Mathematical demonstration of

Riemann hypothesis

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Abstract: A possible demonstrations of the riemann hypothesis based on every infinite complex solution of zeta function of Riemann *ζ(s)* with it is conjugated root, can also be used to be the solution of a quadratic equation of real coefficients which admits it as zeros, using the root theorem of Viete.

Demonstration I:

Riemann zeta function is defined by the Dirichlet series [1] ;[2]

$$\zeta = \sum_{n=1}^{\infty} \frac{1}{n^s} \quad s = \sigma + it$$

This functions has only simple zeros, called trivial zeros at points $\sigma = -2v, v = 1, 2, 3, \ldots$ Every non-trivial zero in the function $\zeta(s)$ are complex numbers which have symmetry property with respect to the real axis *t* = 0 and to the vertical line $\sigma = \frac{1}{2}$ and they are on the called critical line $0 \leq \sigma \leq 1$. For $\sigma > 1$ the function $\zeta(s) \neq 0$.

The Riemann hypothesis states that all non-trivial zeros of zeta function have real part $\sigma = \frac{1}{2}$:[3],

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = 0 \quad s = \sigma + it \Rightarrow \sigma = 1/2$$

For that reason we establish that if an equation *P(x)* =0 with real coefficient admits a complex root $x' = \sigma + it$ of order k, it also admits, with the same order, its conjugate root $x'' = \sigma - it$. [1]

Every infinite complex solution of zeta function of Riemann *ζ(s)* with its conjugated root, can also be used to be the solution of a quadratic equation of real coefficients which admits it as zeros, using the root theorem of Viete:

$$x' = \sigma + it \ , \ x'' = \sigma - it$$

$$x' + x'' = (\sigma + it) + (\sigma - it) = 2\sigma = b \Longrightarrow \sigma = \tfrac{b}{2}$$

$$x' . x'' = (\sigma + it) . (\sigma - it) = \sigma^2 + t^2 = c$$

$$x^2 - bx + c = 0 \text{ siendo } b = 2\sigma \ c = \sigma^2 + t^2$$

$$x'^2 - 2\sigma x' + \sigma^2 + t^2 = 0 \longrightarrow x' = \frac{2\sigma + \sqrt{4\sigma^2 - 4(\sigma^2 + t^2)}}{2} = \sigma + it = s$$

$$x''^2 - 2\sigma x'' + \sigma^2 + t^2 = 0 \longrightarrow x'' = \frac{2\sigma - \sqrt{4\sigma^2 - 4(\sigma^2 + t^2)}}{2} = \sigma - it$$

This equation can also be written as:

x^2−bx+c=0 for b=x´+x´´ and c=x´·x´´

x'^2−*bx´+c=0* for *c=x´(b−x´)* so the equality

belongs and identity x'^2−bx´+x´(b−x´)≡ 0

so this identity has to be verified whatever the values

might be for *b* and *x´* . For that reason, *b*=1 and

x´= *s*= $\sigma + it$ for $\sigma = \frac{b}{2}$ so it should be verified for $\sigma = \frac{1}{2}$, so:

$$(\tfrac{1}{2} + it)^2 - 1(\tfrac{1}{2} + it) + (\tfrac{1}{2} + it)(\tfrac{1}{2} - it) = 0$$

$$\tfrac{1}{4} + 2\tfrac{1}{2}it + i^2t^2 - \tfrac{1}{2} - it + \tfrac{1}{4} - i^2t^2 = 0$$

and this must be verified whatever *t* might be and for $\sigma = \frac{1}{2}$

so, *x´*= *s*= $\frac{1}{2}$ + *it*

x''^2–bx"+c=o for *c=x" (b−x")* so the equality

belongs and identity x''^2−bx"+x" (b−x")≡ 0

so this identity has to be verified whatever the values

might be for *b* and *x".* For that reason, *b*=1 and

$x'' = \sigma - it$ for $\sigma = \frac{b}{2}$ so it should be verified for $\sigma = \frac{1}{2}$, so:

$$(\tfrac{1}{2}-it)^2-1(\tfrac{1}{2}-it)+(\tfrac{1}{2}+it)(\tfrac{1}{2}-it)=0$$

$$\tfrac{1}{4}-2\tfrac{1}{2}it+i^2t^2-\tfrac{1}{2}+it+\tfrac{1}{4}-i^2t^2=0$$

and this must be verified whatever t might be and for $\sigma = \frac{1}{2}$

so, $x''=\frac{1}{2}-it$.

