Energy density of a vacuum observed by background radiation

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In the paper zero-point energy density of free photons is estimated for an empty space surrounded by – and observed by – a bath of thermal background photons. Interpreting the results, the outline of the cosmological arrow of time is suggested. [1]

I. INTRODUCTION

The problem of understanding vacuum energy within scopes of established physical theories has been troubling physics already for many decades. Even before 1998, when our Universe was observed to be undergoing an *accelerated* expansion, the question of zero-point energy of various quantum field theories has sparkled plenty ideas with no widely recognized resolution [2]. Initially attracting purely theoretical search for a mechanism of cancellation of this potentially mathematical artifact of QFTs, the evidence for the existence of the so-called dark energy has moved the problem to the front of physics research.

Interpreting experimental evidence through well-tested and overall brilliant theory of general relativity leads us to believe that dark energy is most likely of a cosmological-constant type, possibly varying with time. Paradoxically – using the otherwise well-established techniques of quantum theories describing constituents of our universe – our closest estimate of such quantity is very many orders of magnitude too large [3]. A more naïve and often mentioned calculation results in an even worse over-estimation at over 120 orders of magnitude.

The goal of this paper is to present a vacuum model thought to be relevant for the majority of space within our universe. Calculations are performed by standard techniques only, with the purpose of supporting the core idea that zero-point energies of various quantum fields could add up to meaningfully describe the structure of dark energy. The focus is on the free photon field, and its propagator is the main object. The use and reinterpretations of a propagator are central to this work.

Throughout the paper the term *vacuum* is to be understood dynamically – it is the interacting, 'physical' vacuum. It models space appearing empty to direct observation, i. e. it contains no real particle and any signal passes through necessarily unaffected. In other words: a background, *observing* signal, must not interact with the vacuum in a visible way – however not so for any lack of coupling. It is this dynamic vacuum allowing vacuum fluctuations, that is believed to be a suitable model candidate for dark constituents of our universe. It should not be confused with the static free-theory state $|0\rangle$.

II. OBSERVED VACUUM

Instead of dealing with *vacuum per se* and trying to understand zero-point energy of quantum fields without context, let us try to focus on its behavior in voids between galaxies as background signals pass through it. These signals in principle consist of various cosmic particles and rays, anything emitted by nearby galaxies or clusters of galaxies, cosmic neutrino background, and cosmic microwave background radiation. For simplicity we first neglect all but the latter, considering the effect of background neutrinos only towards the end of the paper.

Next, we arrive at the key point. A background photon has a finite coherence length and can therefore be thought as present at some point for a finite time τ. Any transient phenomenon within vacuum must therefore already disappear by itself in time τ, or else some background photon might interact with it, observe it. In such a turn-out photon's momentum would have been changed by the vacuum, meaning the latter can no longer be considered as *empty*, i. e. as void of any real particle. Note that such a photon need not be further absorbed in some experimental apparatus and e. g. some conscious observation to take place. The very moment the background photon is diverted, an observation of *something* can be considered to have taken place. Our task remains to appropriately restrict phenomena within vacuum in order for such potential observations to not be manifestable. In other words, we want the background photons to observe *nothing*. For CMBR at temperature $T = 2.7$ K, time τ is around a few picoseconds.

In the following, the vacuum energy density will be estimated using three methods with an increasing level of details. The coupling between real background photons and virtual photons – vacuum fluctuations transiently populating space, is never microscopically modeled. Rather, the consideration of the previous paragraph is explicitly factored into equations by hand. In short, we continue answering what vacuum can consist of, assuming the background photons notice nothing in it.

A. Time-energy uncertainty

As we would like to describe phenomena lasting less than time τ, the first estimate can be derived from the time-energy uncertainty relation. An observation taking place for an amount of time δt can determine the energy of an observed system with an accuracy of δE satisfying the uncertainty inequality

$$\delta E \cdot \delta t > \frac{1}{2} \cdot 2\pi\hbar,$$

with the reduced Planck constant $\hbar$. Note that this is not so much a Heisenberg-like relation, rather it is a consequence of a finite bandwidth resolution of time-limited signals. If the observed energy is to be zero, $\langle E\rangle = 0$, mean fluctuations $\sqrt{\langle E^2\rangle} = \sqrt{\langle E^2\rangle - \langle E\rangle^2} = \delta E$ must not exceed $\dfrac{\pi\hbar}{\delta t}$. For a vacuum observed by the background this means an energy $\frac{\pi\hbar}{\tau}$ per polarization can be hidden in volume $\frac{4\pi}{3}(\frac{c\tau}{2})^3$, yielding energy density

$$\mathcal{E} \approx 12 \cdot \frac{\hbar c}{(c\tau)^4},$$

with the speed of light c. In the current Universe this amounts to around $2{\cdot}10^{-4}\,\frac{\mathrm{eV}}{\mathrm{mm}^3}$. For comparison, the critical density is measured at approximately $5\,\frac{\mathrm{eV}}{\mathrm{mm}^3}$, around 70 % of which is posited to be dark energy. There currently seem to be no full agreement on the measurement across different methods, and it thus continues to be one of the central tasks of physical cosmology.

