A Resolution to the Vacuum Catastrophe

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Abstract - This paper presents a theoretical estimate for the vacuum energy density which turns out to be near zero and thus much more palatable than an infinite or a very large theoretical value obtained by imposing an ultraviolet frequency cut-off. This result helps address the "vacuum catastrophe" and the "cosmological constant problem".

The vacuum energy density (or zero-point energy density, i.e. energy E per volume V) of a free quantum field is given by [1],

$$\hat{\rho}_{vac} = \frac{(-1)^{2j}(2j+1)}{2}\int_{-\infty}^{\infty}\frac{1}{(2\pi)^3}\sqrt{m^2+\mathbf{k}^2}d^3k = \frac{(-1)^{2j}(2j+1)}{2}\int_{0}^{\infty}\frac{4\pi}{(2\pi)^3}\sqrt{m^2+k^2}k^2dk \quad (1)$$

in natural units where the reduced Planck constant $\hbar$ = the speed of light in vacuum $c = 1$. Note that m is the particle mass associated with a specific field, j is the spin, $\mathbf{k}$ is the wave vector and k is the wave number. For a massless field (i.e. photon and gluon, with $m = 0$ and $j = 1$), we can further simplify (1) to

$$\hat{\rho}_{vac} = \frac{(-1)^{2j}(2j+1)}{2}\frac{1}{2\pi^2}\int_{0}^{\infty}k^3dk \quad (2)$$

For massive fields (i.e. electron & quarks ($j = \frac{1}{2}$), W & Z ($j = 1$) and Higgs ($j = 0$), and with $m \neq 0$), equation (2) only provides an approximation which gets better for smaller m (e.g. electron, up and down quarks with negative vacuum energy density contributions).

At first glance, the integral in (2) looks highly divergent, however using the results obtained in [2], a value for this integral can be calculated to be $\frac{1}{20}$ and hence the expectation value of the vacuum energy density *does not diverge* and is nearly zero. The vacuum energy density contributions from all the fields can then be summed up which results in a positive near zero value (given here in metric units by multiplying (2) with values of $\hbar$ and c)

$$\bar{\rho}_{vac} \approx 10^{-29}\ J/m^3 \quad (3)$$

This value agrees with the large scale cosmological observations which set an upper bound of approximately $10^{-11}J/m^3$ on the vacuum energy density [3]. It also agrees with the predictions of current cosmological models and all observational data to date which essentially has found that vacuum has very little energy content leading to a vanishing cosmological constant. This result helps address the "vacuum catastrophe" and the "cosmological constant problem".

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References

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