CONJECTURES ABOUT THE DIFFERENCE OF THE SEQUENCE OF RADICALS OF FIRST GRADE POLYNOMIAL SEQUENCES

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What is the definition of radical of a number?

The fundamental theorem of arithmetic, also called the unique factorization theorem or the unique-prime-factorization theorem, states that every integer greater than 1 either is a prime number itself or can be represented as the product of prime numbers and that, moreover, this representation is unique, up to (except for) the order of the factors. So let $n = p_1^a * p_2^b * p_3^c ....$ then

$\mathrm{rad}(n) = p_1 * p_2 * p_3 .....$

What is a first grade polynomial sequence:

Is the sequence s(n)=a*n+b where n goes from 0 to Infinity and b<a are two positive natural numbers.

Sequence s(n)=3n+2 (OEIS number A016789) can be expressed as:

2,5,8,11,14,17,20,23,26,29,32,35,38,41,44,47,50,53,56,59,62,65,68,71,74,77,80,83,86,89,92,95,98,101,104,107,110,113,116,119,122,125,128,131,134,137,140,143,146,149,152,155,158,161,164,167,170,173,176,179,182,185,...

Applying rad function to every item we have:

2,5,2,11,14,17,10,23,26,29,2,35,38,41,22,47,10,53,14,59,62,65,34,71,74,77,10,83,86,89,46,95,14,101,6,107,110,113,58,119,122,5,2,131,134,137,70,143,146,149,38,155,158,161,82,167,170,173,22,179,182,185,...

Applying now the difference operator that change every item with the difference between next and current item we have:

3,-3,9,3,3,-7,13,3,3,-27,33,3,3,-19,25,-37,43,-39,45,3,3,-31,37,3,3,-67,73,3,3,-43,49,-81,87,-75,81,3,3,-55,61,3,-117,-3,129,3,3,-67,73,3,3,-111,117,3,3,-79,85,3,3,-151,157,3,3,-91,97,3,3,-187,193,3,3,-103,109,3,3,-207,213,3,3,-115,121,-217,13,27,189,3,3,-127,133,3,3,...

If you apply the same steps to sequence s(n)=3n+1 (OEIS number A016777) you will get:

1,5,3,3,-11,17,3,-17,9,17,3,3,-27,33,3,-39,19,29,3,3,-59,65,3,3,-35,41,3,3,-63,69,3,3,-87,93,3,3,-95,101,3,-107,51,65,3,3,-99,105,3,3,-71,77,3,3,-147,153,3,-153,73,-51,143,3,-135,14

1,3,3,-179,185,3,3,-179,185,3,3,-107,113,3,3,-171,177,3,3,-119,125,-237,243,-251,...

You can extend to sequences like s(n)=4n+1(OEIS A016813), s(n)=4n+2(OEIS A016825), s(n)=4n+3(OEIS A004767) and the outputs are:

4,-2,10,4,4,-16,24,4,4,4,-26,-8,46,4,4,4,4,4,4,-74,82,4,4,4,4,4,4,4,-74,-28,-6,124,4,4,4,4,4,-98,106,4,4,-152,160,4,4,4,-164,172,4,4,4,4,4,4,4,-206,214,4,4,4,-206,214,4,4,-170,178,4,4,4,4,4,-268,276,-260,268,4,4,4,4,4,-256,264,-218,226,4,4,4,4,4,-338,346,-242,250,...

4,4,4,-8,16,4,4,4,4,4,4,-36,-4,52,4,4,4,4,4,4,4,-56,64,-80,88,4,4,4,4,4,-80,88,4,4,4,4,-116,124,4,-152,160,4,4,4,4,4,4,4,-128,136,4,4,4,4,4,4,4,-152,160,-216,224,-236,244,4,4,4,-236,244,4,4,4,4,-248,256,4,-200,208,4,4,4,4,4,4,-308,88,232,-276,284,4,4,4,4,4,-332,340,...

4,4,4,4,4,-20,28,4,4,4,4,4,4,4,-38,46,4,-56,64,4,4,4,4,-62,70,4,4,4,4,4,4,4,-116,124,4,-122,130,4,4,4,4,-110,-22,144,4,4,4,4,4,4,-134,142,4,4,4,4,4,4,4,-236,244,4,4,4,4,4,4,-216,38,190,4,4,4,4,4,4,4,-206,214,4,4,4,4,4,-332,340,-308,316,4,-326,334,4,-356,364,4,-254,262,...

