CONJECTURES ABOUT THE DIFFERENCE OF THE SEQUENCE OF RADICALS OF THE NATURAL NUMBERS

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What is the definition of radical of a number?

The fundamental theorem of arithmetic, also called the unique factorization theorem or the unique-prime-factorization theorem, states that every integer greater than 1 either is a prime number itself or can be represented as the product of prime numbers and that, moreover, this representation is unique, up to (except for) the order of the factors. So let $n = p_1{}^a * p_2{}^b * p_3{}^c....$ then

$\mathrm{rad}(n) = p_1 * p_2 * p_3.....$

Suppose to apply rad function to the natural numbers. In OEIS you will find this sequence with number A007947 but I started it with second term or 2 because 1 is excluded from the arithmetic theorem having different properties:

2,3,2,5,6,7,2,3,10,11,6,13,14,15,2,17,6,19,10,21,22,23,6,5,26,3,14,29,30,31,2,33,34,35,6,37,38,39,10,41,42,43,22,15,46,47,6,7,10,51,26,53,6,55,14,57,58,59,30,61,...

This sequence is quite unpredictable but more can be conjectured on the difference sequence. The operator difference simply substitute an item in the sequence with the difference between the following item and the item itself for every item; it's like a derivative of the sequence. In OEIS you will find this sequence with number A076334:

1,-1,3,1,1,-5,1,7,1,-5,7,1,1,-13,15,-11,13,-9,11,1,1,-17,-1,21,-23,11,15,1,1,-29,31,1,1,-29,31,1,1,-29,31,1,1,-21,-7,31,1,-41,1,3,41,-25,27,-47,49,-41,43,1,1,-29,31,1,-41,-19,63,1,1,...

At the first sight it seems that this step added more variability with the introduction of negative items. But I found an algorithm that assign a structure to this sequence.

Algorithm:

1) read an item, if this is 1 write it and goto step 1, if not **it will be a negative item** that starts a subsequence, save it and goto step 2

2) add every new item to the subsequence **until the sum of the subsequence equals the number of items of that subsequence**. In this case write the subsequence and goto step 1

The sequence above can be written, following the algorithm as:

1,{-1,3},1,1,{-5,1,7},1,{-5,7},1,1,{-13,15},{-11,13},{-9,11},1,1,{-17,-1,21},{-23,11,15},1,1,{-29,31},1,1,{-29,31},1,1,{-29,31},1,1,{-21,-7,31},1,{-41,1,3,41},{-25,27},{-47,49},{-

41,43},1,1,{-29,31},1,{-41,-19,63},1,1,...

So the conjectures about this structure stated until now are the following:

Conjecture 1

The algorithm can be applied forever to this sequence.

Conjecture 2

The consecutives 1s outside the subsequences can be **one or two**.

Conjecture 3

If you search the position of the prime numbers of the original sequence in the derived one, **all corresponds to the first one outside the subsequences or to the first item of a subsequence.**