha\beta} + \kappa^2 \phi^2 A_\alpha A_\beta & \kappa \phi^2 A_\alpha \\ \kappa \phi^2 A_\beta & \phi^2 \end{pmatrix} \quad (2.4)$$

where the vector potential is scaled by constant κ for later purposes (a good choice turns out to be $\kappa = 4\sqrt{\pi G}$). The signature is (+—).

Using the metric (2.4) and the definitions (2.3) together with the cylinder condition in (2.2) one gets three terms after pulling out the y -integral

$$S = - \int d^4x \sqrt{-g} \phi \left(\frac{R}{16\pi G} + \frac{1}{4} \phi^2 F_{\alpha\beta} F^{\alpha\beta} + \frac{2}{3\kappa^2} \frac{\partial^\alpha \phi \partial_\alpha \phi}{\phi^2} \right) \quad (2.5)$$

²This method has been called pouring "stone soup" from a can [7].

³Even though this is commonly considered old-fashioned.

where G is defined as $G \equiv \hat{G} / \int dy$.

Extending the Kaluza-Klein method to cases of different kind of matter have met with difficulties in spite of long history of attention and research. In this situation there are two types of alternatives (i) to wait for more general class of theories in dimensions higher than five, or (ii) organize matter differently, i.e. take the quantum numbers provided by the Kaluza-Klein theory. In addition, one must remember that mass, as well as charge and spin, come from the black hole solutions of Einstein's equations in empty space.

The boson sector of the KK world consists now of the graviton, photon and a massless scalar, having spins $j=2, 1, 0$, respectively, and charge 0. These are associated with representations of the Lorentz group. Recall that we have one more spacetime symmetry available for model building in the next section 3.

The case of considering the standard model gauge symmetry $SU(3) \times SU(2) \times U(1)$ is reviewed in [3, 4]. It turns out that in the 4+D dimensional theory that compactifying the $D > 1$ extra dimensions that matter fields have to be introduced to get an energy-momentum tensor, or that the 4+D dimensional action is not the minimal Einstein-Hilbert action. In this article we limit to the case $D = 1$. Also, it turns out that the case of composite gauge bosons are needed in some models.

3 Supersymmetry

Supersymmetry is a transformation between bosons, with integer spin, and fermions, with half-integer spin [8]. An operator Q which generates transformations between fermions and bosons is an anti-commuting spinor

$$Q|\text{boson}\rangle = |\text{fermion}\rangle, \quad Q|\text{fermion}\rangle = |\text{boson}\rangle \quad (3.1)$$

Q and its hermitian conjugate Q^\dagger carry spin 1/2. Therefore supersymmetry must be a spacetime symmetry. The generators Q and Q^\dagger satisfy the following algebra

$$\begin{aligned} \{Q, Q^\dagger\} &= P^\mu \\ \{Q, Q\} &= \{Q^\dagger, Q^\dagger\} = 0 \\ [P^\mu, Q] &= [P^\mu, Q^\dagger] = 0 \end{aligned} \quad (3.2)$$

where P^μ is the four momentum generator of space-time translations.

It is believed here that supersymmetry, a spacetime symmetry, should not be compromised.

In the N=1 supersymmetric model there is the graviton G and its spin $\frac{3}{2}$ superpartner gravitino \tilde{G}

$$G = \begin{pmatrix} \rightarrow \\ \leftarrow \end{pmatrix} \quad \text{and} \quad \tilde{G} = \begin{pmatrix} \rightarrow \\ \leftarrow \end{pmatrix} \quad (3.3)$$

This the graviton supermultiplet. Secondly, there are the massless fields the photon γ and its neutral spin $\frac{1}{2}$ superpartner, the photino, denoted \tilde{m}^0 . They form the vector supermultiplet

$$\gamma = \begin{pmatrix} \rightarrow \\ \leftarrow \end{pmatrix} \quad \text{and} \quad \tilde{m}^0 = \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}, \quad (3.4)$$

The third superpair is the spin $\frac{1}{2}$ fermion m and scalar superpartners \tilde{s} . Here I have to introduce charge for the superpair since it is provided by the Reissner-Nordström black hole solutions of (2.1), and we have the electromagnetic vector potential A_μ in (2.5). This chiral supermultiplet therefore is m^+ and its scalar superpartner $\tilde{s}_{1,2}^+$

$$m^+ = \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \quad \text{and} \quad \tilde{s}_{1,2}^+ \quad (3.5)$$

In the next section it turns out that the charge needed is $\frac{1}{3}$ of electron charge. In (3.3) – (3.6) the horizontal and vertical arrows refer to helicity and spin, respectively, and + and 0 refer to charge in units of $\frac{1}{3}$ electron charge. The \tilde{m}^0 is a Majorana fermion. The R-parity for fields is simply $P_R = (-1)^{2(\text{spin})}$. The m^+ and \tilde{m}^0 are assumed to have zero, or light mass of the order of the first generation quark and lepton mass scale.

