

CONJECTURES ABOUT MIN/MAX PRIME DIVISOR OF THE NATURAL NUMBERS AND PRIME NUMBERS

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Applying functions over prime divisors to the sequence of natural numbers and taking the differences of this sequence you can split this in subsequences by the sum equals length rule so that you can associate each subsequence to each gap between consecutive prime numbers

Premise

All this research has been done using Mathematica©, a symbolic computation environment that uses a programming language called Wolfram Language.

Some useful function

Define $\text{rad}(n)$ as the product of all prime numbers that are divisors of n . Now define sumrad the sum of all prime numbers that are divisors of n , $\text{minrad}(n)$ as the lower of such prime numbers, $\text{maxrad}(n)$ the greater of such prime numbers, $\text{minmaxdiff}(n)$ the difference of the twos if positive, minrad otherwise, amean1 the floor of the arithmetic mean of the prime divisors and amean2 the ceiling of the arithmetic mean, gmean1 the floor of the geometric mean of the prime divisors and gmean2 the ceiling of the geometric mean:

```
prodrad[n_Integer]:=Times@@(First/@FactorInteger[n])
sumrad[n_Integer]:=Plus@@(First/@FactorInteger[n])
minrad[n_Integer]:=Min[First/@FactorInteger[n]]
maxrad[n_Integer]:=Max[First/@FactorInteger[n]]
minmaxdiff[n_Integer]:=If[minrad[n]!=maxrad[n],
    maxrad[n]-minrad[n],
    minrad[n]]
]
amean1[n_Integer]:=Floor[Mean[First/@FactorInteger[n]]]
amean2[n_Integer]:=Ceiling[Mean[First/@FactorInteger[n]]]
gmean1[n_Integer]:=Floor[GeometricMean[First/@FactorInteger[n]]]
gmean2[n_Integer]:=Ceiling[GeometricMean[First/@FactorInteger[n]]]
```

Above function $\text{FactorInteger}(n)$ returns a list of couples. Each couple is done by a prime number p and an exponent number e where p^e is a divisor of n but not $p^{(e+1)}$. As an example:

```
In[120]= FactorInteger[60]
```

```
Out[120]= {{2, 2}, {3, 1}, {5, 1}}
```

Now apply the functions to the natural number starting with 2:

```
In[121]= r1=minrad/@Range[2,300]
r2=maxrad/@Range[2,300]
r3=minmaxdiff/@Range[2,300]
r4=amean1/@Range[2,300]
r5=amean2/@Range[2,300]
r6=gmean1/@Range[2,300]
r7=gmean2/@Range[2,300]
r8=sumrad/@Range[2,300]
```

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Out[121]= {2, 3, 2, 5, 2, 7, 2, 3, 2, 11, 2, 13, 2, 3, 2, 17, 2, 19, 2, 3, 2, 23, 2, 5, 2, 3, 2, 29, 2,
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3, 2, 11, 2, 5, 2, 3, 2, 149, 2, 151, 2, 3, 2, 5, 2, 157, 2, 3, 2, 7, 2, 163, 2, 3, 2, 167,
2, 13, 2, 3, 2, 173, 2, 5, 2, 3, 2, 179, 2, 181, 2, 3, 2, 5, 2, 11, 2, 3, 2, 191, 2, 193,
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2, 3, 2, 251, 2, 11, 2, 3, 2, 257, 2, 7, 2, 3, 2, 263, 2, 5, 2, 3, 2, 269, 2, 271, 2, 3,
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5, 31, 2, 11, 17, 7, 3, 37, 19, 13, 5, 41, 7, 43, 11, 5, 23, 47, 3, 7, 5, 17, 13, 53, 3,
11, 7, 19, 29, 59, 5, 61, 31, 7, 2, 13, 11, 67, 17, 23, 7, 71, 3, 73, 37, 5, 19, 11, 13,
79, 5, 3, 41, 83, 7, 17, 43, 29, 11, 89, 5, 13, 23, 31, 47, 19, 3, 97, 7, 11, 5, 101, 17,
103, 13, 7, 53, 107, 3, 109, 11, 37, 7, 113, 19, 23, 29, 13, 59, 17, 5, 11, 61, 41, 31,
5, 7, 127, 2, 43, 13, 131, 11, 19, 67, 5, 17, 137, 23, 139, 7, 47, 71, 13, 3, 29, 73, 7,
37, 149, 5, 151, 19, 17, 11, 31, 13, 157, 79, 53, 5, 23, 3, 163, 41, 11, 83, 167, 7, 13,
17, 19, 43, 173, 29, 7, 11, 59, 89, 179, 5, 181, 13, 61, 23, 37, 31, 17, 47, 7, 19, 191,
3, 193, 97, 13, 7, 197, 11, 199, 5, 67, 101, 29, 17, 41, 103, 23, 13, 19, 7, 211, 53, 71,
107, 43, 3, 31, 109, 73, 11, 17, 37, 223, 7, 5, 113, 227, 19, 229, 23, 11, 29, 233, 13,
47, 59, 79, 17, 239, 5, 241, 11, 3, 61, 7, 41, 19, 31, 83, 5, 251, 7, 23, 127, 17, 2, 257,
43, 37, 13, 29, 131, 263, 11, 53, 19, 89, 67, 269, 5, 271, 17, 13, 137, 11, 23, 277, 139,
31, 7, 281, 47, 283, 71, 19, 13, 41, 3, 17, 29, 97, 73, 293, 7, 59, 37, 11, 149, 23, 5}
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Out[123]= {2, 3, 2, 5, 1, 7, 2, 3, 3, 11, 1, 13, 5, 2, 2, 17, 1, 19, 3, 4, 9, 23, 1, 5, 11, 3, 5, 29, 3, 31, 2,
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15, 137, 21, 139, 5, 44, 69, 2, 1, 24, 71, 4, 35, 149, 3, 151, 17, 14, 9, 26, 11, 157, 77, 50,
3, 16, 1, 163, 39, 8, 81, 167, 5, 13, 15, 16, 41, 173, 27, 2, 9, 56, 87, 179, 3, 181, 11, 58,
21, 32, 29, 6, 45, 4, 17, 191, 1, 193, 95, 10, 5, 197, 9, 199, 3, 64, 99, 22, 15, 36, 101, 20,
11, 8, 5, 211, 51, 68, 105, 38, 1, 24, 107, 70, 9, 4, 35, 223, 5, 2, 111, 227, 17, 229, 21,
8, 27, 233, 11, 42, 57, 76, 15, 239, 3, 241, 9, 3, 59, 2, 39, 6, 29, 80, 3, 251, 5, 12, 125,
14, 2, 257, 41, 30, 11, 26, 129, 263, 9, 48, 17, 86, 65, 269, 3, 271, 15, 10, 135, 6, 21, 277,
137, 28, 5, 281, 45, 283, 69, 16, 11, 34, 1, 17, 27, 94, 71, 293, 5, 54, 35, 8, 147, 10, 3}
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Out[124]= {2, 3, 2, 5, 2, 7, 2, 3, 3, 11, 2, 13, 4, 4, 2, 17, 2, 19, 3, 5, 6, 23, 2, 5, 7, 3, 4, 29, 3, 31,
  2, 7, 9, 6, 2, 37, 10, 8, 3, 41, 4, 43, 6, 4, 12, 47, 2, 7, 3, 10, 7, 53, 2, 8, 4, 11, 15,
  59, 3, 61, 16, 5, 2, 9, 5, 67, 9, 13, 4, 71, 2, 73, 19, 4, 10, 9, 6, 79, 3, 3, 21, 83, 4, 11,
  22, 16, 6, 89, 3, 10, 12, 17, 24, 12, 2, 97, 4, 7, 3, 101, 7, 103, 7, 5, 27, 107, 2, 109, 6,
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  4, 9, 137, 9, 139, 4, 25, 36, 12, 2, 17, 37, 5, 19, 149, 3, 151, 10, 10, 6, 18, 6, 157, 40,
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  64, 8, 2, 257, 16, 22, 6, 16, 66, 263, 5, 29, 9, 46, 34, 269, 3, 271, 9, 7, 69, 8, 9, 277,
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  32, 13, 21, 12, 14, 25, 5, 9, 191, 3, 193, 50, 7, 5, 197, 6, 199, 4, 35, 52, 18, 8, 23, 53,
  13, 8, 15, 5, 211, 28, 37, 55, 24, 3, 19, 56, 38, 6, 15, 14, 223, 5, 4, 58, 227, 8, 229,
  10, 7, 16, 233, 6, 26, 31, 41, 9, 239, 4, 241, 7, 3, 32, 6, 16, 16, 17, 43, 4, 251, 4, 17,
  65, 9, 2, 257, 16, 22, 7, 16, 67, 263, 6, 29, 10, 46, 35, 269, 4, 271, 10, 8, 70, 8, 10,
  277, 71, 17, 5, 281, 18, 283, 37, 9, 9, 24, 3, 17, 12, 50, 38, 293, 4, 32, 20, 7, 76, 18, 4}

