DYNAMICS IN RELATIVITY IS THE WAY TO QUANTUM THEORY

Vladimir B. Belayev
Center for Relativity and Astrophysics
wbelayev@yandex.ru

Abstract. Equations of the critical curve are obtained by the nonzero energy integral variation in accordance with principles of the calculus of variations in mechanics. This method is compared with the Fermat's principle and geodesics principle. The force vector acting on the photon in Schwarzschild space-time is found for the weak gravity and corresponds to photon's gravitational mass equal to the twice mass of a material particle of the same energy. Compliance with the law of conservation of energy as a source of gravity leads to the presence of particles with negative gravitational mass and zero kinematic momentum in the results of the annihilation reaction. Accretion of matter onto compact stars results in their absorption of positive energy from the vacuum and the release negative energy in a free deep space. The particles with negative gravitational mass create there antigravitating vacuum with negative pressure. Extended Space Model (ESM) is a generalization of the special theory of relativity at a 5-dimensional space. Rotations in extended space correspond to the motion of a particle in gravity field in the embedded four-dimensional space-time. Within the framework of ESM the photons have a nonzero mass in a gravitational field. We study how a rotation in ESM agrees with photon dynamics in the Schwarzschild field. A non-conservation of energy in gravitational systems is interpreted by the ESM as the rotation of the energy-momentum vector in 5-dimensional space. The comparison bubble cosmic structures of a type gravastar with electron an estimate of its radius is made.

1. INTRODUCTION

It is known, that between the mechanical and optical phenomena there is a certain likeness, which historically was exhibited that a set of the optical phenomena managed uniformly well to be described both within the framework of wave, and within the framework of the corpuscular theories. In particular, motion of a beam of light in an inhomogeneous medium in many respects similar to motion of a material particle in a potential field [1]. In the given activity, we shall take advantage of this connection to describe the gravitational phenomena.

The Fermat principle is the basis of geometric optics in media. It is also formulated for Riemannian space-time [2, 3]. In [4, 5] it is proposed a variational principle of the stationary energy integral of a light-like particle, which does not lead to violation of the isotropy of light path and agrees with Fermat's principle for static gravitational fields. It is also applicable to non-stationary gravitational fields in which the particle motion is free. This approach is the choice of Lagrangian of the particle and the definition of canonical momenta and forces in accordance with Lagrange's mechanics. A correspondence is established between the physical energy and momentum of the particle, determined from non-gravitational interactions, and the contravariant canonical momentum vector.

In [6-8] it is investigated a generalization of special theory of relativity in a 5-dimensional space $G(1,4)$ with a metric (+ - - - -) having an additional coordinate s. In ESM, in addition to the rotations in plane (TX) relating to the Lorentz transformations, the rotations in planes (TS) and (XS) are considered. In this model the approach is used, where the 5-th coordinate is the interval in (1+3)-

dimensions [9]. Movement along additional 5-th coordinate corresponds to the presence of particles rest-mass in (1+3)D. This is the case when a photon, get into gravitational field, gains a nonzero mass. At the same time, it is being localized [10], while in the Minkowski space it is compared to the an infinite plane wave. In this paper we study how (TS)-rotation agrees with photon dynamics in the Schwarzschild field, which is analyzed using the principle of extreme energy of a light-like particle based on Lagrangian mechanics [11].

Assuming that the standard model of cosmology and gravity theory are correct, astronomers have identified phenomena, whose essence is reduced to following statements [12-17]:

1) The main part of the Universe mass (more than 0.9) is made of dark matter and dark energy, which is associated with physical vacuum.

2) These dark substances do not emit electromagnetic radiation and do not interact with him, or exhibit such properties very weakly but have gravity.

3) The space vacuum has negative pressure, or, in other words, shows properties of an antigravitation, which determines dynamics of Universe extension.

In present work we develop an approach to explanation of above-mentioned phenomena. Accretion of matter onto compact stars results in the birth electron-positron pair with the appearance of additional particles from the vacuum having positive gravitational energy. Subsequent annihilation of $e^{+}e^{-}$ pair release particles with negative gravitational mass [11], which are thrown in free deep space and create there antigravitating vacuum with negative pressure. The possibility of existence of body with a negative mass was considered and in general relativity [18].The particles with positive energy form halo of compact stars and their possible contribution to dark matter is considered.

The photons according to ESM having positive rest-mass in a gravity field are concentrated around massive stars and black holes. Various modes that proposes existing nonzero mass to a photon are discussed in review [19]. In quantum nonlinear medium photons interact with each other so strongly that they begin to act as though they have mass, and they bind together to form molecules [20].

A gravitational model gravastar, or gravitational condensed star [21, 22] was offered as alternative to black holes. Such object corresponds to the solution of the Einstein equation, which outside of area occupied by masses, coincides with Schwarzchild solution. Inside of gravastar there is other, nonsingular solution, so that metric is as a whole received nonsingular. Gravastar has a structure similar to a structure of a bubble. This bubble has a dense rigid envelope, which is under tension because of liquid substance, pressed apart it from within. The general thin-wall formalism is developed and applied to the investigation of the motion of various bubbles arising in the course of phase transitions in the very early Universe [23, 24]. Regular superconducting solution for interior of the Kerr-Newman spinning particle for parameters of electron represents a highly oblated rotating bubble formed by Higgs field which expels the electromagnetic field and currents from interior to domain wall boundary of the bubble [25]. The external fields correspond exactly to Kerr-Newman solution, while interior of the bubble is flat. Along with this, the bubble configuration of atomic nuclei has been discussed [26]. In the present work we will show, that it is possible result for gravastar when transfer of energy, as a source of gravitational field, to the vacuum coincides with properties of microworld.

