An entropy perspective of the pilot-wave theory in quantum mechanics

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Abstract: In this article, a modified picture of de Broglie-Bohm (dBB) picture is presented from the perspective of information exchange. The vacuum zero-point energy (ZPE) fluctuations are incorporated into the entropy because the interactions between the particle and ZPE should be treated similar to measurements in that they result in a pilot-wave. The inclusion of a term in the entropy resulting from mutual exchange of information between the particle and the vacuum (through ZPE) necessarily leads to the dBB picture.

Introduction: Quantum mechanics has been highly successful in the past decades. Indeed, many of its predictions have been successfully verified. However, several different ontological interpretations, which are often left out in the mathematical framework have led to inconsistent explanation of our objective reality. According to the orthodox Copenhagen interpretation [1], the state of a particle is before a measurement is represented by a wave function $\psi(x, t)$ with the probability given by the square of the wavefunction. The time-evolution of the wavefunction is governed by the Schrödinger equation. But, at the time of the measurement, the wavefunction suddenly "collapses" implying that there is no objective reality. This led to the famous quote "Is there moon when nobody looks?" [2]. The traditional approach is often supplemented with the ensemble interpretation [3]. As explained by Ballentine, the ensemble interpretation is often confused with measurement on a beam of particles rather than viewing the ensemble as a set of copies in the initial configuration space of the particle. In this sense, "the ensemble of quantum mechanics" should not be misinterpreted with "the ensemble of statistical mechanics". Heisenberg also combined Aristotlian notions to consider the particle as if it is potentially present everywhere [4]. Originally, in 1923 de Broglie proposed the first pilot-wave theory, which considers a quantum object such as an electron as a localized, vibrating particle moving in concert with a spatially extended, particle centered pilot wave [5]. Based on his physical picture, de Broglie successfully predicted single-particle effects such as diffraction and interference. Nevertheless, he did not detail the wave–particle interaction. Several years after de Broglie, David Bohm demonstrated that the guide wave could be viewed as a "phase" that provides information regarding the velocity of an initial configuration within the configuration space.
According to the stochastic interpretation, the state of the particle is described by some initial conditional probability density, whose gradual changes with time are governed by the second Kolmogorov equation [6].

In this sense, the "de Broglie-Bohm" (dBB) picture presents the best plausible interpretation of quantum mechanics. But, it uses the "action" in a way to describe pilot waves. In this article, a possibly new picture based on entropy maximization is suggested.

Theory: Consider a particle that moves "randomly" along the x-axis over a lattice of points with a step size $\delta x = c\Delta t$, where c is speed of light and Δt is the finite-time interval. By the time-energy uncertainty principle, virtual electron-positron pairs of mass m occur in the vacuum fluctuation during a time $\Delta t = \hbar/_{2mc^2}$. Here, this time interval should be viewed as the minimum amount of time required to change the energy (and hence information) of the particle by one standard deviation. If the probability of the particle stepping right is given by Γ, one may write the average displacement after X steps taken in a time t as $\langle x_X \rangle = (2\Gamma - 1)\delta x X = \frac{p}{m} t$, where *p* is the momentum and *m* is the mass of the particle. Thus, one could express $\langle x_X \rangle = (2\Gamma - 1)ct = \frac{p}{m} t$ or $\Gamma = \frac{1}{2}\left(\frac{p}{mc} + 1\right)$. Now, the binary Bernoulli entropy (S_{Binary}) of such a random walk is known to be

$$S_{Binary} = -\Gamma log\Gamma - (1-\Gamma)\log(1-\Gamma) \quad (1).$$

In equation (1), we attain an unbiased distribution ($\Gamma = 1/2$) only when $p = 0$ or the particle is at rest. For non-relativistic cases ($p \ll mc$), we can approximate the change in this entropy in time $\Delta t = \hbar/2mc^2$ using a Taylor series as

$$\Delta S_{Binary} = \frac{2}{\hbar}\left(mc^2 - \frac{p^2}{2m}\right) \quad (2).$$

The entropy may be further expanded to include the impetus (or momentum) imparted by the interaction between the particle and the vacuum zero-point energy Vo, which leads to the expression

$$\Delta S_{Binary} = \frac{2}{\hbar}\left(mc^2 - \left(\frac{p^2}{2m} + V_0\right)\right) \quad (3).$$

The evolution of the particle is determined by setting the time-derivative of entropy to be zero.

$$\frac{dS}{dt} = \frac{p}{2m}\frac{\partial p}{\partial t} + \frac{\partial V_o}{\partial t} = 0 \quad (4).$$

In the above equation, the fluctuation of vacuum-particle interaction is included in the so-called "free particle" case. In other words, there is no true "free particle" because any extended objected that exists (and thereby has extension) must interact the surroundings to be in its place. In this sense, a measurement is constantly being made through the vacuum-particle interaction. Now, using the second equation with a wavefunction and substituting the traditional operators, one can see that

$$\frac{-\hbar^2}{2m}\frac{\partial}{\partial x}\frac{\partial}{\partial t}\frac{\partial}{\partial x}\psi(x,t) + \frac{\partial V_o}{\partial t}\psi(x,t) = 0 \quad (5).$$

In equation (5), swapping time and space derivatives,

$$\frac{\partial}{\partial t}\left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)\right) + \frac{\partial V_o}{\partial t}\psi(x,t) = 0 \quad (6).$$

In case of a measurement, we change the particle's entropy by exchanging information with it. Thus, the net change in the particle's energy must be included in case of any other potential. We may write a general potential as an energy term including the vacuum-particle interaction.

$$V_{net} = V + V_o \quad (7).$$

Thus, we obtain

$$\frac{\partial}{\partial t}\left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)\right) + \frac{\partial V_{net}}{\partial t}\psi(x,t) = 0 \quad (8).$$

Using the fact that the total energy is conserved (including the vacuum-particle interaction), the total time-derivative of energy is zero.

This leads to the following

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V_{net}\psi(x) = E\psi(x)$$

(9).

Here, E is the total energy of the particle before measurement. This reproduces the standard time-independent Schrodinger wave equation. Thus, we could attempt to solve for the momentum of a free particle using the above equation itself.

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V_{net}\psi(x) = E\psi(x)$$

(10).

We obtain the wavefunction as:

$$\psi(x) = e^{\pm ikx} \qquad k = \sqrt{2m(E - V_{net})}$$

(11).

Or

$$\psi(x) = Re^{\pm Yx}$$

(12).

Here,

We changed $R(x) = e^{\pm ijx}$ $where\ j = \sqrt{2mE}$ and $Y = \sqrt{2mV_{net}}$

For a free particle, one can clearly see that (11) the term $e^{\pm zx}$serves as the pilot-wave arising from zero-point vacuum-particle interactions, where z is a constant. Thus, it is possible to use the entropy picture to see that the so-called guide or pilot wave in the dBB picture arises from entropy or mutual information exchange between particle and vacuum.

Conclusions: Using the entropy picture, one can interpret the pilot-wave in the de Broglie Bohm picture as arising mainly from vacuum-particle interactions.

References:

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