The behaviour of light in gravitational fields

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Abstract

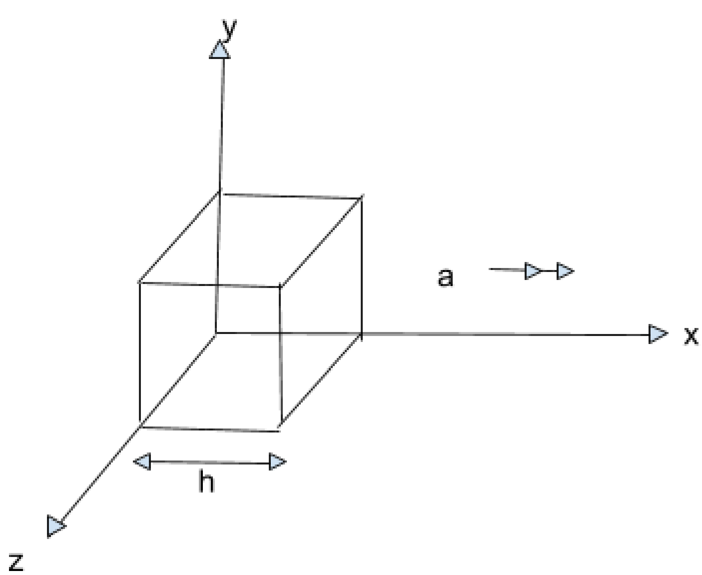
We discuss the equivalence principle and how this relates to the gravitational fields of large bodies. We derive formulas relating time dilation and acceleration with respect to accelerating reference frames. We conclude with an assessment of the possibility of the existence of an astronomical body or 'Black Hole' capable of halting the velocity of light.

Introduction

This paper considers the gravitational effect of accelerating reference frames on the speed of light and specifically addresses the question as to whether it is possible to halt or reverse the velocity of light with gravitational fields.

Discussion

Consider a cubical container of length **h** which is sufficiently distant from any gravitational fields for the effects of these to be ignored.
Let us define our coordinate system with the three spatial dimensions **x,y,z** and the time dimension **t** so that the container is placed at the origin at **t=0**.
Let the container be accelerating with an acceleration of **a** along the **x** axis as shown.
Let this defined reference frame be denoted as **K** and the accelerating reference frame as **K'**.



Any occupants of the container will be subject to a gravitational field with an acceleration due to gravity of **a**. This is known as the equivalence principle. [1]

Furthermore, we can say that any uniform gravitational field whether created by an object with mass[1] or an accelerating frame of reference can be eliminated by the selection of an appropriate accelerating reference frame.
We will assume that the container has an unlimited energy supply such that it can maintain a constant gravitational field as measured by **K'**. Any occupants of the container will be assumed to possess the necessary apparatus to measure the strength of the gravitational field. This may consist of measuring the rate of acceleration of a released mass. Providing that this rate of acceleration remains constant as measured by **K',** we can conclude that the gravitational field is constant relative to **K'.**

If we now consider a particle in the container starting at the origin moving in the direction of the positive **x** axis with a velocity **u** as seen from the **K** reference frame and endeavour to answer the following question.

"What value of **a** will be required to reverse the velocity of the particle within the container as seen from **K'**?".

For values of **u** which are small relative to the speed of light **c**, we can use Newtonian mechanics to give the approximate answer. The velocity of the container after a time **t** will be given by **at**. The velocity of the particle as seen by **K'** will therefore be **u-at**. If the particle reaches zero velocity from the **K'** reference frame we have **t=u/a**. The distance traveled by the container as measured by the **K** reference frame

after a time **t** will be given by $\int\int a\,dt = \frac{1}{2} a t^2$

The distance traveled by the particle as measured by the **K** reference frame after a time **t** will be given by **ut**.
We therefore have; $ut - \frac{1}{2} a t^2 = u \frac{u}{a} - \frac{1}{2} a \left(\frac{u}{a}\right)^2 = \frac{u^2}{2a}$ as the distance traveled by the particle after a time **t** as measured by the **K'** reference frame.
Where $\frac{u^2}{2a} < h$ the gravitational field experienced by the **K'** reference frame will be of sufficient strength to reverse the path of the particle within the container.
We therefore have the condition $a > \frac{u^2}{2h}$ which must be satisfied in order for this to be the case.

If we replace the particle with a photon, the speed of the particle becomes **c**. We can immediately see that if this were the case, it would not be possible to create an acceleration of sufficient strength to reverse the velocity of the photon as seen from **K'** because this would mean that the far wall of the container would have to be moving faster than light as seen from the **K** reference frame. We will now show mathematically that this is the case.

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[1] This assumes that the reference frame is sufficiently small for any tidal forces to be insignificantly small.