4 Preon Model

In the standard model the unification of gauge interactions is of the order 10^{16} GeV. It is also the unified theory proton decay mediating X-boson mass lower limit, corresponding to a proton lifetime of 10^{32} years. Denote this energy by Λ_{cr} . Independent of the energy scale, we accept in the present model only the interactions provided by the five dimensional Kaluza-Klein theory (2.2): gravity and the electromagnetic interaction. The strong and weak interactions must build hadrons at low energies, down to scale of the pion mass. At higher energies like above Λ_{cr} the strong gauge interaction goes to zero. I now make the assumption that the non-abelian gauge interactions operate below Λ_{cr} as usually, but above Λ_{cr} they do not contribute. These requirements can be fulfilled by a preon model in which the quarks and leptons consist of three preons, having charges $\frac{1}{3}$ and 0. Above Λ_{cr} the quarks and leptons ionize, or make a phase transition to unbound phase. The preons are introduced in the vector and chiral supermultiplets (3.5) and (3.6).

A model for quark and lepton constituents was introduced in [10, 11, 12, 13].⁴ I believe this structure of the preon model brings clarity as compared to the case of traditional approach to supersymmetry and grand unification. The neutral gauge bosons are obtained from (3.4) and (3.5) as the Z^0 and photon. Charged gauge bosons and fermions are there, but to be taken by

⁴ Supersymmetry was anticipated in passing in [11].

hand, because of Reissner-Nordström solutions of Einstein's equations. This set up is an alternative to supersymmetric theories in 10 or 11 dimensions, which have turned out to be more complex than expected from the point of view of the standard model.

I presume that quantum gravity, when available, will organize the preons in bound states in three generations. Alternatively, there may be a new very strong gauge interaction between the preons, like e.g. in [16, 17, 18, 19].⁵ In those cases introducing supersymmetry as indicated above fails.

Assuming a generic attractive interaction, or potential, the preons combine freely without extra assumptions into standard model fermion composite states. They form a three member combinatorial system, modulo three [12]. For the same charge preons fermionic permutation antisymmetry factor ϵ_{ijk} must be included. These arguments lead heuristically to four bound states made of preons, which form the first generation quarks (q) and leptons (l) (dropping the tildes)

$$\begin{aligned}
u_k &= \epsilon_{ijk} m_i^+ m_j^+ m^0 \\
\bar{d}_k &= \epsilon_{ijk} m_i^+ m_j^0 m^0 \\
e &= \epsilon_{ijk} m_i^- m_j^- m_k^- \\
\bar{\nu} &= \epsilon_{ijk} \bar{m}_i^0 \bar{m}_j^0 \bar{m}_k^0
\end{aligned}
\tag{4.1}$$

The strong and weak interactions are built to operate between the three preon bound states in (4.1) as gauge boson mediated transitions between them. More details are given in [10, 12] and references therein.

Bound states of scalar constituents do not make a spectrum like fermions. A neutral, very light two body bound state is expected to exist

$$a_i^0 = \tilde{s}_i^+ \tilde{s}_i^-, \quad i = 1, 2 \tag{4.2}$$

Scalar bound states can also be formed from the fermions

$$\begin{aligned}
b^0 &= m^+ m^- \\
c^0 &= m^0 m^0 \\
h^\pm &= m^\pm m^0
\end{aligned}
\tag{4.3}$$

The states (4.2) and (4.3) (and other possible states including mixtures) are candidates for the Higgs, axion and the like, which are important in spontaneously broken symmetries of the standard model. Finally, the model allows an unbound scalar charge $\frac{1}{3}$ field.

5 Cosmological Inflation

Several models of inflation have been proposed some time ago and experimental results from the sky have become more and more accurate. It was noted in

⁵A different kind of supersymmetric preon model has been presented in [20, 21].

[22, 23] that quantum corrections to general relativity are important in the early universe. They lead to R^2 , with R being the curvature of spacetime, corrections in the Einstein-Hilbert action. In situations where curvature is large these corrections lead to an effective cosmological constant causing an inflationary de Sitter era. In addition, predictions for corrections to the microwave background were obtained in detailed calculations. The simplest Starobinsky action is

$$S_{Staro} = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} \left(R + \frac{R^2}{6m^2} \right) \quad (5.1)$$

where m ($\sim 3 \cdot 10^{13}$ GeV) is the inflaton mass as the only parameter. Note that it is entirely based on gravitational interactions but it is non-renormalizable. Starobinsky inflation is equivalent to Higgs inflation in supergravity because both models lead to indistinguishable predictions. The potential of the Starobinsky inflation in terms of the canonical inflaton field ϕ

$$V(\phi) = \frac{3}{4} M_{\text{Pl}}^2 m^2 \left[1 - \exp \left(-\sqrt{2/3} \phi / M_{\text{Pl}} \right) \right]^2 \quad (5.2)$$

The characteristic features of this scalar potential are: it is bounded from below, it has an absolute minimum at $\phi = 0$ and it has a plateau which leads to slow roll inflaton in the inflationary period. The inflaton potential drives the inflation and its quantum fluctuations generate deviations from flatness, isotropy and homogeneity.

