On the Significance of Planck's Constant

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Abstract

Planck's constant h has dimensions of energy (ergs) × time (seconds) called "actions." Action is a four-dimensional constant and is the same size for all observers in special relativity, even if those observers disagree as to the size of the energy and time components of that action. The various guessed at formalisms of non-relativistic quantum mechanics introduce h arbitrarily, empirically. But being inconsistent with special relativity would seem to render that *ad hoc* introduction out of context. In this brief article we shall avoid this inconsistency and instead directly generalise the Planck-Einstein relation $E = hf$ to reveal a new mass-free paradigm of Relativity where tidal effects are local, which will leave little of the classical physics that preceded it. Hence, we historically avoid the misguided advent of non-relativistic quantum mechanics altogether and finally reveal the true significance of h. The maths employed here can be found in [1] and [2] as well as all references used.

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1. Introduction

In this article we use relativistic mass to generalise $E = hf$, before we reason our way to the ultimate conclusion that mass never *really* existed. What we will end up with are the foundations for a mass-free paradigm of Relativity. It will be shown that there is a natural evolution from classical physics to Relativity where the significance of Planck's four-dimensional constant will be apparent.

2. The Light

We begin with the fact that many practical applications of special relativity are found in the theory of relativistic mechanics. The basis of relativistic mechanics is the following four equations

$$M = m_0/\sqrt{1 - (v/c)^2} \qquad (1)$$

which is a function of velocity v where M is the relativistic mass, m_0 the rest mass and c the speed of light in a vacuum. Multiplying both sides of Eq. (1) by the velocity vector $\boldsymbol{v}$ gives us the expression for relativistic momentum

$$\boldsymbol{p} = M\boldsymbol{v} = m_0\boldsymbol{v}/\sqrt{1 - (v/c)^2} \qquad (2)$$

and multiplying both sides of Eq. (1) by c^2 gives the total energy of a particle

$$E = Mc^2 = m_0c^2/\sqrt{1 - (v/c)^2} \qquad (3)$$

Ignoring M in Eqs. (2) and (3) and combining the two equations gives us

$$E = \sqrt{(m_0c^2)^2 + (pc)^2}$$

where we have ignored the negative root, which we shall justify below. *Of these four equations only Eqs. (2) and (3) are theoretically necessary where we arbitrarily ignore M, and as the basis of relativistic dynamics they are routinely confirmed in elementary-particle physics.*

We now introduce the energy of a photon $E = hf$ where f is frequency, and equating this with the expression above we get

$$hf = \sqrt{(m_0c^2)^2 + (pc)^2}$$

Now, if $m_0 = 0$ then using $f = c/\lambda$ we derive de Broglie's hypothesis

$$\lambda = h/p \qquad (4)$$

where λ is wavelength, and if $p = 0$ we similarly derive the Compton wavelength

$$\lambda_c = h/m_0 c \qquad (5)$$

Historically this much was known before 1925, with the possible exception of the derivation of Eq. (5) as presented here. To generalise $E = hf$ we first rewrite Eq. (5) in terms of m_0 and then substituting for m_0 in Eq. (1) we obtain the expression

$$\lambda = h/M\sqrt{c^2 - v^2} \qquad (6)$$

But as M is also a function of velocity v we have

$$\lambda = \frac{h}{\frac{m_0\sqrt{c^2 - v^2}}{\sqrt{1 - (v/c)^2}}} \qquad (6a)$$