2. (TS)-ROTATION IN EXTENDED SPACE MODEL

In Minkowski space $M(1,3)$ a 4-vector of energy and momentum

$$\tilde{p} = \left(\frac{E}{c}\ ,\ p_x\ ,\ p_y\ ,\ p_z\right)$$

is associated to each particle [2]. In the extended space $G(1,4)$ [6-8] it is completed to 5-vector

$$\overline{p} = \left(\frac{E}{c},\ p_x\ ,\ p_y\ ,\ p_z\ ,\ mc\right)\ ,$$

where m is a rest mass of the particle. In blank space in a fixed reference system there are two types of various objects with zero and nonzero masses. In space $G(1,4)$ to them there corresponds 5-vectors

$$\overline{p}_f = \left(\frac{\hbar\omega}{c}\ ,\ \frac{\hbar\omega}{c},\ 0\right)$$

and

$$\overline{p}_m = (mc\ ,\ 0\ ,\ mc)\ . \tag{1}$$

For simplicity we have recorded these vectors in (1 + 2)-dimensional space. The vector $\overline{p}_f$ describes a photon with energy $\hbar\omega$ and with speed c. The vector $\overline{p}_m$ describes a fixed particle. Next we will consider the motion of a photon.

At hyperbolic rotations on an angle ϕ_{TS} in the plane (TS)

$$\frac{E'}{c} = \frac{E}{c}\cosh\phi_{TS} + p_s\sinh\phi_{TS}\ ,\qquad P' = P\ ,$$
$$p'_s = p_s\cosh\phi_{TS} + \frac{E}{c}\sinh\phi_{TS} \tag{2}$$

the photon vector (1) will be transformed as follows:

$$\left(\frac{\hbar\omega}{c}\ ,\ \frac{\hbar\omega}{c}\ ,\ 0\right) \rightarrow \left(\frac{\hbar\omega}{c}\cosh\phi\ ,\ \frac{\hbar\omega}{c}\ ,\ \frac{\hbar\omega}{c}\sinh\phi\right) =$$
$$\left(\frac{\hbar\omega}{c}n\ ,\ \frac{\hbar\omega}{c}\ ,\ \frac{\hbar\omega}{c}\sqrt{n^2-1}\right). \tag{3}$$

In ESM this rotation is associated with the photon's motion in a medium in enclosed three-dimension space with refraction index $n > 1$. In such areas the speed of light is reduced. The parameter n relates the speed of light in vacuum with the speed of light in a medium v as

$$n = \frac{c}{v}\ . \tag{4}$$

According to the concept that the 5-th coordinate is interval in (1+3)-dimensions the photon gains a rest mass

$$M = \frac{\hbar\omega}{c^2}\sinh\phi = \frac{\hbar\omega\sqrt{n^2-1}}{c^2}$$

in gravity field of embedded space-time.

In addition to the (TS)-rotation of 5-momenta (2) in ESM there is (XS)-rotation

$$\frac{E'}{c} = \frac{E}{c}\ ,\qquad P' = P\cosh\phi_{XS} + p_s\sinh\phi_{XS}\ ,$$
$$p'_s = p_s\cosh\phi_{XS} + P\sinh\phi_{XS}\ .$$

With the help of these transformations from the components of photon 5-momentum $\overline{p}_f$ in a flat extended space, one can pass to the components of its 4-momentum in an arbitrary 4-dimensional space [11]:

$$\left(\frac{\hbar\omega}{c}\ ,\ \frac{\hbar\omega}{c}\ ,\ 0\right) \rightarrow \left(\frac{\hbar\omega}{c}\mathrm{F}^T(\mathrm{x}_i)\ ,\ \frac{\hbar\omega}{c}\mathrm{F}^P(\mathrm{x}_i)\ ,\ \frac{\hbar\omega}{c}\mathrm{F}^S(\mathrm{x}_i)\right)\ ,$$

where $\mathrm{F}^T(\mathrm{x}_i)$, $\mathrm{F}^P(\mathrm{x}_i)$, $\mathrm{F}^S(\mathrm{x}_i)$ are functions of coordinates. Transformations are not communicative at specified angles of rotation ϕ_{TS} and ϕ_{XS}:

$$(TS) - (XS) \neq (XS) - (TS)\ .$$

In the case of a material particle, a transformation (TX) is added to them.

3. PRINCIPLE OF THE STATIONARY ENERGY INTEGRAL OF PHOTON

In [4, 6] it is proposed a variational principle of the stationary energy integral of photon without violation of Lorentz-invariance. In it the interval in pseudo-Riemann space-time with metrical coefficients $\tilde{g}_{11}$:

$$ds^2 = \tilde{g}_{ij} dx^i dx^j$$

after substitutions

$$\tilde{g}_{11} = \rho^2 g_{11} \ , \qquad \tilde{g}_{1k} = \rho g_{1k} \ , \qquad \tilde{g}_{kq} = g_{kq}$$

is rewritten in form

$$ds^2 = \rho^2 g_{11} dx^{12} + 2\rho g_{1k} dx^1 dx^k + g_{kq} dx^k dx^q \ .$$

Here ρ is some quantity, which is assumed to be equal 1. Putting down x^1 as time, coordinates with indexes $k, q = 2{,}3{,}4$ as space coordinates and considering ρ as energy of light-like particle with $ds = 0$ we present it as

$$\rho = \left(g_{11} \frac{dx^1}{d\mu}\right)^{-1} \left\{ -g_{1k} \frac{dx^k}{d\mu} + \sigma \left[(g_{1k} g_{1q} - g_{11} g_{kq}) \frac{dx^k}{d\mu} \frac{dx^q}{d\mu} \right]^{1/2} \right\} \ , \tag{5}$$

where σ is ± 1 and μ is affine parameter.