Out[126]= {2, 3, 2, 5, 2, 7, 2, 3, 3, 11, 2, 13, 3, 3, 2, 17, 2, 19, 3, 4, 4, 23, 2, 5, 5, 3, 3, 29, 3, 31, 2, 5,
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  8, 59, 4, 61, 8, 5, 2, 9, 5, 67, 6, 9, 5, 71, 3, 73, 9, 4, 7, 9, 5, 79, 4, 3, 10, 83, 4,
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  277, 17, 10, 5, 281, 7, 283, 12, 7, 7, 17, 3, 17, 7, 18, 13, 293, 4, 18, 9, 6, 18, 18, 4}

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Out[128]= {2, 3, 2, 5, 5, 7, 2, 3, 7, 11, 5, 13, 9, 8, 2, 17, 5, 19, 7, 10, 13, 23, 5, 5, 15, 3, 9, 29,
  10, 31, 2, 14, 19, 12, 5, 37, 21, 16, 7, 41, 12, 43, 13, 8, 25, 47, 5, 7, 7, 20, 15, 53, 5,
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  103, 15, 15, 55, 107, 5, 109, 18, 40, 9, 113, 24, 28, 31, 16, 61, 24, 10, 11, 63, 44, 33,
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  39, 149, 10, 151, 21, 20, 20, 36, 18, 157, 81, 56, 7, 30, 5, 163, 43, 19, 85, 167, 12, 13,
  24, 22, 45, 173, 34, 12, 13, 62, 91, 179, 10, 181, 22, 64, 25, 42, 36, 28, 49, 10, 26, 191,
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  109, 48, 5, 38, 111, 76, 18, 30, 42, 223, 9, 8, 115, 227, 24, 229, 30, 21, 31, 233, 18, 52,
  61, 82, 26, 239, 10, 241, 13, 3, 63, 12, 46, 32, 33, 86, 7, 251, 12, 34, 129, 25, 2, 257,
  48, 44, 20, 32, 133, 263, 16, 58, 28, 92, 69, 269, 10, 271, 19, 23, 139, 16, 28, 277, 141,
  34, 14, 281, 52, 283, 73, 27, 26, 48, 5, 17, 36, 100, 75, 293, 12, 64, 39, 14, 151, 36, 10}
```

These sequences appear quite unpredictable but more can be conjectured on the difference sequence. The operator difference simply substitute an item in the sequence with the difference between the following item and the item itself for every item; it's like a derivative of the sequence.

```
In[129]:= s1=Differences [r1]
s2=Differences [r2]
s3=Differences [r3]
s4=Differences [r4]
s5=Differences [r5]
s6=Differences [r6]
s7=Differences [r7]
s8=Differences [r8]