The energy density $\mathcal{E}$ depends on the temperature of the background radiation, and decreases with the scale factor in the same way as a real radiation energy density does. It nonetheless describes a cosmological constant type of energy, as the scale-factor dependency should be seen as an indirect one: the above result has been derived as the energy density of space itself – with the 'size' of background photons serving merely as a measure. Note further, that approximating a single background photon with the volume $\sim(c\tau)^3$, the space is roughly tiled by them without much overlay. A single microscopic region of space can therefore be assumed to only be observed by one background photon at a time.

In the following any such notion of volume is no longer used: the energy density will be computed without such averaging, focusing on the allowed duration τ of vacuum fluctuations only.

B. Vacuum fluctuations

To describe a part of dark energy, we now focus on understanding the free-photon sector of vacuum. Vacuum fluctuations can be explored through a massless scalar quantum field $\phi = \phi\,(\vec{r},\,t)$. In the following, fluctuations of ϕ persisting some time t are modeled with

$$\phi\,(\,\cdot\,,\,t)\;\phi\,(0,\,0),$$

i. e. originating at the origin. *Spatial amplitude of a time t enduring vacuum fluctuation*, δ_t, is then expressed by

$$\delta_t(\vec{r}) = \langle 0|\,\phi\,(\vec{r},\,t)\;\phi\,(0,\,0)\,|0\rangle\,.$$

In the free-photon case this describes one component. As $t > 0$, the fields are already time-ordered, and δ_t can be evaluated as the Feynman propagator. The massless propagator in position space in natural units reads [4]

$$\delta_t(\vec{r}) = G^{\mathrm{F}}(\vec{r},\,t) \equiv -\frac{1}{4\pi^2}\,\frac{1}{t^2 - \|\vec{r}\|^2 - i\,0^+},$$

with 0^+ denoting the prescription for integration in a complex plane. Next, with standard units restored, δ_t in momentum space – i. e. the *spectral amplitude* – reads

$$\tilde{\delta}_t(\mathbf{k}) = \frac{e^{-ickt}}{2i\,c\,(k - i\,0^+)},$$

where $k = \|\mathbf{k}\|$. This result can be derived with a spatial Fourier transformation of G^{F}, or an inverse temporal Fourier transformation of $\tilde{G}^{\mathrm{F}}$. Associated integrals converge upon appropriate Wick rotations. The expression for $\tilde{\delta}_t$ also arises by taking a zero-mass limit of Fourier transformed massive propagators.

The vacuum energy density can now be estimated as

$$\mathcal{E} = 2 \cdot \int_0^\tau \frac{\mathrm{d}t}{\tau}\,\frac{t}{\tau}\left|\left(\frac{\mathcal{V}(\mathcal{B}_{\frac{2\pi}{ct}})}{\mathcal{V}(\mathcal{B}_{\frac{2\pi}{c\tau}})}\right)^{-1}\int\limits_{\mathbf{k}\,\in\,\mathcal{B}_{\frac{2\pi}{ct}}}\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\,\frac{\tilde{\delta}_t(\mathbf{k})}{\frac{\tau}{36}}\,\varepsilon_{\mathbf{k}}\right|.$$

The overall factor 2 counts the two independent EM polarizations. Vacuum fluctuations persisting for time t are taken into account up to the cut-off τ imposed by the observing background signal. Further, the contribution of a particular VF is weighted proportionally to its duration. In order for the single-mode zero-point energy $\varepsilon_{\mathbf{k}} = \frac{1}{2}\hbar\omega_{\mathbf{k}} = \frac{1}{2}\hbar ck$ not to exceed the amount $\frac{\pi\hbar}{t}$ as suggested by the time-energy uncertainty relation, $\mathbf{k}$ is only taken to be applicable within a finite ball of radius $\frac{2\pi}{ct}$, denoted by $\mathcal{B}_{\frac{2\pi}{ct}}$. Similarly to the duration weight, the contribution is further reduced by the volume $\mathcal{V}$ of the integration region. Finally, the spectral amplitude of VFs is normalized by $\frac{\tau}{36}$ in order to serve as a probability amplitude, i. e. fixing an average number of *two* fluctuations to be present in a volume $\frac{4\pi}{3}(\frac{c\tau}{2})^3$. The result of this section is obtained by a straight-forward integration,