The Starobinsky model predicts for spectral tilt n_s and tensor-scalar ratio the values $n_s = 1 - 2/N$ and $r = 12/N^2$, where N is the number e-folds. The 2018 CMB data from the Planck satellite [24] give $r < 0.064$ (95 percent confidence) and $n_s = 0.9649 \pm 0.0042$ (68 percent confidence level; $n_s = 1$ means scale independent power spectrum).

Starting from the early model of supersymmetry, the Wess-Zumino model [25], one is interested to know whether the CMB data can be tried on it. The data disfavor simple models of inflation with monomial potential ϕ^n . Instead potentials with concave regions like $\phi^2(v - \phi)^2$ may provide reasonable inflation if $v \gg M_{\text{Pl}}$ and $\phi_0 \sim v/4$. This form can be interpreted as coming from the minimal Wess-Zumino model with superpotential W and scalar potential V as follows for real fields Φ [26, 27]

$$W = \frac{1}{2} \mu \Phi^2 - \frac{1}{3} \lambda \Phi^3, \quad V = \left| \frac{\partial W}{\partial \Phi} \right|^2 \quad (5.3)$$

The W-Z model field Φ is complex, and it can be written as modulus and phase $\Phi = \frac{1}{\sqrt{2}} \phi \exp(i\theta)$. The scalar potential becomes now

$$V = A(\phi^4 - 2 \cos(\theta) v \phi^3 + v^2 \phi^2) \quad (5.4)$$

This reduces to hilltop form when $\theta = 0$: $V = A(\phi^2(v - \phi)^2)$. For the phenomenological analysis a two field form of $\Phi = (\psi + i\sigma)/\sqrt{2}$ is used. The parameters n_s and r were calculated using perturbation theory, quantum field

theory techniques and numerically integrating two-point scalar field perturbations in Fourier space. The model gives for $N = 50$ foldings and $v = (5-10)M_{\text{Pl}}$ with initial conditions near $\sigma = 0$ axis results, which are very close to what the Starobinsky model gave.

6 Conclusions

The present supersymmetric Kaluza-Klein preon model is based on the proposal that the physical domain of supersymmetry is the preon level instead of quark and lepton level. Consequently both the fields and the superpartners are in the basic supermultiplets building up quarks and leptons. Supersymmetric models possess diffeomorphism invariance, and they are D=10 low energy limits of string theory, but it has turned out that the construction of the standard model is difficult in 10 and 11 dimensions. Therefore the present model has rich enough structure for quantitative study, and it is hoped to provide an alternative way towards quantum gravity.

Summarizing, the model

1. is built on Kaluza-Klein classical unification of gravity and electromagnetism, the latter emerging from the five dimensional gravity without sources,
2. contains matter-gauge unification in terms of a spacetime symmetry rather than gauge symmetry - e.g. $SU(5)$ or other group - unification as traditionally. Weak and strong interactions are, on log scale, late time interactions to provide for astrophysics, chemistry and biology,
3. is an economic way to build the standard model fermions, a possible mechanism for three generations is indicated,
4. has no minimal supersymmetric standard model superpartner issue, i.e. no squarks or sleptons etc. exist,⁶
5. is, due to unbroken supersymmetry, more constrained than models with broken super- and grand unified symmetry,
6. has fewer elementary fields and therefore rests on simpler vacuum structure than main stream theories,
7. includes the parameter $\Lambda_{cr} \sim 10^{16}$ GeV, which is in conformity with the energy scale of the coupling constant unification energy of traditional models,
8. dark matter finds a natural explanation as being formed of primordial black holes, because during the early high density inflationary period gravity is the dominant interaction obeying Einstein-like quantum equation without sources,

⁶These superpartners can be constructed within the model though, but that would be against the original ambition.

9. has no information paradox because of preon quantum numbers are conserved by black holes, and
10. or rather the supersymmetry property of it, underlays description of inflation: the supergravity equivalent Starobinsky and the supersymmetric Wess-Zumino models of inflation agree with the Planck 2018 CMB data.

The fundamental question of quantizing spacetime itself, like e.g. in loop quantum gravity [28] or causal dynamical triangulation [29], is beyond the scope of this article.

From global supersymmetry the next, and more difficult, step is to go to local supersymmetry [30]. It is hoped that the present preon model provides a new avenue towards better understanding of the roles of all four interactions. This article is intended to serve as an affirmative feasibility study of a research proposition, which is hoped to receive community response.⁷ I hope to return to these questions elsewhere.

What would change because of this model? All calculations with broken supersymmetry would be in doubt. The minimal supersymmetric model might be of no use. The present model would strengthen models or theories having unbroken supersymmetry and abelian interactions. In such models there is less freedom, fewer parameters and simpler vacuum. Implications to 10 and 11 dimensional theories would have to be reconsidered.

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⁷It is recognized that preons are not favored at present. They are mentioned e.g. in [31, 32] but no more in [33].

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⁸The model was conceived in November 1974 at SLAC. I proposed that the c-quark would be a gravitational excitation of the u-quark. The idea was opposed by the community and was therefore not written down until five years later.

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