Out[129]= {1, -1, 3, -3, 5, -5, 1, -1, 9, -9, 11, -11, 1, -1, 15, -15, 17, -17, 1, -1, 21, -21, 3, -3,
  1, -1, 27, -27, 29, -29, 1, -1, 3, -3, 35, -35, 1, -1, 39, -39, 41, -41, 1, -1, 45, -45,
  5, -5, 1, -1, 51, -51, 3, -3, 1, -1, 57, -57, 59, -59, 1, -1, 3, -3, 65, -65, 1, -1, 69,
  -69, 71, -71, 1, -1, 5, -5, 77, -77, 1, -1, 81, -81, 3, -3, 1, -1, 87, -87, 5, -5, 1,
  -1, 3, -3, 95, -95, 1, -1, 99, -99, 101, -101, 1, -1, 105, -105, 107, -107, 1, -1, 111,
  -111, 3, -3, 1, -1, 5, -5, 9, -9, 1, -1, 3, -3, 125, -125, 1, -1, 129, -129, 5, -5, 1,
  -1, 135, -135, 137, -137, 1, -1, 9, -9, 3, -3, 1, -1, 147, -147, 149, -149, 1, -1, 3, -3,
  155, -155, 1, -1, 5, -5, 161, -161, 1, -1, 165, -165, 11, -11, 1, -1, 171, -171, 3, -3,
  1, -1, 177, -177, 179, -179, 1, -1, 3, -3, 9, -9, 1, -1, 189, -189, 191, -191, 1, -1,
  195, -195, 197, -197, 1, -1, 5, -5, 3, -3, 1, -1, 9, -9, 209, -209, 1, -1, 3, -3, 5, -5,
  1, -1, 11, -11, 221, -221, 1, -1, 225, -225, 227, -227, 1, -1, 231, -231, 3, -3, 1, -1,
  237, -237, 239, -239, 1, -1, 3, -3, 11, -11, 1, -1, 249, -249, 9, -9, 1, -1, 255, -255,
  5, -5, 1, -1, 261, -261, 3, -3, 1, -1, 267, -267, 269, -269, 1, -1, 3, -3, 275, -275, 1,
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Out[130]= {1, -1, 3, -2, 4, -5, 1, 2, 6, -8, 10, -6, -2, -3, 15, -14, 16, -14, 2, 4, 12, -20, 2, 8, -10,
4, 22, -24, 26, -29, 9, 6, -10, -4, 34, -18, -6, -8, 36, -34, 36, -32, -6, 18, 24, -44, 4,
-2, 12, -4, 40, -50, 8, -4, 12, 10, 30, -54, 56, -30, -24, -5, 11, -2, 56, -50, 6, -16, 64,
-68, 70, -36, -32, 14, -8, 2, 66, -74, -2, 38, 42, -76, 10, 26, -14, -18, 78, -84, 8, 10,
8, 16, -28, -16, 94, -90, 4, -6, 96, -84, 86, -90, -6, 46, 54, -104, 106, -98, 26, -30,
106, -94, 4, 6, -16, 46, -42, -12, 6, 50, -20, -10, -26, 2, 120, -125, 41, -30, 118, -120,
8, 48, -62, 12, 120, -114, 116, -132, 40, 24, -58, -10, 26, 44, -66, 30, 112, -144, 146,
-132, -2, -6, 20, -18, 144, -78, -26, -48, 18, -20, 160, -122, -30, 72, 84, -160, 6, 4,
2, 24, 130, -144, -22, 4, 48, 30, 90, -174, 176, -168, 48, -38, 14, -6, -14, 30, -40, 12,
172, -188, 190, -96, -84, -6, 190, -186, 188, -194, 62, 34, -72, -12, 24, 62, -80, -10, 6,
-12, 204, -158, 18, 36, -64, -40, 28, 78, -36, -62, 6, 20, 186, -216, -2, 108, 114, -208,
210, -206, -12, 18, 204, -220, 34, 12, 20, -62, 222, -234, 236, -230, -8, 58, -54, 34, -22,
12, 52, -78, 246, -244, 16, 104, -110, -15, 255, -214, -6, -24, 16, 102, 132, -252, 42,
-34, 70, -22, 202, -264, 266, -254, -4, 124, -126, 12, 254, -138, -108, -24, 274, -234,
236, -212, -52, -6, 28, -38, 14, 12, 68, -24, 220, -286, 52, -22, -26, 138, -126, -18}
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108, -96, 1, 9, -17, 47, -47, -7, 8, 48, -21, -9, -24, 0, 122, -125, 38, -29, 120, -122,
3, 53, -63, 13, 122, -116, 118, -134, 39, 25, -67, -1, 23, 47, -67, 31, 114, -146, 148,
-134, -3, -5, 17, -15, 146, -80, -27, -47, 13, -15, 162, -124, -31, 73, 86, -162, 8, 2,
1, 25, 132, -146, -25, 7, 47, 31, 92, -176, 178, -170, 47, -37, 11, -3, -23, 39, -41, 13,
174, -190, 192, -98, -85, -5, 192, -188, 190, -196, 61, 35, -77, -7, 21, 65, -81, -9, -3,
-3, 206, -160, 17, 37, -67, -37, 23, 83, -37, -61, -5, 31, 188, -218, -3, 109, 116, -210,
212, -208, -13, 19, 206, -222, 31, 15, 19, -61, 224, -236, 238, -232, -6, 56, -57, 37, -33,
23, 51, -77, 248, -246, 7, 113, -111, -12, 255, -216, -11, -19, 15, 103, 134, -254, 39,
-31, 69, -21, 204, -266, 268, -256, -5, 125, -129, 15, 256, -140, -109, -23, 276, -236,
238, -214, -53, -5, 23, -33, 16, 10, 67, -23, 222, -288, 49, -19, -27, 139, -137, -7}
```

```
Out[132]= {1, -1, 3, -3, 5, -5, 1, 0, 8, -9, 11, -9, 0, -2, 15, -15, 17, -16, 2, 1, 17, -21, 3, 2, -4,
1, 25, -26, 28, -29, 5, 2, -3, -4, 35, -27, -2, -5, 38, -37, 39, -37, -2, 8, 35, -45, 5,
-4, 7, -3, 46, -51, 6, -4, 7, 4, 44, -56, 58, -45, -11, -3, 7, -4, 62, -58, 4, -9, 67,
-69, 71, -54, -15, 6, -1, -3, 73, -76, 0, 18, 62, -79, 7, 11, -6, -10, 83, -86, 7, 2,
5, 7, -12, -10, 95, -93, 3, -4, 98, -94, 96, -96, -2, 22, 80, -105, 107, -103, 14, -16,
109, -105, 6, 1, -7, 22, -18, -9, 8, 20, -9, -6, -11, -1, 123, -125, 21, -17, 125, -126,
8, 21, -30, 5, 128, -128, 130, -135, 21, 11, -24, -10, 15, 20, -32, 14, 130, -146, 148,
-141, 0, -4, 12, -12, 151, -117, -12, -25, 12, -13, 161, -142, -15, 36, 125, -163, 9,
-5, 3, 11, 151, -162, -5, 0, 25, 14, 134, -176, 178, -174, 25, -20, 9, -9, 2, 10, -19,
3, 183, -189, 191, -144, -42, -3, 193, -192, 194, -196, 32, 16, -33, -11, 16, 29, -39,
-6, 8, -11, 207, -184, 10, 17, -30, -22, 17, 36, -17, -32, 9, -1, 209, -219, 0, 53, 170,
-219, 221, -219, -3, 8, 218, -227, 20, 4, 11, -33, 231, -236, 238, -235, -3, 28, -25,
9, 1, 0, 27, -40, 248, -247, 13, 47, -56, -6, 255, -241, 6, -16, 10, 50, 197, -258, 24,
-20, 37, -12, 235, -266, 268, -262, -2, 62, -61, 1, 268, -207, -53, -13, 277, -264,
266, -247, -27, -1, 16, -22, 15, -5, 38, -13, 256, -289, 28, -13, -12, 68, -57, -15}
```