$$\begin{aligned}\mathcal{E} &= \frac{36\,\hbar}{4\,\pi^2\,\tau^6}\int_0^\tau \mathrm{d}t\;t^4\left|\int_0^{\frac{2\pi}{ct}} k^2\,\frac{e^{-ickt}}{i}\,\mathrm{d}k\right| \\ &= \frac{36\,\hbar c}{4\,\pi^2\,\tau^6\,c^4}\int_0^\tau \mathrm{d}t\;t\cdot\bigl|4\pi(\pi - i)\bigr| \\ &= \frac{36\sqrt{1+\pi^2}}{2\,\pi}\,\frac{\hbar c}{(c\tau)^4} \approx 18\cdot\frac{\hbar c}{(c\tau)^4} = 3\cdot 10^{-4}\,\frac{\mathrm{eV}}{\mathrm{mm}^3}.\end{aligned}$$

Due to translational symmetry, $\mathcal{E}$ is an energy density at any point in space, even though the derivation is strictly speaking only laid out for fluctuations around the origin. The result is of the same form as – and is comparable to – the one obtained in the previous section.

Two cut-offs have been introduced in this approach, related to restrictions imposed by the presence of the background radiation. Note however, there is no absolute cut of momenta: as $t \to 0$, the allowed $\mathbf{k}$ region grows and $k \to \infty$. Nonetheless, distinct $\mathbf{k}$-modes are *bosonic* forms of energy, pronouncing such one-or-zero treatments incomplete. In our next and final attempt we therefore utilize methods of statistical description. These can as well be employed for the temporal part, replacing its τ related cut-off. The background is here onward thus understood primarily as a thermal bath.

C. Thermal vacuum

In this section, a vacuum in thermal equilibrium with the background (CMB) radiation, is explored. Fluctuations of the field ϕ are modeled in the same way as before, i. e. $\phi\,(\,\cdot\,,\,t)\;\phi\,(0,\,0)$. With the goal of losing the notion of time, we now however characterize them by their *energy*, denoted by ε. Note that this is achieved through substitution, $t \mapsto \dfrac{\pi\hbar}{\varepsilon}$, and does not introduce any additional Fourier transformation. With standard units and a normalization constant $\tilde{\delta} \approx 244$, the amplitude of a fluctuation of energy ε in momentum space reads

$$\tilde{\delta}_\varepsilon(\mathbf{k}) = \frac{\tilde{\delta}\, e^{-i\pi\hbar ck/\varepsilon}}{2i\,\hbar c\,(k - i\,0^+)}.$$

Its unit is that of inverse energy. It can thus serve as *density of energy states* (DoS), which is also how it is applied to the problem in the following. The internal energy of a system in thermal equilibrium can be expressed as

$$U = \int \mathrm{d}E\, f(E)\, d(E)\, E,$$

where d is the DoS and $f \in \{f^{\mathrm{FD}},\, f^{\mathrm{BE}}\}$ a Fermi-Dirac or Bose-Einstein probability distribution function. The average number of fluctuations is similarly computed through $\int \mathrm{d}E\, f(E)\, d(E)$, and the constant $\tilde{\delta}$ is determined such, that again *two* differently polarized VFs are found on average per unit volume, now $\frac{4\pi}{3}(\frac{\beta\hbar c}{2})^3$.

Energy density of *thermal vacuum* now reads

$$\begin{aligned}\mathcal{E} &= \int \mathrm{d}\varepsilon\, f^{\mathrm{FD}}_\mu(\varepsilon)\, d(\varepsilon)\;\varepsilon \\ &= \int \mathrm{d}\varepsilon\, f^{\mathrm{FD}}_\mu(\varepsilon)\, d_\varepsilon \cdot \left| \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\, f^{\mathrm{BE}}(\varepsilon_\mathbf{k})\, \tilde{\delta}_\varepsilon(\mathbf{k})\, \varepsilon_\mathbf{k} \right|.\end{aligned}$$

Here $d_\varepsilon = \frac{2\mu}{\varepsilon}$ is the duration-related degeneracy, with the factor 2 counting EM-polarizations, and $\varepsilon_\mathbf{k} = \frac{\hbar ck}{2}$ is again the single-mode zero-point energy. Furthermore, the chemical potential stands $\mu = \frac{1}{2\beta} = \frac{k_{\mathrm{B}}T}{2}$ as per equipartition theorem, with Boltzmann constant k_{B}. The temperature T matches that of the background and is the sole temperature appearing in the equation. As expected, the bosonic degrees of freedom, i. e. single $\mathbf{k}$ modes, are distributed according to Bose-Einstein statistics. On the contrary, single ε modes of VFs are *fermionic*, which may for now be supported by the following two remarks. Firstly, this segment is associated with the temporal characterization, and time itself is rather 'anti-symmetric'. Secondly, just as a pair of fermions can behave bosonically, so too can an infinite collective of bosons behave fermionically. In other words: at some point in space-time, a fluctuation of a particular energy ε and fixed polarization is either present or it is not.