The identities should be verified for *b* and *b*=1 and that is possible if and only if $\sigma = \frac{1}{2}$. So, all the non-trivial zeros in the zeta function of Riemann *ζ(s)* must be on the straight line $\sigma = \frac{1}{2}$. If it is not so, the identities contradict their own definition that states that: ´an identity is an equality which is verified whatever the values attributed to letters may be´

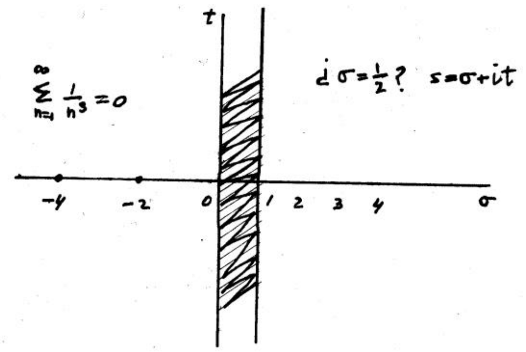


Figure 1:

$\sigma < 0$ exist only the called trivial zeros, in points -2, -4, …

$0 \leq \sigma \leq 1$, called critical line, all the non-trivial zeros are on that line, and it is not known if they are on the straight line $\sigma = \frac{1}{2}$. Every solution equals *s*= $\sigma + it$

For $\sigma > 1$ there isn´t any zero.

Demonstration II:

Beginning with the equation x^2−*bx* + *c* = *0* for *b* = *x´*+ *x″* and *c*= *x´ x″*

x'^2−*bx´*+*c*=*0* *c*=*x´*(*b*−*x´*) for b and x´.If b=1 we have the following equation:

x'^2-*1x´*+*x´*(*1*-*x´*)=*0* and this expression is an identity x'^2-*1x´*+*x´*(*1*-*x´*)$\equiv 0$.

This has to be verified whatever the value of x´ may be, so it has to be also verified for every value s=$\sigma + it$, that is for every value of the solution of the Riemann zeta equation.

If we substitute *x´* for σ +it, we have the following:

$(\sigma + it)^2 - 1(\sigma + it) + (\sigma + it)(\sigma - it) \equiv 0$

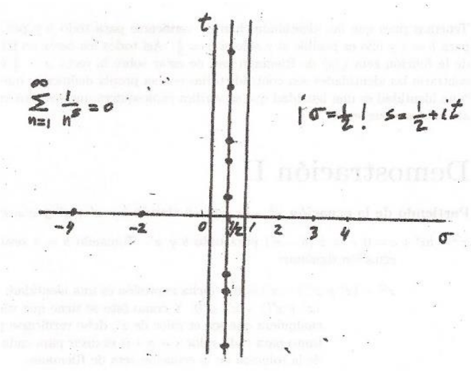
$\sigma^2 + 2\sigma it + i^2t^2 - \sigma - it + \sigma^2 - i^2t^2 \equiv 0 \Rightarrow i^2t^2 + 2\sigma it + 2\sigma^2 \equiv i^2t^2 + it + \sigma$ and being an identity, both members rmust be identical, as the definition of an identity. We can have the following system:

$i^2t^2 - i^2t^2 \equiv 0$ for all σ

$2\sigma it - it \equiv 0$ for $\sigma = \frac{1}{2}$

$2\sigma^2 - \sigma \equiv 0$ for $\sigma = 0$ and $\sigma = \frac{1}{2}$

$\sigma = 0$ does not verify the system, so the system is verified if and only if $\sigma = \frac{1}{2}$. For that reason, we have that *x´*= *s*= $\frac{1}{2}$+*it,* as we wanted to demonstrate. Dedekin function and similar, whose are *s*=$\sigma + it$, have the same demonstration.



FINAL FIGURE:

$\sigma < 0$ exist only the called trivial zeros, in points -2, -4, …

$0 \leq \sigma \leq 1$, called critical line, all the non-trivial zeros are on that line, and it has been demonstrated that they are all on the straight line $\sigma = \frac{1}{2}$. Then, all the solutions are *s*= $\frac{1}{2}$ *+it.*

For $\sigma > 1$ there isn´t any zero.

References:

[1] I. M. Vinográdov, *Enciclopedia de las matematicas / Encyclopedia of mathematics*, no. v. 5. Grupo Anaya Comercial, 2004.

[2] E. Bombieri, "Problems of the Millennium : The Riemann Hypothesis," *Millenn. Prize Probl.*, vol. 1, no. 2004, pp. 3–9, 2005.

[3] A. Bouvier and M. George, *Diccionario de matemáticas*. Akal, 1984.