```

Out[133]= {1, -1, 3, -2, 4, -5, 1, 1, 7, -8, 10, -8, -1, -2, 15, -14, 16, -15, 1, 2, 16, -20, 2, 3, -5,
  2, 24, -25, 27, -29, 5, 3, -4, -3, 34, -26, -3, -4, 37, -37, 39, -36, -3, 9, 34, -44, 4,
  -3, 6, -2, 45, -50, 5, -3, 6, 5, 43, -55, 57, -44, -12, -3, 7, -3, 61, -57, 3, -8, 66,
  -68, 70, -53, -16, 7, -2, -3, 73, -75, -1, 19, 61, -79, 7, 12, -7, -9, 82, -85, 6, 3, 4,
  8, -13, -9, 94, -92, 2, -3, 97, -93, 95, -95, -3, 23, 79, -104, 106, -103, 14, -15, 108,
  -105, 6, 2, -8, 23, -19, -8, 7, 21, -10, -5, -12, -1, 123, -125, 21, -16, 124, -125,
  7, 22, -31, 6, 127, -127, 129, -134, 20, 12, -25, -9, 14, 21, -33, 15, 129, -145, 147,
  -140, -1, -3, 11, -12, 151, -116, -13, -24, 11, -12, 160, -141, -15, 36, 124, -163, 9,
  -5, 3, 12, 150, -161, -6, 1, 24, 15, 133, -175, 177, -173, 24, -19, 8, -9, 2, 11, -20,
  4, 182, -188, 190, -143, -43, -2, 192, -191, 193, -195, 31, 17, -34, -10, 15, 30, -40,
  -5, 7, -10, 206, -183, 9, 18, -31, -21, 16, 37, -18, -32, 9, -1, 209, -218, -1, 54, 169,
  -219, 221, -219, -3, 9, 217, -227, 20, 5, 10, -32, 230, -235, 237, -234, -4, 29, -26,
  10, 0, 1, 26, -39, 247, -247, 13, 48, -56, -7, 255, -241, 6, -15, 9, 51, 196, -257, 23,
  -19, 36, -11, 234, -265, 267, -261, -2, 62, -62, 2, 267, -206, -54, -12, 276, -263,
  265, -246, -28, 0, 15, -21, 14, -5, 38, -12, 255, -289, 28, -12, -13, 69, -58, -14}

Out[134]= {1, -1, 3, -3, 5, -5, 1, 0, 8, -9, 11, -10, 0, -1, 15, -15, 17, -16, 1, 0, 19, -21, 3, 0, -2,
  0, 26, -26, 28, -29, 3, 0, 0, -3, 35, -31, 0, -3, 38, -38, 40, -39, -1, 3, 41, -45, 5, -4,
  4, -2, 48, -51, 5, -4, 4, 0, 52, -56, 58, -54, -3, -2, 6, -4, 63, -62, 3, -4, 67, -69,
  71, -65, -5, 3, 2, -4, 75, -76, 0, 6, 74, -80, 6, 0, 0, -5, 85, -86, 6, -3, 3, 0, 0, -7,
  95, -94, 2, -2, 98, -97, 99, -98, -1, 6, 97, -105, 107, -105, 6, -7, 110, -109, 6, -3,
  -1, 4, 0, -7, 8, 0, 0, -4, -2, -2, 124, -125, 9, -6, 126, -127, 7, 0, -8, 2, 132, -132,
  134, -135, 7, 0, 0, -9, 10, 0, -8, 4, 141, -146, 148, -145, 1, -2, 7, -8, 153, -145, 0,
  -9, 9, -10, 161, -154, -4, 7, 155, -164, 10, -8, 2, 2, 164, -168, 0, -1, 9, 0, 166, -176,
  178, -176, 8, -7, 7, -8, 8, -4, -5, 1, 186, -189, 191, -180, -8, -2, 194, -193, 195,
  -196, 11, 0, 0, -10, 10, 0, -6, -3, 9, -11, 208, -201, 4, 0, 0, -12, 12, 0, 0, -10, 10,
  -8, 217, -220, 0, 12, 212, -223, 225, -223, 0, 1, 226, -229, 11, -5, 5, -9, 233, -236,
  238, -237, -1, 8, -6, 1, 9, -8, 8, -12, 248, -248, 12, 0, -9, -4, 255, -251, 10, -11, 4,
  7, 247, -259, 12, -10, 10, -5, 258, -266, 268, -266, 1, 10, -9, -2, 272, -261, -7, -5,
  277, -275, 277, -272, -5, 0, 10, -14, 15, -11, 11, -5, 281, -290, 14, -9, -3, 12, 0, -14}

Out[135]= {1, -1, 3, -2, 4, -5, 1, 1, 7, -8, 10, -9, 0, -2, 15, -14, 16, -15, 1, 0, 18, -20, 2, 1, -3, 1,
  25, -25, 27, -29, 4, 0, 0, -3, 34, -30, 0, -3, 37, -37, 39, -38, -1, 3, 40, -44, 4, -3,
  4, -2, 47, -50, 5, -4, 4, 0, 51, -55, 57, -53, -3, -3, 7, -4, 62, -61, 3, -4, 66, -68,
  70, -64, -5, 3, 2, -4, 74, -75, -1, 7, 73, -79, 6, 0, 0, -5, 84, -85, 6, -3, 3, 0, 0, -7,
  94, -93, 2, -2, 97, -96, 98, -97, -1, 6, 96, -104, 106, -104, 6, -7, 109, -108, 6, -3,
  -1, 4, 0, -7, 7, 1, 0, -4, -3, -1, 123, -125, 10, -6, 125, -126, 7, 0, -8, 2, 131, -131,
  133, -134, 7, 0, 0, -9, 10, 0, -8, 4, 140, -145, 147, -144, 1, -2, 7, -8, 152, -144, 0,
  -9, 9, -10, 160, -153, -4, 7, 154, -163, 9, -7, 2, 2, 163, -167, 0, -1, 9, 0, 165, -175,
  177, -175, 8, -7, 7, -8, 8, -4, -5, 1, 185, -188, 190, -179, -8, -2, 193, -192, 194,
  -195, 11, 0, 0, -10, 10, 0, -6, -3, 9, -11, 207, -200, 4, 0, 0, -12, 12, 0, 0, -10, 10,
  -8, 216, -219, 0, 12, 211, -222, 224, -222, 0, 1, 225, -228, 11, -5, 5, -9, 232, -235,
  237, -236, -2, 9, -6, 1, 9, -8, 8, -12, 247, -247, 12, 0, -9, -5, 255, -250, 10, -11, 4,
  7, 246, -258, 12, -10, 10, -5, 257, -265, 267, -265, 1, 10, -9, -2, 271, -260, -7, -5,
  276, -274, 276, -271, -5, 0, 10, -14, 14, -10, 11, -5, 280, -289, 14, -9, -3, 12, 0, -14}

```