Because our velocities will be large compared to **c**, we need to apply the Lorentz Transformation for the time dimension [2] thus:

t'= $\frac{t}{\sqrt{1-u^2/c^2}}$

Where **t** is the measure of a time interval as measured by a clock placed in the **K** reference frame, and **t'** is the measure of the same time interval as measured by a clock placed in the **K'** reference frame.

We therefore have; $\frac{du}{dt}$ **>** $\frac{lu^2}{2h}$ where $l = 1 - u^2/c^2$ as the required condition.
If we use natural units for time and distance, we can write **c**=1.
If we also set **h**=1/2 for simplicity, we have;

$$\frac{du}{dt} > u^2 - u^4$$

If we plot this on a velocity/time graph, the right hand side of this inequality will be a straight line of gradient zero. We can therefore conclude that there can be no solution to this inequality for any positive values of **u**.

For completeness, we will now consider the situation from the **K'** reference frame and show that the equivalence principle still holds true for gravitational fields produced by the close proximity to a large mass, where the acceleration due to gravity approaches the strength required to halt the velocity of light. For this, we need to include gravitational time dilation as described in General Relativity [1].
If we consider a similar scenario with a photon moving along the positive **x** axis and a clock placed at the 'floor' of the container measuring a time **t'** and a second clock placed a distance **d** from the 'floor' of the container along the positive **x** axis measuring a time **t**. We shall now derive an expression giving the amount of time dilation relative to the two clocks as a function of **a** and **d.**

I.e. $\gamma = f(a,d)$ where γ is the time dilational coefficient such that **t**= γ **t'**

From the **K'** reference frame starting at **t**=0,
we see the photon travel a distance **d** which will take a time **d/c**. However, the distance **d** will then have increased an amount $\frac{1}{2}$ **a** t^2 or $\frac{1}{2}$ **a** $\frac{d^2}{c^2}$ and so on.

The total increase in distance will be given by adding successive multiples of $\frac{1}{2}$ **a** $\frac{d^2}{c^2}$ which can be expressed as an infinite mathematical series thus:

$$\frac{1}{2}\,a\,\frac{d^2}{c^2} + \frac{1}{4}\,a^2\,\frac{d^4}{c^4} + \frac{1}{8}\,a^3\,\frac{d^8}{c^8} + \ldots\ldots + \frac{1}{2^n}\,a^n\,\frac{d^{2^n}}{c^{2^n}}$$

Because γ is defined as the ratio of the two time measurements which are directly proportional to the distances, it follows that $\gamma = \dfrac{d}{d+\sum_{n=1}^{n=\infty} \frac{a^n}{2^n}\frac{d^{2^n}}{c^{2^n}}}$

Using natural units for distance and time we have;

$$\gamma = \frac{d}{d+\sum_{n=1}^{n=\infty} \frac{a^n d^{2^n}}{2^n}}$$

From this we can see that as **d**$\rightarrow \infty$ $\gamma \rightarrow 0$

Setting the distance **d**=1, this simplifies to:

$$\gamma = \frac{1}{1+\sum_{n=1}^{n=\infty} \frac{a^n}{2^n}}$$

From this we can see that as **a**$\rightarrow \infty$ $\gamma \rightarrow 0$

We can also say that $0 < \gamma < 1$ and that because t and t' exist in the same reference frame and **t=**γ **t',** then **t'=t/**γ **.**

Importantly, because γ can never become zero, we can say that the propagation of light can never become zero.

Conclusion

From this we can draw the following conclusion.
It is not possible to halt the velocity of light with a gravitational field and consequently slow the measurement of time to zero. This statement is independent of whether the gravitational field is created by the close proximity of a large mass, or by the selection of an accelerating reference frame.

It was previously assumed that an object of sufficiently large mass would produce a gravitational field of sufficient strength to halt the velocity of light being emitted by the object. Such an object was commonly known as a ‘Black Hole’. We can conclude that because there can not be an equivalent accelerating frame of reference that such an object could not be formed.

Furthermore, because the acceleration due to gravity of a large mass is proportional to the mass of the object, the mass of the object would also have to be infinite in order to halt the direction of light.

Original studies of theoretical astronomical bodies of sufficient size and density required to reverse the direction of light were based on the concept of ‘escape velocity’. However, because any velocity would be sufficient for a body to escape the gravitational pull of another larger body provided that that velocity could be maintained, it does not seem that the concept of ‘escape velocity’ is worthy of further consideration. In order for a smaller untethered body with mass to be stationary in the location of a larger mass, the smaller mass must necessarily be accelerating at the acceleration due to gravity at the specific location.

The correct consideration is therefore ‘acceleration due to gravity’ and as we have shown, it is not possible to produce an acceleration due to gravity of sufficient strength to halt the velocity of light.

References

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