The partial derivatives with respect to coordinates are written as

$$\frac{\partial \rho}{\partial x^\lambda} = -\frac{1}{2u_1 u^1} \frac{\partial g_{ij}}{\partial x^\lambda} u^i u^j \ , \tag{6}$$

where $u^i = dx^i / d\mu$ is four-velocity vector. The partial derivatives with respect to components u^λ are

$$\frac{\partial \rho}{\partial u^\lambda} = -\frac{u_\lambda}{u_1 u^1} \ . \tag{7}$$

With $g_{11} = 0$ and $g_{1k} \neq 0$ even if for one k the energy takes form

$$\rho = \frac{g_{kq} u^k u^q}{2u_1 u^1} \ .$$

In this case the partial derivatives of ρ coincide with (6) and (7).

For the free moving a particle Lagrangian is taken in form

$$L = -\rho \tag{8}$$

and conforms to relation [27]:

$$\rho = u^\lambda \frac{\partial L}{\partial u^\lambda} - L \ .$$

Partial derivatives of Lagrangian give the canonical momenta

$$p_\lambda = \frac{\partial L}{\partial u^\lambda} = \frac{u_\lambda}{u^1 u_1} \tag{9}$$

and forces

$$F_\lambda = \frac{\partial L}{\partial x^\lambda} = \frac{1}{2u^1 u_1} \frac{\partial g_{ij}}{\partial x^\lambda} u^i u^j \ . \tag{10}$$

Components of the associated vector of the canonical momenta are

$$p^\lambda = \frac{u^\lambda}{u^1 u_1} \ . \tag{11}$$

Physical energy and momenta of photon with frequency ω in Minkowski space-time with affine parameter $\mu = ct$ form contravariant 4-vector of momenta $\pi^i = (\hbar\omega / c) u^i$. For arbitrary affine parameter it is rewritten as

$$\pi^i = \frac{\hbar\omega}{c}\frac{u^i}{u^1} \quad . \qquad (12)$$

And in pseudo-Riemannian space-time similar energy and momenta of the photon will be put in line with the components of the contravariant vector of momenta. A certain fixed value of the photon's frequency ω_0 is given by the corresponding equality $\omega = \omega_0/u_1$. Comparing expressions (11) and (12), we obtain

$$\pi^i = \frac{\hbar\omega_0}{c}p^i \quad . \qquad (13)$$

This one provides Lagrangian of the photon $L_{ph} = \hbar\omega_0 L$. The components of vector

$$F^k = g^{k\lambda}F_\lambda \qquad (14)$$

associated to (10), with this approach, are proportional to gravity forces:

$$Q^i = \hbar\omega_0 F^i \quad , \qquad (15)$$

which acts on the photon. That is, although the non-straight motion of a particle in space-time according to General relativity is due to its curvature, identified with the gravitational field, we, studying the motion in the coordinate frame of reference, consider the analogy with the action of forces on the particle. That is, although the indirect motion of a particle in space-time according to General relativity is due to its curvature, identified with the gravitational field, we, studying the motion in the coordinate frame of reference, consider the analogy with the action of forces on the particle.

Taking into account equation (8) a motion equations are found by using Hamilton's principle from variation of energy integral

$$S = \int_{\mu_0}^{\mu_1} L d\mu = -\int_{\mu_0}^{\mu_1} \rho d\mu \quad ,$$

where μ_0, μ_1 are values of the affine parameter in points, which are linked by found extremal curve. Energy ρ is non-zero, its variations leave interval to be light-like, and application of standard variational procedure yields Euler-Lagrange equations

$$\frac{d}{d\mu}\frac{\partial\rho}{\partial u^\lambda} - \frac{\partial\rho}{\partial x^\lambda} = 0 \quad . \qquad (16)$$

The obtained equations of the isotropic critical curve can be rewritten as

$$\frac{dp_\lambda}{d\mu} - F_\lambda = 0 \quad . \qquad (17)$$

4. ENERGY AND MOMENTUM OF PARTICLE TRANSFERRED TO GRAVITY FIELD

In accordance with conservation laws, the vector of energy and momentum of a system that includes a particle and the gravitational field generated by it, denoted by $\bar{p}^k$, can be written as the sum of the momentum and energy of the particle itself p^k and transmitted it to the gravitational field $\overset{\leftrightarrow}{p}^k$. The vector $\bar{p}^k$ changes under the influence of the force from the source of gravity:

$$\frac{d\bar{p}^k}{d\mu} = \frac{dp^k}{d\mu} + \frac{d\overset{\leftrightarrow}{p}^k}{d\mu} = F^k \quad .$$

Passing in equations (17) to the associated canonical momenta and forces, we obtain

$$F^k = \frac{dp^k}{d\mu} + g^{k\lambda}\frac{dg_{\lambda i}}{d\mu}p^i \quad .$$

Comparing two expressions for F^k and passing to the partial derivatives of metrical coefficients we find the rate of exchange of energy and momentum between particle and gravitational field

$$\frac{d\overset{\leftrightarrow}{p}^k}{d\mu} = g^{k\lambda}\frac{\partial g_{\lambda i}}{\partial x^j}u^j p^i \quad .$$

When considering the dynamics of a single particle, this vector is an analogue of the pseudotensor used in the laws of conservation in tensor form.

From the conservation laws it follows that the force acting on the system, including the particle and the gravitational field generated by it, is equal in magnitude and opposite in sign to the force acting on the system of the source of gravitation from the side of the particle system. This is equivalent to fulfilling Newton's third law. Its adherence to the Newtonian limit of gravity means the equality of the passive and active gravitational masses.