```
Out[136]= {1, -1, 3, 0, 2, -5, 1, 4, 4, -6, 8, -4, -1, -6, 15, -12, 14, -12, 3, 3, 10, -18, 0, 10, -12,
6, 20, -19, 21, -29, 12, 5, -7, -7, 32, -16, -5, -9, 34, -29, 31, -30, -5, 17, 22, -42,
2, 0, 13, -5, 38, -48, 11, -7, 13, 9, 28, -49, 51, -28, -23, -8, 16, -2, 51, -48, 7, -12,
57, -66, 68, -34, -31, 13, -3, 0, 61, -72, -4, 40, 40, -71, 10, 23, -13, -19, 76, -79, 10,
5, 9, 15, -25, -19, 92, -88, 5, -7, 94, -79, 81, -88, 0, 40, 52, -102, 104, -91, 22, -31,
104, -89, 4, 3, -15, 45, -37, -14, 1, 52, -19, -11, -28, 7, 115, -125, 44, -26, 111, -115,
10, 43, -61, 11, 118, -109, 111, -125, 36, 23, -49, -19, 29, 41, -65, 29, 110, -139, 141,
-130, -1, 0, 16, -18, 139, -76, -25, -49, 23, -25, 158, -120, -24, 66, 82, -155, 1, 11,
-2, 23, 128, -139, -22, 1, 49, 29, 88, -169, 171, -159, 42, -39, 17, -6, -8, 21, -39, 16,
165, -186, 188, -94, -78, -12, 188, -181, 183, -192, 63, 33, -67, -14, 24, 59, -79, -11,
15, -13, 194, -156, 19, 35, -61, -43, 33, 73, -35, -58, 12, 12, 181, -214, -1, 107, 112,
-203, 205, -199, -9, 10, 202, -215, 34, 9, 21, -56, 213, -229, 231, -228, -10, 60, -51,
34, -14, 1, 53, -79, 244, -239, 22, 95, -104, -23, 255, -209, -4, -24, 12, 101, 130, -247,
42, -30, 64, -23, 200, -259, 261, -252, 4, 116, -123, 12, 249, -136, -107, -20, 267, -229,
231, -210, -46, -1, 22, -43, 12, 19, 64, -25, 218, -281, 52, -25, -25, 137, -115, -26}
```

At the first sight it seems that this step added more variability with the introduction of negative items. Now apply a simple rule to each sequence. **Collect items in a subsequence until the sum of items equals its length so write it and restart with a new one.** This algorithm can be described with this function:

```
In[137]= splitseq[l_List] := Module[{i, parz = {}, res = {}},
  For[i = 1, i <= Length[l], i++,
    AppendTo[parz, l[[i]]];
    If[Total[parz] == Length[parz],
      AppendTo[res, parz];
      parz = {}
    ]
  ];
  Return[res]
]
```

So you can apply the above function to the sequences and you will get:

```
In[138]= t1=splitseq[s1]
t2=splitseq[s2]
t3=splitseq[s3]
t4=splitseq[s4]
t5=splitseq[s5]
t6=splitseq[s6]
t7=splitseq[s7]
t8=splitseq[s8]
```

```

Out[138]= {{1}, {-1, 3}, {-3, 5}, {-5, 1, -1, 9}, {-9, 11}, {-11, 1, -1, 15}, {-15, 17}, {-17, 1, -1, 21},
{-21, 3, -3, 1, -1, 27}, {-27, 29}, {-29, 1, -1, 3, -3, 35}, {-35, 1, -1, 39},
{-39, 41}, {-41, 1, -1, 45}, {-45, 5, -5, 1, -1, 51}, {-51, 3, -3, 1, -1, 57},
{-57, 59}, {-59, 1, -1, 3, -3, 65}, {-65, 1, -1, 69}, {-69, 71}, {-71, 1, -1, 5, -5, 77},
{-77, 1, -1, 81}, {-81, 3, -3, 1, -1, 87}, {-87, 5, -5, 1, -1, 3, -3, 95},
{-95, 1, -1, 99}, {-99, 101}, {-101, 1, -1, 105}, {-105, 107}, {-107, 1, -1, 111},
{-111, 3, -3, 1, -1, 5, -5, 9, -9, 1, -1, 3, -3, 125}, {-125, 1, -1, 129},
{-129, 5, -5, 1, -1, 135}, {-135, 137}, {-137, 1, -1, 9, -9, 3, -3, 1, -1, 147},
{-147, 149}, {-149, 1, -1, 3, -3, 155}, {-155, 1, -1, 5, -5, 161}, {-161, 1, -1, 165},
{-165, 11, -11, 1, -1, 171}, {-171, 3, -3, 1, -1, 177}, {-177, 179},
{-179, 1, -1, 3, -3, 9, -9, 1, -1, 189}, {-189, 191}, {-191, 1, -1, 195}, {-195, 197},
{-197, 1, -1, 5, -5, 3, -3, 1, -1, 9, -9, 209}, {-209, 1, -1, 3, -3, 5, -5, 1, -1, 11, -11, 221},
{-221, 1, -1, 225}, {-225, 227}, {-227, 1, -1, 231}, {-231, 3, -3, 1, -1, 237},
{-237, 239}, {-239, 1, -1, 3, -3, 11, -11, 1, -1, 249}, {-249, 9, -9, 1, -1, 255},
{-255, 5, -5, 1, -1, 261}, {-261, 3, -3, 1, -1, 267}, {-267, 269}, {-269, 1, -1, 3, -3, 275},
{-275, 1, -1, 279}, {-279, 281}, {-281, 1, -1, 5, -5, 15, -15, 1, -1, 291}}

Out[139]= {{1}, {-1, 3}, {-2, 4}, {-5, 1, 2, 6}, {-8, 10}, {-6, -2, -3, 15}, {-14, 16}, {-14, 2, 4, 12},
{-20, 2, 8, -10, 4, 22}, {-24, 26}, {-29, 9, 6, -10, -4, 34}, {-18, -6, -8, 36}, {-34, 36},
{-32, -6, 18, 24}, {-44, 4, -2, 12, -4, 40}, {-50, 8, -4, 12, 10, 30}, {-54, 56},
{-30, -24, -5, 11, -2, 56}, {-50, 6, -16, 64}, {-68, 70}, {-36, -32, 14, -8, 2, 66},
{-74, -2, 38, 42}, {-76, 10, 26, -14, -18, 78}, {-84, 8, 10, 8, 16, -28, -16, 94},
{-90, 4, -6, 96}, {-84, 86}, {-90, -6, 46, 54}, {-104, 106}, {-98, 26, -30, 106},
{-94, 4, 6, -16, 46, -42, -12, 6, 50, -20, -10, -26, 2, 120}, {-125, 41, -30, 118},
{-120, 8, 48, -62, 12, 120}, {-114, 116}, {-132, 40, 24, -58, -10, 26, 44, -66, 30, 112},
{-144, 146}, {-132, -2, -6, 20, -18, 144}, {-78, -26, -48, 18, -20, 160},
{-122, -30, 72, 84}, {-160, 6, 4, 2, 24, 130}, {-144, -22, 4, 48, 30, 90}, {-174, 176},
{-168, 48, -38, 14, -6, -14, 30, -40, 12, 172}, {-188, 190}, {-96, -84, -6, 190},
{-186, 188}, {-194, 62, 34, -72, -12, 24, 62, -80, -10, 6, -12, 204},
{-158, 18, 36, -64, -40, 28, 78, -36, -62, 6, 20, 186}, {-216, -2, 108, 114},
{-208, 210}, {-206, -12, 18, 204}, {-220, 34, 12, 20, -62, 222}, {-234, 236},
{-230, -8, 58, -54, 34, -22, 12, 52, -78, 246}, {-244, 16, 104, -110, -15, 255},
{-214, -6, -24, 16, 102, 132}, {-252, 42, -34, 70, -22, 202},
{-264, 266}, {-254, -4, 124, -126, 12, 254}, {-138, -108, -24, 274},
{-234, 236}, {-212, -52, -6, 28, -38, 14, 12, 68, -24, 220}}