Now you can apply the following algorithm to each of these sequences:

Algorithm for sequence s(n)=a*n+b:

1) read an item, if this is equal to **a**, write it and go to step 1, if not, **it will be a negative item** that starts a subsequence, save it and go to step 2

2) add every new item to the subsequence **until the sum of the subsequence equals the number of items of that subsequence multiplied by a**. In this case write the subsequence and go to step 1

Above sequences can be written, following the algorithm as:

3,{-3,9},3,3,{-7,13},3,3,{-27,33},3,3,{-19,25},{-37,43},{-39,45},3,3,{-31,37},3,3,{-67,73},3,3,{-43,49},{-81,87},{-75,81},3,3,{-55,61},3,{-117,-3,129},3,3,{-67,73},3,3,{-111,117},3,3,{-79,85},3,3,{-151,157},3,3,{-91,97},3,3,{-187,193},3,3,{-103,109},3,3,{-207,213},3,3,{-115,121},...

{1,5},3,3,{-11,17},3,{-17,9,17},3,3,{-27,33},3,{-39,19,29},3,3,{-59,65},3,3,{-35,41},3,3,{-63,69},3,3,{-87,93},3,3,{-95,101},3,{-107,51,65},3,3,{-99,105},3,3,{-71,77},3,3,{-147,153},3,{-153,73,-51,143},3,{-135,141},3,3,{-179,185},3,3,{-179,185},3,3,

4,{-2,10},4,4,{-16,24},4,4,4,{-26,-8,46},4,4,4,4,4,4,{-74,82},4,4,4,4,4,4,4,{-74,-28,-6,124},4,4,4,4,4,{-98,106},4,4,{-152,160},4,4,4,{-164,172},4,4,4,4,4,4,4,{-206,214},4,4,4,{-206,214},4,4,{-170,178},4,4,4,4,4,{-268,276},{-260,268},4,4,4,4,4,...

4,4,4,{-8,16},4,4,4,4,4,4,{-36,-4,52},4,4,4,4,4,4,4,{-56,64},{-80,88},4,4,4,4,4,{-80,88},4,4,4,4,{-116,124},4,{-152,160},4,4,4,4,4,4,4,{-128,136},4,4,4,4,4,4,4,{-152,160},{-216,224},{-236,244},4,4,4,{-236,244},4,4,4,4,{-248,256},4,{-200,208},4,4,4,4,4,4,...

4,4,4,4,4,{-20,28},4,4,4,4,4,4,4,{-38,46},4,{-56,64},4,4,4,4,{-62,70},4,4,4,4,4,4,4,{-116,124},4,{-122,130},4,4,4,4,{-110,-22,144},4,4,4,4,4,4,{-134,142},4,4,4,4,4,4,4,{-236,244},4,4,4,4,4,4,{-216,38,190},4,4,4,4,4,4,4,{-206,214},4,4,4,4,4,{-332,340},{-308,316},...

So the conjectures about this structure stated until now are the following:

Conjecture 1

The algorithm can be applied forever to this sequence.

Conjecture 2

The consecutives a, outside the subsequences, are in the range from 1 to r(k+1) where r is sequence Prime(n)^2-2 (A049001):

2, 7, 23, 47, 119, 167, 287, 359, 527, 839, 959, 1367, 1679,...

To determine the value of k consider the length of the first primes that are divisors of a. For example:

- for every odd number, k=0 then r(k+1)=r(1)=2
- for ever even number not dividing 3, k=1 then r(k+1)=r(2)=7
- for every number dividing 6 but not 5, k=2 then r(k+1)=r(3)=23
- for every number dividing 30 but not 7, k=3 then r(k+1)=r(4)=47
- ...

Conjecture 3

If you search the position of the prime numbers of the original sequence in the derived one, if a is odd **all correspond to the first one outside the subsequences or to the first item of a subsequence.**

If a is even, outside the subsequences every position can be found but **inside the subsequences only first item can be found**.