5. COMPARISON OF NULL GEODESICS, ENERGY INTEGRAL VARIATION AND FERMAT PRINCIPLES

Let us clear whether proposed variational method conforms to Fermat's principle, which for stationary gravity field [2] is formulated as follows

$$\delta \int \frac{1}{g_{11}}(dl + g_{1k}dx^k) = 0 \quad ,$$

where dl is element of spatial distance along the ray

$$dl^2 = \left(\frac{g_{1p}g_{1q}}{g_{11}} - g_{pq}\right)dx^p dx^q \quad .$$

Comparing the integrand

$$df = \frac{1}{g_{11}}(dl + g_{1k}dx^k) \tag{18}$$

with equation (5) we write

$$\frac{df}{d\mu} = \rho u^1 \quad . \tag{19}$$

Null geodesics and extreme energy integral curves for light-like particle are identical in static space-time [5, 11]. In [28] the generalized Fermat's principle is proposed and it is shown that obtained curves are null geodesics. It is applied Pontryagin's minimum principle of the optimal control theory and obtained an effective Hamiltonian for the light-like particle motion in a curved spacetime. The dynamical equations for this Hamiltonian are

$$Q = u^1$$

and

$$\frac{d}{d\mu}\left(\frac{\partial Q}{\partial \dot{x}^q}\right) - \frac{\partial Q}{\partial x^q} - \frac{\partial Q}{\partial x^1}\frac{\partial Q}{\partial \dot{x}^q} = 0 \quad . \tag{20}$$

Function Q coincides with following from (18) expression for $df/d\mu$ under condition that the metric coefficients also depend on time. Following from (19) expression for energy $\rho = Q/u^1$ substituted in Euler-Lagrange equations (16) yields

$$\frac{1}{u^1}\frac{d}{d\mu}\left(\frac{\partial Q}{\partial u^q}\right) - \frac{1}{(u^1)^2}\frac{\partial Q}{\partial u^q}\frac{du^1}{d\mu} - \frac{1}{u^1}\frac{\partial Q}{\partial x^q} = 0 \quad . \tag{21}$$

The Euler-Lagrange equation for the time coordinate obtained from energy integral variation principle is reduced to the form

$$\frac{du^1}{d\mu} + \frac{u^1}{2u_1}\frac{\partial g_{ij}}{\partial x^1}u^i u^j = 0 \quad .$$

Comparing its with equations (7) we write

$$\frac{du^1}{d\mu} = (u^1)^2 \frac{\partial(Q/u^1)}{\partial x^1} = u^1 \frac{\partial Q}{\partial x^1} \ .$$

Substituting this expression in (21) and multiplying it by u^1 gives equations (20), which confirms the consistency of principle of an extremal energy integral of light-like particle and generalized Fermat's principle.

6. PHOTON'S DYNAMICS IN SCHWARZSCHILD SPACE-TIME

6.1. SPHERICAL COORDINATES

A centrally symmetric gravity field in the free space is described by the Schwarzschild metric. At spherical coordinates $x^i = (\tau, r, \theta, \varphi)$ with $\tau = ct$ its line element is

$$ds^2 = \left(1 - \frac{\alpha}{r}\right) d\tau^2 - \left(1 - \frac{\alpha}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \ , \tag{22}$$

where α is constant. To find the photon motion, we solve the Euler-Lagrange equations, which give for Lagrangian (8) a solution that is identical with the geodesics. In plane $\theta = \pi/2$ equations (17) with canonical momenta (9) and forces (10) yield [4, 5, 11]:

$$\frac{d\tau}{d\mu} = 1 \ , \qquad \frac{d\varphi}{d\mu} = \frac{B}{r^2}\left(1 - \frac{\alpha}{r}\right) \ , \tag{23}$$

where B is constant. Substituting these values in equation $ds = 0$ we find

$$\frac{dr}{d\mu} = \pm\left[\left(1 - \frac{\alpha}{r}\right)^2 - \left(\frac{B}{r}\right)^2 \left(1 - \frac{\alpha}{r}\right)^3\right]^{1/2} \ . \tag{24}$$

The value of the coordinate velocity in the remote frame is

$$v = \sqrt{\left(r\frac{d\varphi}{dt}\right)^2 + \left(\frac{dr}{dt}\right)^2} = c\left(1 - \frac{\alpha}{r}\right) \ .$$

In 4D space-time for the Schwarzschild field the canonical momenta are

$$p_1 = 1 \ , \qquad p_2 = \mp \frac{1}{\left(1 - \frac{\alpha}{r}\right)} \sqrt{1 - \frac{B^2}{r^2}\left(1 - \frac{\alpha}{r}\right)} \ ,$$

$$p_3 = 0 \ , \qquad p_4 = -B \ .$$

Nonzero components of the contravariant vector of momenta are given by

$$p^1 = \left(1 - \frac{\alpha}{r}\right)^{-1} \ ,$$

$$p^2 = \pm\sqrt{1 - \frac{B^2}{r^2}\left(1 - \frac{\alpha}{r}\right)} \ , \qquad p^4 = \frac{B}{r^2} \ .$$

The physical energy and momentum are matched exactly with the contravariant vector, since in the limit of the Minkovsky space it has momentum components with a sign coinciding with the direction of motion.

6.2. RESTANGULAR COORDINATES

Considering the non-radial motion, in order to avoid the appearance of a fictitious component of momenta and force due to the sphericity of the coordinate system we use the Schwarzschild metric in rectangular coordinates [4, 5, 11]. To the isotropic form of metric one can go from its spherical form (22) with the help of the transformation

$$r=\left(1+\frac{\alpha}{4\bar{r}}\right)^2\bar{r}\quad, \qquad (25)$$

and it is written as

$$ds^2=c^2\left(\frac{1-\frac{\alpha}{4\bar{r}}}{1+\frac{\alpha}{4\bar{r}}}\right)^2 dt^2- \left(1+\frac{\alpha}{4\bar{r}}\right)^4(dx^2+dy^2+dz^2)\quad, \qquad (26)$$

where (t,x,y,z) is rectangular frame and $\bar{r}=\sqrt{x^2+y^2+z^2}$.