Out[140]= {{1}, {-1, 3}, {-4, 6}, {-5, 1, 0, 8}, {-10, 12}, {-8, -3, 0, 15}, {-16, 18}, {-16, 1, 5, 14},
{-22, 4, 6, -8, 2, 24}, {-26, 28}, {-29, 6, 7, -13, -1, 36}, {-20, -7, -7, 38}, {-36, 38},
{-34, -7, 19, 26}, {-46, 6, -4, 11, -3, 42}, {-52, 5, -1, 11, 11, 32}, {-56, 58},
{-32, -25, -2, 6, 1, 58}, {-52, 5, -15, 66}, {-70, 72}, {-38, -33, 15, -13, 7, 68},
{-76, 0, 36, 44}, {-78, 7, 29, -15, -17, 80}, {-86, 3, 15, 7, 17, -31, -13, 96},
{-92, 3, -5, 98}, {-86, 88}, {-92, -7, 47, 56}, {-106, 108}, {-100, 25, -29, 108},
{-96, 1, 9, -17, 47, -47, -7, 8, 48, -21, -9, -24, 0, 122}, {-125, 38, -29, 120},
{-122, 3, 53, -63, 13, 122}, {-116, 118}, {-134, 39, 25, -67, -1, 23, 47, -67, 31, 114},
{-146, 148}, {-134, -3, -5, 17, -15, 146}, {-80, -27, -47, 13, -15, 162},
{-124, -31, 73, 86}, {-162, 8, 2, 1, 25, 132}, {-146, -25, 7, 47, 31, 92},
{-176, 178}, {-170, 47, -37, 11, -3, -23, 39, -41, 13, 174}, {-190, 192},
{-98, -85, -5, 192}, {-188, 190}, {-196, 61, 35, -77, -7, 21, 65, -81, -9, -3, -3, 206},
{-160, 17, 37, -67, -37, 23, 83, -37, -61, -5, 31, 188}, {-218, -3, 109, 116},
{-210, 212}, {-208, -13, 19, 206}, {-222, 31, 15, 19, -61, 224}, {-236, 238},
{-232, -6, 56, -57, 37, -33, 23, 51, -77, 248}, {-246, 7, 113, -111, -12, 255},
{-216, -11, -19, 15, 103, 134}, {-254, 39, -31, 69, -21, 204},
{-266, 268}, {-256, -5, 125, -129, 15, 256}, {-140, -109, -23, 276},
{-236, 238}, {-214, -53, -5, 23, -33, 16, 10, 67, -23, 222}}

```



```
Out[141]= {{1}, {-1, 3}, {-3, 5}, {-5, 1, 0, 8}, {-9, 11}, {-9, 0, -2, 15}, {-15, 17}, {-16, 2, 1, 17},
{-21, 3, 2, -4, 1, 25}, {-26, 28}, {-29, 5, 2, -3, -4, 35}, {-27, -2, -5, 38},
{-37, 39}, {-37, -2, 8, 35}, {-45, 5, -4, 7, -3, 46}, {-51, 6, -4, 7, 4, 44}, {-56, 58},
{-45, -11, -3, 7, -4, 62}, {-58, 4, -9, 67}, {-69, 71}, {-54, -15, 6, -1, -3, 73},
{-76, 0, 18, 62}, {-79, 7, 11, -6, -10, 83}, {-86, 7, 2, 5, 7, -12, -10, 95},
{-93, 3, -4, 98}, {-94, 96}, {-96, -2, 22, 80}, {-105, 107}, {-103, 14, -16, 109},
{-105, 6, 1, -7, 22, -18, -9, 8, 20, -9, -6, -11, -1, 123}, {-125, 21, -17, 125},
{-126, 8, 21, -30, 5, 128}, {-128, 130}, {-135, 21, 11, -24, -10, 15, 20, -32, 14, 130},
{-146, 148}, {-141, 0, -4, 12, -12, 151}, {-117, -12, -25, 12, -13, 161},
{-142, -15, 36, 125}, {-163, 9, -5, 3, 11, 151}, {-162, -5, 0, 25, 14, 134}, {-176, 178},
{-174, 25, -20, 9, -9, 2, 10, -19, 3, 183}, {-189, 191}, {-144, -42, -3, 193},
{-192, 194}, {-196, 32, 16, -33, -11, 16, 29, -39, -6, 8, -11, 207},
{-184, 10, 17, -30, -22, 17, 36, -17, -32, 9, -1, 209}, {-219, 0, 53, 170},
{-219, 221}, {-219, -3, 8, 218}, {-227, 20, 4, 11, -33, 231}, {-236, 238},
{-235, -3, 28, -25, 9, 1, 0, 27, -40, 248}, {-247, 13, 47, -56, -6, 255},
{-241, 6, -16, 10, 50, 197}, {-258, 24, -20, 37, -12, 235},
{-266, 268}, {-262, -2, 62, -61, 1, 268}, {-207, -53, -13, 277},
{-264, 266}, {-247, -27, -1, 16, -22, 15, -5, 38, -13, 256}}
```