Rather than cancelling terms we *observe* that if $v = 0$ in Eq. (6a) we get the Compton wavelength. Thus when $v = 0$ the M used in Eq. (6) corresponds to m_0 in Eq. (6a), which leaves the case of $v > 0$ where the M used in Eq. (6) now corresponds to

$$m_0/\sqrt{1 - (v/c)^2} \qquad (\#)$$

in Eq. (6a). Therefore, if Eq. (6a) is the Compton wavelength when $v = 0$, then holding m_0 fixed and *discarding* (#) when $v > 0$ gives us the generalized Compton wavelength

$$\lambda_{GC} = h/m_0\sqrt{c^2 - v^2} \qquad (7)$$

With the "free invention" of Eq. (7) all that now remains of Eq. (1) is (#) whose absence is accounted for below. But first rewriting Eq. (7) in terms of m_0 and substituting into Eq. (2) we find a *qualitatively* different expression where frequency has replaced the concept of rest mass

$$\boldsymbol{p} = hf\boldsymbol{v}/(c^2 - v^2)$$

Excluding Eq. (3), then, leaves (#) as the only expression using rest mass, and since (#) in itself is meaningless it is incumbent upon us to account for its absence. Therefore, starting with Eq. (4)

$$\lambda = h/p$$

and substituting Newton's definition of momentum $p = mv$ for p gives us the basis of wave mechanics or de Broglie equation

$$\lambda = h/mv$$

And substituting (#) for m puts (#) in context giving the relativistic expression

$$\lambda = \frac{h\sqrt{1-(v/c)^2}}{m_0 v} \qquad v > 0$$

Finally rewriting this expression in terms of m_0 and substituting into the magnitude of Eq. (2) we find

$$p = \frac{hv\sqrt{1-(v/c)^2}}{\lambda v\sqrt{1-(v/c)^2}} = \frac{h}{\lambda}$$

It follows that the absence of (#) entails the absence of: 1) de Broglie's equation, 2) $p = mv$ and 3) Eq. (2). Consistency now dictates we substitute Eq. (7) into Eq. (3) and then to use the wave vector $\boldsymbol{k}$ where $k = 2\pi/\lambda$, the Dirac constant $\hbar = h/2\pi$, and the angular frequency $\omega = 2\pi f = kc$, to give us our sought for generalisation of $E = hf$

$$\begin{aligned} &\boldsymbol{p} = \hbar\boldsymbol{k} \\ &E = \hbar\omega c^2/(c^2 - v^2) \\ &\boldsymbol{p} = \hbar\omega\boldsymbol{v}/(c^2 - v^2) \qquad v > 0 \end{aligned}$$

The absence of Eq. (2) now necessitates the use of these irreducible relations, and hence the need to translate rest mass to angular frequency ω where

$$\omega = 2\pi c/\lambda_{GC} = m_0 c\sqrt{c^2 - v^2}/\hbar \qquad v \geq 0$$

To demonstrate that the relations are consistent with that which came before we have

$$\boldsymbol{p} = \frac{\hbar\omega\boldsymbol{v}}{(c^2 - v^2)} = \frac{m_0\boldsymbol{v}c\sqrt{c^2 - v^2}}{(c^2 - v^2)} = \frac{m_0\boldsymbol{v}c}{\sqrt{c^2 - v^2}} = \frac{m_0\boldsymbol{v}}{\sqrt{1 - (v/c)^2}}$$

Now rather than assigning a zero-rest mass to the photon we use the rest mass of a particle to find the relativistic angular frequency of that particle. Furthermore as $v \to c$ relative to an observer we see that $\omega \to 0$, and this reflects time-dilation.

In contrast ignoring Eq. (1) and arbitrarily substituting the Compton wavelength directly into Eq. (2) gives us

$$\boldsymbol{p} = \frac{hf\boldsymbol{v}}{\sqrt{c^2 - v^2}}$$

However, this expression is erroneous as f is not velocity-dependent and remains unchanged as $v \to c$ relative to an observer.

We now conclude that the basis of relativistic dynamics ($v > 0$) is consistent with the de Broglie relations ($v = 0$), and if Eq. (2) does not exist, then it is these relations that are really *being confirmed in elementary-particle physics.*

Clearly the coining of the term "photon" by Lewis in 1926 is inappropriate as the term refers to the smallest unit of *radiant* energy, but the relations just derived imply the term should be extended to include the energy of *matter* as well. To avoid confusion we shall simply define the Light as the electromagnetic energy and momentum of a particle of *radiation*, or a particle of *matter*. Wave-particle ambiguity is avoided.