We will consider the motion in the plane $z=0$ and seek the force acting on the particle at a point $(t,x,0,0)$ that corresponds to the value of the angular coordinate $\varphi=0$ in the spherical frame. Coordinate transformations in the plane are

$$x=\bar{r}\cos\varphi,\qquad y=\bar{r}\sin\varphi\quad .$$

The nonzero spatial components of the 4-velocity are

$$\bar{u}^2=\frac{dx}{d\mu}=\frac{d\bar{r}}{d\mu},\qquad \bar{u}^3=\frac{dy}{d\mu}=\frac{d\varphi}{d\mu}\bar{r}\quad .$$

The transformation (25) implies the relation

$$dr=\left(1-\frac{\alpha^2}{16\bar{r}^2}\right)d\bar{r}\quad . \qquad (27)$$

Equations (24)-(25) yield

$$\bar{u}^1=1\quad,\qquad \bar{u}_1=\left(\frac{1-\frac{\alpha}{4\bar{r}}}{1+\frac{\alpha}{4\bar{r}}}\right)^2\quad, \qquad (28)$$

$$\bar{u}^2=\pm\frac{\left(1-\frac{\alpha}{4\bar{r}}\right)}{\left(1+\frac{\alpha}{4\bar{r}}\right)^3}\left[1-\frac{B^2\left(1-\frac{\alpha}{4\bar{r}}\right)^2}{\bar{r}^2\left(1+\frac{\alpha}{4\bar{r}}\right)^6}\right]^{1/2}\quad, \qquad (29)$$

$$\bar{u}^3=\frac{B\left(1-\frac{\alpha}{4\bar{r}}\right)^2}{\bar{r}\left(1+\frac{\alpha}{4\bar{r}}\right)^6}\quad . \qquad (30)$$

Substitution of these velocities in (11) gives components of associated vector of the canonical momenta

$$\bar{p}^1=\left(\frac{1+\frac{\alpha}{4\bar{r}}}{1-\frac{\alpha}{4\bar{r}}}\right)^2\quad,$$

$$\bar{p}^2=\pm\frac{1}{\left(1-\frac{\alpha}{16\bar{r}^2}\right)}\left[1-\frac{B^2\left(1-\frac{\alpha}{4\bar{r}}\right)^2}{\bar{r}^2\left(1+\frac{\alpha}{4\bar{r}}\right)^6}\right]^{1/2}\quad,$$

$$\bar{p}^3=\frac{B}{\bar{r}\left(1+\frac{\alpha}{4\bar{r}}\right)^4}\quad .$$

Passing back from the variable $\bar{r}$ to r, we write, in accordance with equation (13), the value of the photon energy and momentum in a remote coordinate frame

$$E=\hbar\omega_0\left(1-\frac{\alpha}{r}\right)^{-1}\quad,$$

$$\bar{P}=[(\bar{p}^2)+(\bar{p}^3)]^{1/2}=\frac{1}{\left(1-\frac{\alpha}{16\bar{r}^2}\right)}\frac{\hbar\omega_0}{c}\quad,$$

where ω_0 is the photon frequency at infinity at the world line with unlimited r. Moving to the scale of the length of spherical frame in view of equation (27) we obtain $P=\hbar\omega_0/c$.

Using the analogy of geometrical optics with gravity [6-8, 28] the refraction index (4) is given by $n = \left(1 - \frac{\alpha}{r}\right)^{-1}$.

Turning to ESM we write four-momentum after rotation in the plane (TS) in space $G(1,4)$ (3):

$$\left(\frac{E}{c}, P, p_s\right) = \left(\frac{\hbar\omega}{c\left(1-\frac{\alpha}{r}\right)}, \frac{\hbar\omega}{c}, \frac{\hbar\omega}{c}\frac{[\alpha(2r-\alpha)]^{1/2}}{r-\alpha}\right) .$$

We obtain the coincidence of the energy and momentum in the embedded four-dimensional space-time in ESM with the result given by variational principle of the stationary energy integral of photon in the Schwarzschild field.

6.3. FORCES AND GRAVITY MASS OF PHOTON

At spherical coordinates the non-zero components of canonical forces vector (10) and associate vector F^k (14) are

$$F_2 = \frac{\alpha}{r^2\left(1-\frac{\alpha}{r}\right)} - \frac{B^2}{r^3} + \frac{\alpha B^2}{2r^4} ,$$

$$F^2 = -\frac{\alpha}{r^2} + \frac{B^2}{r^3}\left(1-\frac{\alpha}{r}\right)\left(1-\frac{\alpha}{2r}\right) . \qquad (31)$$

For a gravitational constant G, active gravitational mass M and $\alpha = 2GM$ the first term of F^2 yields (15) for the radial motion $(B = 0)$ twice Newton gravity force acting on a photon

$$Q^2 = -\hbar\omega_0 \frac{\alpha}{r^2} .$$

It corresponds to the passive gravitational mass of the photon

$$m_{gp} = 2\hbar\omega_0 . \qquad (32)$$

Considering the non-radial motion we use the Schwarzschild metric in rectangular coordinates (26). Substituting nonzero 4-velocity components (28)-(30) in (10), we find nonzero component of the force vector (15) acting on the photon:

$$\overline{Q}^2 = -\hbar\omega_0 \frac{\alpha\left(1-\frac{\alpha}{8\overline{r}}\right)}{\overline{r}^2\left(1+\frac{\alpha}{4\overline{r}}\right)^5\left(1-\frac{\alpha}{4\overline{r}}\right)} .$$

Taking into account transformation (25) it is rewritten as

$$\overline{Q}^2 = -\hbar\omega_0 \frac{\alpha\left(1-\frac{\alpha}{8\overline{r}}\right)}{r^2\left(1-\frac{\alpha^2}{16\overline{r}^2}\right)} .$$

Its magnitude does not depend on the direction of motion of the photon. This formula differs from force in spherical coordinates (31) because the expression for the canonical force (10) is non-covariant, that is, with this approach gravity force acting on the photon depends on the choice of the coordinate system. However, in the limit of weak gravity these expressions asymptotically converge and give Newton's law of gravitation with passive gravitational mass of the photon $2\hbar\omega_0$ (32). One conforms to the light deflection in central gravity field, which is twice value being given by the Newton gravity theory.