The final evaluation – the calculation of thermal free-photon zero-point energy density – can now be carried out as

$$\begin{aligned}\mathcal{E} = &\int \mathrm{d}\varepsilon\, \frac{1}{e^{\beta(\varepsilon-\mu)}+1}\, \frac{2\mu}{\varepsilon} \cdot \\ &\cdot \left| \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\, \frac{1}{e^{\beta\hbar ck/2}-1}\, \frac{\tilde{\delta}\, e^{-i\pi\hbar ck/\varepsilon}}{2i\,\hbar c\,(k - i\,0^+)}\, \frac{\hbar ck}{2} \right| \\ = &\int_0^\infty \mathrm{d}\varepsilon\, \frac{1}{e^{\beta\varepsilon - 1/2}+1}\, \frac{1}{\beta\varepsilon} \cdot \left| \int_0^\infty \frac{\mathrm{d}k\, k^2}{8\pi^2}\, \frac{\tilde{\delta}\, e^{-i\pi\hbar ck/\varepsilon}}{e^{\beta\hbar ck/2}-1} \right| \\ = &\int_0^\infty \mathrm{d}\varepsilon\, \frac{1}{e^{\beta\varepsilon - 1/2}+1}\, \frac{1}{\beta\varepsilon} \cdot \frac{2\,\tilde{\delta}}{(\beta\hbar c)^3\pi^2} \left| \zeta^{\mathrm{H}}\left(3, 1 + \frac{2\pi i}{\beta\varepsilon}\right) \right| \\ \approx\, &0.8 \cdot \frac{\hbar c}{(\beta\hbar c)^4} = 3 \cdot 10^{-4}\, \frac{\mathrm{eV}}{\mathrm{mm}^3},\end{aligned}$$

confirming the previous result. Again, the critical energy density of the Universe is approximately $5\,\frac{\mathrm{eV}}{\mathrm{mm}^3}$. Note that in this approach the single-modes of fluctuations have still been taken as on-shell and a derivation of a more qft-complete result is left to the reader. This approach also calls for an updated view of what a potential observation of something real in vacuum would be – fluctuations of low energy are taken as possibly lasting longer than τ. In the initial consideration any potential interaction was taken as changing the observing signal's momentum. This is however not necessarily the case, since there is a non-zero amplitude for signal's non-scattered traversal through space even with interactions present. In this sense the analysis here has been performed up to first order of perturbation theory, where non-scattered traversal is equivalent to the trivial Feynman diagram.

III. INTERPRETATION & MONOLOGUE

Let us first point out that the results of the three employed methods match well – in a sense even suspiciously well. The main prediction for present day free-photon vacuum energy density, $3\cdot 10^{-4}\,\frac{\mathrm{eV}}{\mathrm{mm}^3}$, accounts for $6\cdot 10^{-5}$ of Universe's total density. This fraction further matches the portion of *real* free-photon energy density, namely CMB radiation. In this way, the structure of the vacuum resembles the structure of the observable universe, supporting the claim that *a universe is nothing but nucleated vacuum and some more vacuum.* Dark energy then represents the uncondensed part. In terms of 'particles', the free-photon sector can be described as a one-dimensional free Fermi gas of vacuum fluctuations, with Fermi energy $E_{\mathrm{F}} = 0$. This is due to VF's spherically symmetrical structure, with only one dimension remaining physically relevant, and effective quadratic dispersion arising from $|\mathrm{d}^3\mathbf{k}\,\tilde{\delta}_\mathbf{k}\varepsilon_\mathbf{k}| \rightsquigarrow \mathrm{d}k\, k^2$. A zero Fermi energy is also consistent with the energy of radiation fluctuations being zero at absolute zero. Furthermore, there are *two* independent photon fluctuations due to two independent EM polarizations – matching the internal degree of freedom of spin-$\frac{1}{2}$ particles. This is also the final remark on why fluctuations have been treated as fermions. Other parts of dark energy, arising due to massive fields, are of course expected to behave differently.