We mentioned above that we would justify our ignoring the negative root in the derivation of

$$E^* = \pm\sqrt{(m_0c^2)^2 + (pc)^2}$$

The first thing to note is that no such expression exists now that we have the Light. In hindsight, then, Dirac's equation from 1928 assumes that E^* does exist, and it associates the negative root with *antimatter*, which leaves the question of why we don't observe an equal amount of *antimatter* in the universe? However, looking at

$$E = \hbar\omega c^2/(c^2 - v^2) \qquad\qquad v \geq 0$$

we observe that E is positive in the interval $0 < v < c$ corresponding to *matter*, and E is negative in the interval $c < v < c\sqrt{2}$, which must correspond to *antimatter*. Thus E shows the whereabouts of *antimatter* to be in a mirror-universe reflected by the speed of c. Furthermore, when $v = 0$ and $v = c\sqrt{2}$ simultaneously we have $E = \pm\hbar\omega$, which is a difference of $E = 2\hbar\omega$

corresponding to the energy of pair creation/destruction. Special relativity implies that since *matter* is ‘massless’ it must be moving at speed c in its own rest frame. As *matter* is the origin in its own rest frame we can extend x-y axes from that ‘origin’ to have a plane. We now represent *matter* moving at speed c up a vertical time axis t but with no unit lengths marked, which begins at *O*. Rotating the axis around *O* perpendicularly gives us the time axis $t\,i$ where $i = \sqrt{-1}$ and by symmetry *matter*’ (with x’-y’ axes perpendicular to x-y giving a perpendicular plane) moving along this axis at speed *c*. The relative speed of these perpendicular planes is thus $\sqrt{c^2 + c^2} = c\sqrt{2}$. However, the energy of *matter*’ is in the interval $c < v \leq c\sqrt{2}$ implying *matter*’ corresponds to *antimatter*. By special relativity, then, the number of second’s (or any other unit of time) t' passing for *antimatter* relative to *matter* as one second passes for *matter* is given by the reciprocal of the time-dilation formula with $v = c\sqrt{2}$

$$t' = \sqrt{1 - (2c^2/c^2)} = i$$

Therefore, generalising the energy of a photon by completely exhausting the equations of relativistic mechanics gives us the Light

Matter	$E_+ = \hbar\omega c^2/(c^2 - v^2)$ $\boldsymbol{p}_+ = \hbar\omega\boldsymbol{v}/(c^2 - v^2)$	$v > 0$
Radiation	$E_0 = \hbar\omega$ $\boldsymbol{p}_0 = \hbar\boldsymbol{k}$	(de Broglie relations)
Antimatter	$E_- = i\hbar\omega c^2/(c^2 - v^2)$ $\boldsymbol{p}_- = i\hbar\omega\boldsymbol{v}/(c^2 - v^2)$	$v > 0$

A new mass-free paradigm of Relativity begins with the Light. What does the Light entail for the mass-based physics used today?

3. Equivalence Identity

As a particle theory the Standard Model ‘unifies’ electromagnetism, weak, and strong nuclear interactions. Thus the Light entails a fundamental revision of the Standard Model as the Standard Model must be derived anew beginning with the Light in this new paradigm of Relativity. And that brings us to the ‘force’ of gravity.

Newton’s second law of motion ($F = ma$) contains all three of Newton’s laws of motion, i.e., the third law ($F = -F$); and first law ($F = 0$), or law of inertia. Inertial frames are those in which the law of inertia holds.