Obtained gravitational mass of the light-like particle is independent on the direction of its motion. The gravitational mass of a photon for low gravity is equal to doubled mass of a material particle, equivalent to its energy. This corresponds to the result of Tolman [30] for active gravity mass of photon. He applied solutions of Einstein's equation for the electromagnetic field in the case of weak gravity to analyze the gravitational interaction of a light packet or beam with a material particle.

This result can have the following application. At annihilation of an electron and positron the energy determined from non-gravitational interactions and the momentum are preserved. We will consider how the gravitational mass of system changes. Although it is not known exactly whether the gravitational mass of the positron is positive or negative, some estimates give its positive value [31]. Proceeding from this assumption the total gravitational mass of an electron and positron $2m_e$ is twice less than the gravitational mass of the formed gamma quanta $4m_e$. This raises the question of mass conservation [32]. If to consider energy as a gravitation source, it means that on condition of its preservation at annihilation besides gamma quanta this process has to be allocated the particles g^- which are carrying away negative energy as a source of gravitational field, that is, having negative gravitational mass. Process of annihilation will look as follows

$$e^+ + e^- \to 2\gamma + 2g^- \quad . \tag{33}$$

The particles g^- with gravitational mass

$$m_{g_-} = -m_e$$

do not have a kinetic momentum and therefore their detection by standard means of particle registration, for example, a bubble chamber, is not possible. With the passage of light beams through the area with negative energy needs to produce the effect of defocusing, opposites focusing of light by the gravitational lens [33].

7. GRAVASTAR STRUCTURE

The gravastar model was offered [21, 22] alternatively to black holes. It is considered as final object, which is formed in because of processes of elementary particles disintegrations. This is static spherically symmetrical field with the metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{h(r)} + r^2(d\theta + \sin^2\theta d\varphi^2) \quad . \tag{34}$$

In the case of an isotropic medium and a constant density ε in the inner region of gravastars the solution of Einstein's equations yields a density $c^2\varepsilon = -p$. This corresponds to the de Sitter metric with coefficients

$$f(r) = Ch(r) = C\left(1 - \frac{r^2}{R_D^2}\right) \quad . \tag{35}$$

Here R_D is the radius of a curvature of the de Sitter world and C is arbitrary constant. Radius of a curvature is within the limits corresponding to the shell of gravastar $r_1 < R_D < r_2$ and is $R_D = \sqrt{(r_1)^3/\alpha}$, where α is a radius of Black Hole with equal mass.

For the spherically symmetric static gravastar-like object with negative central pressure it is found [33] that shell cannot be perfect fluids. Anisotropic pressures in the "crust" are unavoidable. The anisotropic Tolman − Oppenheimer − Volkoff equation can be used to find them [34]. Existence of rotating gravastars cannot be ruled out by invoking its instability [35]. LIGO's observations of gravitational waves from colliding objects have been found either to not be consistent with the gravastar concept [36, 37], but it does not deny the existence of gravastar in principle.

Accretion of matter onto compact stars causes gamma radiation [38] and creates conditions for the birth of gamma-ray electron-positron pair. This reaction, the inverse of (33), occurs when extracting pairs of particles g^- and opposite to them in "a gravitational charge" particle g^+ from a vacuum. Having negative gravitational mass, particle g^- is absorbed immediately leaving g^+ with a positive gravitational mass:

$2\gamma \to e^+ + e^- + 2g^+$.

The subsequent collision of a positron with an electron will cause annihilation with the release of a pair of g^-. Gravastar and Black Hole will attract particles g^+. On g^- the compact stars act like White Hole on particles with positive energy. They push these particles into free deep space. This process results in the compact stars's absorption of positive energy from the vacuum compact star and the release negative energy around. In this sense, it is reversed by Hawking radiation, which leads to the evaporation of Black Holes. The particles g^- initiate the dynamics of cosmological expansion controlled by antigravity [40]. The particles g^+ can form a halo of compact stars. Their ability to be part of dark matter depends on whether they have a rest mass. If it is absent, they will dissipate faster in the outer space.

8. ENERGY DENSITY IN SPHERICAL GRAVITATIONAL SYSTEMS

The gravitational mass of a spherical body[2] described by metric (35) with radius r_1 is given by

$$M = 4\pi \int_0^{r_1} \varepsilon r^2 dr \quad , \tag{36}$$

where ε is proper density of matter. Integration is performed here in case of the element of volume $dV_c = 4\pi r^2 dr$, which corresponds to the coordinate frame, whereas in its proper frame the given element of space volume will be $dV_p = 4\pi r^2 h^{-1}(r) dr$. Condition $h(r) < 1$ means that the gravitational mass of body is less than the sum of individual gravitational masses its constituent elements. This interprets as the transfer of energy, as a source of gravitational field, to the vacuum.[5] The whole mass of spherical body is defined as follows:

$$M_p = 4\pi \int_0^{r_1} \varepsilon r^2 h^{-1}(r) dr \quad . \tag{37}$$

In terms of ESM in static case the energy-momentum vector $\overline{p}$ with total density of matter $\varepsilon_p = \varepsilon h^{-1}(r)$ can be represented as 5-vector

$$\overline{p}_{mt} = \left(c\varepsilon h^{-1}(r) \;\; , \; 0 \;\; , \; c\varepsilon h^{-1}(r)\right).$$

Its hyperbolic rotation in the plane (TS) (2) on an angle $\phi_{TS} = \ln\left(h(r)\right)$ yields

$$\overline{p}_{mg} = (c\varepsilon \;\; , \; 0 \;\; , \; c\varepsilon).$$

This rotation corresponds transition from the total density of matter to the density as a source of gravity.