```
Out[142]= {{1}, {-1, 3}, {-2, 4}, {-5, 1, 1, 7}, {-8, 10}, {-8, -1, -2, 15}, {-14, 16}, {-15, 1, 2, 16},
{-20, 2, 3, -5, 2, 24}, {-25, 27}, {-29, 5, 3, -4, -3, 34}, {-26, -3, -4, 37},
{-37, 39}, {-36, -3, 9, 34}, {-44, 4, -3, 6, -2, 45}, {-50, 5, -3, 6, 5, 43}, {-55, 57},
{-44, -12, -3, 7, -3, 61}, {-57, 3, -8, 66}, {-68, 70}, {-53, -16, 7, -2, -3, 73},
{-75, -1, 19, 61}, {-79, 7, 12, -7, -9, 82}, {-85, 6, 3, 4, 8, -13, -9, 94},
{-92, 2, -3, 97}, {-93, 95}, {-95, -3, 23, 79}, {-104, 106}, {-103, 14, -15, 108},
{-105, 6, 2, -8, 23, -19, -8, 7, 21, -10, -5, -12, -1, 123}, {-125, 21, -16, 124},
{-125, 7, 22, -31, 6, 127}, {-127, 129}, {-134, 20, 12, -25, -9, 14, 21, -33, 15, 129},
{-145, 147}, {-140, -1, -3, 11, -12, 151}, {-116, -13, -24, 11, -12, 160},
{-141, -15, 36, 124}, {-163, 9, -5, 3, 12, 150}, {-161, -6, 1, 24, 15, 133}, {-175, 177},
{-173, 24, -19, 8, -9, 2, 11, -20, 4, 182}, {-188, 190}, {-143, -43, -2, 192},
{-191, 193}, {-195, 31, 17, -34, -10, 15, 30, -40, -5, 7, -10, 206},
{-183, 9, 18, -31, -21, 16, 37, -18, -32, 9, -1, 209}, {-218, -1, 54, 169},
{-219, 221}, {-219, -3, 9, 217}, {-227, 20, 5, 10, -32, 230}, {-235, 237},
{-234, -4, 29, -26, 10, 0, 1, 26, -39, 247}, {-247, 13, 48, -56, -7, 255},
{-241, 6, -15, 9, 51, 196}, {-257, 23, -19, 36, -11, 234},
{-265, 267}, {-261, -2, 62, -62, 2, 267}, {-206, -54, -12, 276},
{-263, 265}, {-246, -28, 0, 15, -21, 14, -5, 38, -12, 255}}
```

```
Out[143]= {{1}, {-1, 3}, {-3, 5}, {-5, 1, 0, 8}, {-9, 11}, {-10, 0, -1, 15}, {-15, 17}, {-16, 1, 0, 19},
{-21, 3, 0, -2, 0, 26}, {-26, 28}, {-29, 3, 0, 0, -3, 35}, {-31, 0, -3, 38}, {-38, 40},
{-39, -1, 3, 41}, {-45, 5, -4, 4, -2, 48}, {-51, 5, -4, 4, 0, 52}, {-56, 58},
{-54, -3, -2, 6, -4, 63}, {-62, 3, -4, 67}, {-69, 71}, {-65, -5, 3, 2, -4, 75},
{-76, 0, 6, 74}, {-80, 6, 0, 0, -5, 85}, {-86, 6, -3, 3, 0, 0, -7, 95},
{-94, 2, -2, 98}, {-97, 99}, {-98, -1, 6, 97}, {-105, 107}, {-105, 6, -7, 110},
{-109, 6, -3, -1, 4, 0, -7, 8, 0, 0, -4, -2, -2, 124}, {-125, 9, -6, 126},
{-127, 7, 0, -8, 2, 132}, {-132, 134}, {-135, 7, 0, 0, -9, 10, 0, -8, 4, 141},
{-146, 148}, {-145, 1, -2, 7, -8, 153}, {-145, 0, -9, 9, -10, 161},
{-154, -4, 7, 155}, {-164, 10, -8, 2, 2, 164}, {-168, 0, -1, 9, 0, 166}, {-176, 178},
{-176, 8, -7, 7, -8, 8, -4, -5, 1, 186}, {-189, 191}, {-180, -8, -2, 194}, {-193, 195},
{-196, 11, 0, 0, -10, 10, 0, -6, -3, 9, -11, 208}, {-201, 4, 0, 0, -12, 12, 0, 0, -10, 10, -8, 217},
{-220, 0, 12, 212}, {-223, 225}, {-223, 0, 1, 226}, {-229, 11, -5, 5, -9, 233},
{-236, 238}, {-237, -1, 8, -6, 1, 9, -8, 8, -12, 248}, {-248, 12, 0, -9, -4, 255},
{-251, 10, -11, 4, 7, 247}, {-259, 12, -10, 10, -5, 258}, {-266, 268}, {-266, 1, 10, -9, -2, 272},
{-261, -7, -5, 277}, {-275, 277}, {-272, -5, 0, 10, -14, 15, -11, 11, -5, 281}}
```

```

Out[144]= {{1}, {-1, 3}, {-2, 4}, {-5, 1, 1, 7}, {-8, 10}, {-9, 0, -2, 15}, {-14, 16}, {-15, 1, 0, 18},
{-20, 2, 1, -3, 1, 25}, {-25, 27}, {-29, 4, 0, 0, -3, 34}, {-30, 0, -3, 37}, {-37, 39},
{-38, -1, 3, 40}, {-44, 4, -3, 4, -2, 47}, {-50, 5, -4, 4, 0, 51}, {-55, 57},
{-53, -3, -3, 7, -4, 62}, {-61, 3, -4, 66}, {-68, 70}, {-64, -5, 3, 2, -4, 74},
{-75, -1, 7, 73}, {-79, 6, 0, 0, -5, 84}, {-85, 6, -3, 3, 0, 0, -7, 94},
{-93, 2, -2, 97}, {-96, 98}, {-97, -1, 6, 96}, {-104, 106}, {-104, 6, -7, 109},
{-108, 6, -3, -1, 4, 0, -7, 7, 1, 0, -4, -3, -1, 123}, {-125, 10, -6, 125},
{-126, 7, 0, -8, 2, 131}, {-131, 133}, {-134, 7, 0, 0, -9, 10, 0, -8, 4, 140},
{-145, 147}, {-144, 1, -2, 7, -8, 152}, {-144, 0, -9, 9, -10, 160},
{-153, -4, 7, 154}, {-163, 9, -7, 2, 2, 163}, {-167, 0, -1, 9, 0, 165}, {-175, 177},
{-175, 8, -7, 7, -8, 8, -4, -5, 1, 185}, {-188, 190}, {-179, -8, -2, 193}, {-192, 194},
{-195, 11, 0, 0, -10, 10, 0, -6, -3, 9, -11, 207}, {-200, 4, 0, 0, -12, 12, 0, 0, -10, 10, -8, 216},
{-219, 0, 12, 211}, {-222, 224}, {-222, 0, 1, 225}, {-228, 11, -5, 5, -9, 232},
{-235, 237}, {-236, -2, 9, -6, 1, 9, -8, 8, -12, 247}, {-247, 12, 0, -9, -5, 255},
{-250, 10, -11, 4, 7, 246}, {-258, 12, -10, 10, -5, 257}, {-265, 267}, {-265, 1, 10, -9, -2, 271},
{-260, -7, -5, 276}, {-274, 276}, {-271, -5, 0, 10, -14, 14, -10, 11, -5, 280}}