In special relativity inertial frames extend throughout all space. In general relativity inertial frames are considered *locally* to be freely falling frames moving with neither acceleration nor gravity where the observer experiences weightlessness and tidal forces are considered 'non-local'. The (local) laws of classical physics are the same in all freely falling frames. However, an observer can determine that she is in a real gravitational field, even though her frame is freely falling, by detecting tidal forces. Therefore, if it could be shown *a priori* that $F = ma$ does not exist, then the absence of an inertial frame $(F = 0)$ means our weightless 'at rest' observer could not deny that tidal effects *are* local. This would have no effect upon the Light as it is defined in the subatomic realm where the nonuniformity of the gravitational field is negligible and space-time considered flat. *However*, the absence of inertial frames entails the absence of special relativity, and that just leaves the constant *c* as encoded in the Light from Maxwell's wave theory.

Consider, then, that as the derivation of the Light also consistently accounts for the absence of $p = mv$ from physics, this in turn implies the absence of Newton's second law, for mathematically we have

$$F = \frac{d(mv)}{dt} = m_I a$$

where F is force, m_I is inertial mass and a is acceleration. Following on from Newton's second law we also have

$$W = m_G g$$

where W is the weight of a terrestrial body, m_G is its gravitational mass, and g is the 'local' acceleration of free fall. If we ignore air resistance, then by Galileo's empirical law of falling bodies we put $F = W$ and obtain

$$a = \frac{m_G}{m_I} g$$

General relativity necessitates that a body's acceleration under gravity be independent of its mass. This is finally realised, for $m_G = m_I$ *a priori* as the Light entails 'locally' there *is* only

$$a = g \qquad \text{(Equivalence Identity)}$$

Therefore, if we assume the gravitational constant, G, is used in this mass-free paradigm of Relativity, then all that remains of Newtonian mechanics is G. In contrast to general relativity, then, Relativity transcends a) any notion of mass and force, and b) any local/non-local distinction with regards to gravity. Thus all that now remains to remind us of the preceding classical physics of Newton and Maxwell are the constants G and c respectively. To this extent it is clear that general relativity did not go far enough as Einstein surmised. (Einstein sought to unify light with gravity but failed, and we now know that is because he needed to start with his own discovery of 'light-quanta'.)

Clearly if this is correct, then there never was anything else but this to understand, and what does not follow on from this is erroneous. Therefore, since this much could have been discovered before 1925 the misguided advent of *non*-relativistic quantum mechanics could have been avoided proving Einstein's skepticism about quantum mechanics and insistence on realism correct.

Finally, from the totality of those physics theories used today that just leaves the classical physics of statistical mechanics and thermodynamics unmentioned. But the nonexistence of Newton's three 'laws' of motion entails a revision of these two remaining theories as well.

4. Bekenstein-Hawking formula

Does Relativity subsume these last two remaining vestiges of classical physics? The following suggests that it does. Consider the Bekenstein-Hawking formula [3] for a black hole

$$S_{BH} = (A/4) \times (kc^3/G\hbar)$$

where k is Boltzman's constant from statistical mechanics, and A is the surface area of the event horizon. If we assume this formula holds true in Relativity, then we have a consistent unification of G, c, k, $\hbar$ and *entropy* to remind us of the mass-based paradigm of classical physics that was.

5. What was the nature of $'F'$ and $'m'$ in $F = ma$?

If F is force, then so is the product ma. We understand the meaning of force because it is related to our muscular responses in pushing/pulling bodies, and we understand acceleration via our visual perception of motion. That leaves the question what is mass? [4] In $F = ma$ we have m measuring a body's 'inertia', but in Relativity bodies are massless and only $a(= g)$ remains implying m was only a *projection* based upon our muscular responses.

6. Conclusion

In retrospect the evolution from classical physics to Relativity via the "free invention" of the generalised Compton wavelength was inevitable as it was never realistic to ignore tidal effects 'locally' in a gravity-endowed universe. Hence, the true significance of Planck's four-dimensional constant is that it consistently leaves four-dimensional Relativity as our only description of the *matter/antimatter* universe(s).

References

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