Considering the mass of the gravastar we assume for simplicity that all of it is concentrated in interior area. The solution of Einstein equations for metric (34) with coefficients (35) gives [21, 22] the density of matter $\varepsilon = (3c^2/8\pi G)H_0^2$, where it is denoted $H_0 = 1/R_D$. Substitution of ε in (36) yields gravitational mass

$$M = \frac{c^2}{2G} H_0^2 r_1^3 \quad . \tag{38}$$

The whole mass (37) turn out to be

$$M_p = \frac{3c^2}{2G}\left(-r_1 + \frac{1}{2H_0}\ln\frac{1+H_0 r_1}{1-H_0 r_1}\right) \quad .$$

A particular point of interest for applications to microphysical objects is a possibility of situation when M_p exceeds gravitational mass twice: $M_p = 2M$. For the bubble structures it will take place subject to $H_0 r_1 = 0.998147$. With this condition a particle with the mass of an electron $m_e = 9.1093835 * 10^{-31}$ kg according to equation (38) will have a radius $r_e = 1.3579 * 10^{-57}$ m. This value is less than Planck length by 22 orders of magnitude. It does not include spin, magnetic moment and charge, but this result may be used to estimate the order of the electron radius via gravastar model. Observation of a single electron in a Penning trap [41] suggests the upper limit of the particle's radius to be 10^{-22} m.

9. CONCLUSIONS

Canonical 4-momentum is given by variational principle of the stationary energy integral of photon. The physical energy and momentum of photon are matched exactly with the contravariant 4-momentum, since in the limit of the Minkovsky space it has momentum components with a sign conforms to the direction of motion. For the Schwarzschild space-time the result coinsides with energy and momentum in the embedded four-dimensional space-time obtained by (TS)-turn in ESM that is corresponded to the photon's motion in space with refraction index $n > 1$. One provides localization of the photon and can be considered as the acquisition of rest mass by it.

The identity of the generalized Fermat principles and the stationary energy integral of a light-like particle for velocities is proved. The virtual displacements of coordinates retain path of the light-like particle to be null in the pseudo-Riemann space-time, i.e. not lead to the Lorentz-invariance violation in locality and corresponds to the variational principles of mechanics. The equivalence of the solutions given by the first principle, to the geodesics, means that the use of the second also turns out geodesics. The stationary energy integral principle gives a system of equations that has one equation more. This makes it possible to uniquely determine the affine parameter and energy-momentum vector of the particle.

A definite Lagrangian produces particle canonical momenta and forces acting on it in the coordinate frame. Contravariant forces are mapped to the components of the vector of the gravitational force. The four-force vector is not covariant. The value of the force acting on a particle depends on the choice of the coordinate frame, and therefore the quantities determined through them are meaningful only for weak gravity, for which its values asymptotically converge in the different coordinate frames. The analogy between the mechanics of particle motion in the Schwarzschild space and Newton's gravity theory under this condition allows to determine passive gravitational mass of the photon, which is equal to twice the mass of a material particles of the same energy determined from non-gravitational interactions. This corresponds to the result of Tolman for active gravity mass of photon. This discrepancy suggests that at annihilation of an electron and positron in addition to gamma quanta the particles are released that have zero kinetic energy and momentum and carrying away negative energy as a source of gravitational field, that is, they have negative gravitational mass.

These particles, together with similar particles with positive energy, can be generated by accretion of matter onto compact stars. As a result, positive energy is absorbed and negative energy is released in free space. Areas containing g^- are characterized by negative pressure and exhibits properties of antigravitation. The presence of such regions in the Universe causes its accelerated expansion. The ability of particles g^+ to be part of dark matter depends on whether they can be at rest.

The (TS)-rotation of energy-momentum 5-vector of matter density describes gravitational defect of static mass. It is shown, that gravastar model can be used to describe the properties of the microworld, giving an estimate of the electron radius.