The microscopic structure of dark constituents of our universe should rightfully be expected to be just as rich as its observable part. All known quantum fields are present there, so the ideas of describing dark energy and dark matter as 'single forms of something' seem rather naïve. Note also, that short-lived fluctuations can behave as massless, even if arising from underlying massive fields. Apart from free fields, interactions among them also contribute to the vacuum energy. Since already material materials are known to exhibit a vast array of phenomena – despite only consisting of four fundamental particles – only imagination is the limit to what the vivid Standard model's vacuum can conceive of. As an example, electron-field fluctuations can be considered. Whereas fluctuations in a flat vacuum can be expected to occur as electron-positron *pair* creation and annihilation, an additional form can be present in a bound system. Namely, when a particle is associated with an anti-particle of *another pair*. Such bound chain-like fluctuations then effectively behave as long-lived, forming Cooper-like bosonic pairs. In this way separate fermionic fluctuations are able to condense into a massive superfluid, contributing to dark matter. *Vacuum's behavior depends on the background.* This could further be a reason for why different measurements of the expansion mismatch: the Universe is not completely isotropic.

As the universe ages, expands, and cools, the contribution of free photons to dark energy grows less and less significant, however other sectors could behave differently and grow stronger in emptier space. It is at this point important to note that the neutrino background affects evolution of vacuum in different ways to that of background photons. Despite both being observing radiations, the fact that the latter is bosonic of a single kind while the former is fermionic with multiple flavors, results in a crucial difference. Whereas photons observe by absorption, neutrinos observe by *tadpoling* – namely simply through Z, or with a possible change of form through $W^{\pm}$. It is in this sense, that the vacuum is weakly-interacting. Moreover, passing through space, neutrinos have already been experimentally observed to be changing flavor, which could also be seen as yet another indirect proof of vacuum not being nothing, despite being void of real particles. It can further be expected, that exactly such weak interactions break up the superfluid component of vacuum close to stars and a reason why dark matter is not found nearby. In darker – or better yet, less neutrinic – parts of galaxies and within galaxy-clusters, where vacuum is gravitationally bound yet weakly-less-disrupted, the massive, superfluid, dark-matter component of vacuum is more likely. Furthermore, with density of neutrinos diminishing with the age of a universe, its vacuum is less and less disturbed by them, which could result in an increase of density of dark energy with time. While renormalization of various classes of diagrams is appropriate for understanding high-energy scattering processes, they are *not* to be deemed as just unphysical artifacts of QFTs. There is literally a whole universe down at low-energies yet to be discovered: hidden of course in the dark, for an empty space is a perfect *black* body. It reflects nothing, and absorbs everything – emitting it isotropically as per Huygens' principle. This is finally the fully-interacting picture: a signal passing through space in an unobstructed manner, the way the dressed-photon propagator describes it. With QFT on our hands, we are able to describe the immaterial aether for what it really is.

Finally, a view of the cosmological arrow of time can be offered. It is in a sense both periodic *and* directional. The starting point of description is therefore freely chosen – let us start at the present moment. As dark energy expands the universe, the density of its real constituents drops. With a predicted increase of density of dark energy, the acceleration of expansion accelerates. With nearby clusters of galaxies pushed beyond horizon, it is the vast voids of the cosmic web that are first emptied of any large-scale inhomogeneities. In this way the entropy of a universe drops. A lone void can already be seen as a start of a next new universe. With an expansion getting faster and faster it continues functioning as inflation, while the horizon shrinks and starts heating the new universe within. Any remaining rogue particles serve as nucleation sites, and when the universe is sufficiently heated, new matter starts to form – relic neutrinos further breaking the matter-antimatter symmetry. As matter forms, density of dark energy drops, until the inflationary period stops. Big Bang. The rest is history. The vacuum starts condensing as it cools: first due to fundamental forces and gravity, then through chemistry.

IV. CONCLUSION & DISCUSSION

The aim of this paper was to show that some more understanding could still be developed at the basics of our current and well-tested theories, with a hopefully interesting direction provided. Comments or continuations are therefore very welcome. Please note, that the paper was never intended to pose as a full theory. For one, there is no analysis of Lorentz (in)variance of any proposed equation. The presence of a finite horizon and a finite temperature can be used to determine the frame at rest, in which the energies are computed.

The results themselves do not justify a theory, yet the estimates seem to be the best result for an energy density of a dark energy candidate to date, with no proposition of new particles being made.

Maybe understanding vacuum fluctuations is all the new physics we need?

[1] Wikipedia. https://en.wikipedia.org.
[2] Weinberg. *Rev. Mod. Phys.*, 61, 1989.
[3] Martin. 1205.3365, 2012.
[4] Zhang *et al.* 0811.1261, 2008.