REFERENCES

1. Sivuhin DV. *Optika* [Optics].Moscow, Fizmatlit Publ., 2005, 792 p.
2. Landau LD, Lifshitz EM. *The Classical theory of Fields*. Oxford, Butterworth-Heinemann, 1975, 402 p.
3. Perlick V. Gravitational lensing from a spacetime perspective. *Living Rev. Relativity* 2004 , 7:9.
4. Belayev WB. Application of Lagrange mechanics for analysis of the light-like particle motion in pseudo-Riemann space. *arXiv:0911.0614 [gr-qc]*.
5. Belayev WB. *Dinamika v obschei teorii otnositel'nosti: variacionn'ie metod'i* [The dynamics in general relativity theory: variational methods]. Moscow, URSS Publ., 2017, 216 p.
6. Tsipenyuk DYu, Andreev VA. Structure of extended space, *Bulletin of the Lebedev Physics Institute*, 2000, 6:23-34.
7. Tsipenyuk DYu, Andreev VA. Marriage of electromagnetism and gravity in an extended space model and astrophysical phenomena, Amoroso RL, Kauffman LH, Rowlands P (eds.) *Proc. 8th Vigier symposium, The physics of reality*. Singapore, World Scientific, 2013:171-182.
8. Andreev VA, Tsipenyuk DYu. Problema vvedeniya konechnogo razmera i peremennoy massy fotona [The problem of introducing finite size and variable mass of the photon]. *Inzhenernaya fisika*, 2017, 5:17-28 (in Russ.).
9. Rumer YuB. *Issledovaniya po pyatimernoy optike* [Studies on five-dimensional optics]. Moscow, URSS Publ., 2010, 152 p.
10. Tsipenyuk DYu, Andreev VA. Electromagnetic and gravitational fields in the 5-dimensional extended space model, their localization and interaction with matter. *RENSIT*, 2019, 11(2):93-102.
11. Tsipenyuk DYu, Belayev WB. Extended space model is consistent with the photon dynamics in the gravitational field. *J. Phys.: Conf. Ser*., 2019, 1251:012048.
12. Chernin AD, Dark energy in the nearby Universe: HST data, nonlinear theory, and computer simulations. *Phys. Usp.* 2013, 56:704-709.
13. Ellis J. Dark matter and dark energy: summary and future directions. *Phil. Trans. Roy. Soc. Lond. A* 2003, 361:2607.
14. Akrami Y, Kallosh R, Linde A, Vardanyan V. Dark energy, α-attractors, and large-scale structure surveys. *JCAP*, 2018, 1806:041.
15. Karachentsev ID, Telikova KN. Stellar and dark matter density in the Local Universe. *Astronomical notes*, 2018, 339:615.
16. Shafieloo A, L'Huillier B, Starobinsky AA. Falsifying ΛCDM: Model-independent tests of the concordance model with eBOSS DR14Q and Pantheon. *Phys. Rev. D* 2018, 98:083526.
17. Alam U, Sahni V, Saini TD, Starobinsky AA. Is there supernova evidence for dark energy metamorphosis? *Mon. Not. Roy. Astron. Soc.*, 2004, 354:275.
18. Bondi H. Negative mass in general relativity. Rev. Mod. Phys, 1957, 29:423-428.
19. Rivlin LA. Stopped light: Towards plane-wave-free electrodynamics. *Quantum Electronics*, 2003, 33:777-797.
20. Firstenberg O, Peyronel T, Liang Q-Y, Gorshkov AV, Lukin MD, Vuletić V. Attractive photons in a quantum nonlinear medium, *Nature*, 2013, 502:71-75.
21. Mazur PO, Mottola E. Gravitational vacuum condensate stars, *PNAS* 2004, 101: 9545.
22. Mazur PO, Mottola E. Cosmological dark energy: prospects for a dynamical theory. *Class. Quantum Grav.* 2015, 32:215024.
23. Berezin VA, Kuzmin VA, Tkachev II. Dynamics of bubbles in general relativity. *Phys. Rev. D*, 1987, 36:2919.
24. Pannia FAT, Bergliaffa SEP. Evolution of vacuum bubbles embedded in inhomogeneous spacetimes. *JCAP*, 201703:026.
25. Burinskii A. Superconducting source of the Kerr-Newman electron. Efremov AV, Goloskokov SV (eds.) *Proc. of the XIII Adv. Res. Workshop on HEP*, Dubna, 2010:439-442.

26. Fan X-H, Yong G-C, Zuo W. Probing nuclear bubble configurations by proton-induced reactions *Phys. Rev. C,* 2019, 99:041601(R).
27. Landau LD, Lifshitz EM. *Mechanics.* Oxford, Butterworth-Heinemann, 1982, 224 p.
28. Frolov VP. Generalized Fermat's principle and action for light rays in a curved spacetime. *Phys. Rev. D,* 201388:064039.
29. Okun LB. Photons and static gravity. *Mod. Phys. Lett. A,* 2000,15:1941.
30. Tolman RC. *Relativity Thermodynamics and Cosmology*. New York, Dover Publications, 2011, 528 p.
31. Kalaydzhyan T. Gravitational mass of positron from LEP synchrotron losses. *Sci. Rep.,* 2016, 6:30461.
32. Tsipenyuk DYu. Neizotropnye ob’ekty v rasshirennom prostranstve [Non-isotropic objects in extended space]. *Inzhenernaya fisika*, 2017, 6:20-28 (in Russ.).
33. Tsipenyuk DYu, Andreev VA. Obolochechnye ob’ekty v modeli rasshirennogo prostranstva i mekhanizm obrazovaniya temnoj materii [Shell objects in the model of extended space and the mechanism of formation of dark matter]. Kratkie Soobshcheniya po Fizike, 2004, 9:13-25 (in Russ.).
34. Visser M, Wiltshire DL. Stable gravastars - an alternative to black holes? *Class. Quant. Grav.,* 2004, 21:1135.
35. Cattoen C, Faber T,Visser M. Gravastars must have anisotropic pressures. *Class. Quant. Grav.,* 2005, 22:4189.
36. Chirenti C, Rezzolla L. On the ergoregion instability in rotating gravastars. *Phys. Rev. D,* 2008, 78:084011.
37. Chirenti C, Rezzolla L. Did GW150914 produce a rotating gravastar? *Phys. Rev. D,* 2016, 94:084016.
38. Cardoso V, Franzin E, Pani P. Is the gravitational-wave ringdown a probe of the event horizon? *Phys. Rev. Lett.,* 2016, 116:171101.
39. Bednarek W. Gamma-rays from the vicinity of accreting neutron stars inside compact high-mass X-ray binaries, *A&A,* 2009, 495(3):919 - 929.
40. Tsipenyuk DYu, Andreev VA. Dark matter and condensed bubble object formation in the 5d extended space model *Gravitat. Cosmol.* 2006, 12 (2-3):235-238.
41. Dehmelt HA. Single atomic particle forever floating at rest in free space: new value for electron radius. *Phys. Scr.* 1988, 22:102-110.