Out[145]= {{1}, {-1, 3}, {0, 2}, {-5, 1, 4, 4}, {-6, 8}, {-4, -1, -6, 15}, {-12, 14}, {-12, 3, 3, 10},
{-18, 0, 10, -12, 6, 20}, {-19, 21}, {-29, 12, 5, -7, -7, 32}, {-16, -5, -9, 34},
{-29, 31}, {-30, -5, 17, 22}, {-42, 2, 0, 13, -5, 38}, {-48, 11, -7, 13, 9, 28}, {-49, 51},
{-28, -23, -8, 16, -2, 51}, {-48, 7, -12, 57}, {-66, 68}, {-34, -31, 13, -3, 0, 61},
{-72, -4, 40, 40}, {-71, 10, 23, -13, -19, 76}, {-79, 10, 5, 9, 15, -25, -19, 92},
{-88, 5, -7, 94}, {-79, 81}, {-88, 0, 40, 52}, {-102, 104}, {-91, 22, -31, 104},
{-89, 4, 3, -15, 45, -37, -14, 1, 52, -19, -11, -28, 7, 115}, {-125, 44, -26, 111},
{-115, 10, 43, -61, 11, 118}, {-109, 111}, {-125, 36, 23, -49, -19, 29, 41, -65, 29, 110},
{-139, 141}, {-130, -1, 0, 16, -18, 139}, {-76, -25, -49, 23, -25, 158},
{-120, -24, 66, 82}, {-155, 1, 11, -2, 23, 128}, {-139, -22, 1, 49, 29, 88}, {-169, 171},
{-159, 42, -39, 17, -6, -8, 21, -39, 16, 165}, {-186, 188}, {-94, -78, -12, 188},
{-181, 183}, {-192, 63, 33, -67, -14, 24, 59, -79, -11, 15, -13, 194},
{-156, 19, 35, -61, -43, 33, 73, -35, -58, 12, 12, 181}, {-214, -1, 107, 112},
{-203, 205}, {-199, -9, 10, 202}, {-215, 34, 9, 21, -56, 213}, {-229, 231},
{-228, -10, 60, -51, 34, -14, 1, 53, -79, 244}, {-239, 22, 95, -104, -23, 255},
{-209, -4, -24, 12, 101, 130}, {-247, 42, -30, 64, -23, 200},
{-259, 261}, {-252, 4, 116, -123, 12, 249}, {-136, -107, -20, 267},
{-229, 231}, {-210, -46, -1, 22, -43, 12, 19, 64, -25, 218}}

```

Theorem 1

Let f be a function defined over the factorization of numbers. The sum of the differences of k consecutive values equals the difference between last and first value.

Demonstration

$$v_1 + v_2 + \dots + v_k = f(2) - f(1) + f(3) - f(2) + \dots + f(k) - f(k-1) = f(k) - f(1)$$

Theorem 2

Let f be a function defined over the factorization of numbers. $f(\text{Prime}(i)) = \text{Prime}(i)$ and $f(\text{Prime}(i+k)) - f(\text{Prime}(i)) = \text{Prime}(i+k) - \text{Prime}(i)$

Demonstration

Follows the function definitions.

These function seems to have the property that $f(\text{Prime}(i)+k) - f(\text{Prime}(i)) = k$ iff $\text{Prime}(i)+k$ is prime.

This is not true for other functions over the factorization of numbers:

In[146]=

```
splitseq[Differences[prodrad/@Range[2, 300]]]
```

```
Out[146]= {{1}, {-1, 3}, {1}, {1}, {-5, 1, 7}, {1}, {-5, 7}, {1}, {1}, {-13, 15}, {-11, 13}, {-9, 11},
{1}, {1}, {-17, -1, 21}, {-23, 11, 15}, {1}, {1}, {-29, 31}, {1}, {1}, {-29, 31}, {1},
{1}, {-29, 31}, {1}, {1}, {-21, -7, 31}, {1}, {-41, 1, 3, 41}, {-25, 27}, {-47, 49},
{-41, 43}, {1}, {1}, {-29, 31}, {1}, {-41, -19, 63}, {1}, {1}, {-33, 35}, {1}, {1},
{-65, 67}, {1}, {-59, 23, 39}, {1}, {1}, {-69, -7, 79}, {1}, {-41, 43}, {1}, {1},
{-65, 67}, {-59, 61}, {-45, 47}, {1}, {1}, {-89, 91}, {-83, 19, -23, 91}, {1}, {1},
{-77, 79}, {1}, {1}, {-101, 103}, {1}, {1}, {-97, 99}, {1}, {1}, {-57, -19, 79},
{1}, {-89, -19, 111}, {1}, {-61, -57, 37, 85}, {-125, 127}, {1}, {1}, {-65, 67}, {1},
{-119, 19, 103}, {1}, {1}, {-69, 71}, {1}, {1}, {-137, 139}, {1}, {-125, 53, 75},
{-119, 121}, {-113, 13, 103}, {1}, {-77, 79}, {1}, {1}, {-149, 151}, {-155, 157},
{-81, 83}, {1}, {1}, {-125, -29, 157}, {-113, 29, 87}, {1}, {-139, -13, 155}, {1}, {1},
{-149, 151}, {1}, {1}, {-137, 139}, {1}, {1}, {-93, -73, 169}, {1}, {-185, 187}, {1}, {1},
{-181, 183}, {-131, 133}, {-189, 191}, {1}, {1}, {-101, 103}, {1}, {-137, -43, 183}, {1},
{1}, {-105, 107}, {1}, {1}, {-209, 211}, {1}, {1}, {-109, 111}, {1}, {1}, {-209, 1, 211},
{1}, {-113, 115}, {1}, {1}, {-173, 175}, {-155, 157}, {-117, 119}, {1}, {1}, {-209, 211},
{-219, -19, 119, -87, 211}, {1}, {-185, 187}, {-239, 241}, {-209, 211}, {1}, {1},
{-253, 255}, {1}, {1}, {-129, -43, 175}, {1}, {-197, 199}, {1}, {1}, {-133, 135},
{-239, 241}, {-237, 239}, {1}, {-219, 83, 139}, {1}, {-185, -23, 211}, {1}, {1}, {-141, 143},
{1}, {1}, {-281, 11, 273}, {1}, {-145, 147}, {-251, 253}, {-221, -41, 265}, {1}}
```

As an example $\text{prodrad}(6) - \text{prodrad}(5) = 6 - 5 = 1$ and 6 is not prime.

Now let's analyze deeply sequence $t1$. Every subsequence has the pattern $\{-p+2, c1, -c1, c2, -c2, \dots, cn, -cn, p + \text{subslen} - 2\}$ where $p = \text{Prime}(i)$ and $\text{subslen} = \text{Prime}(i+1) - \text{Prime}(i)$.

We couple each number with the corresponding natural number:

```
In[147]:= z1=Transpose [ {s1,Range [2,299] } ]
```

```
Out[147]= {{1, 2}, {-1, 3}, {3, 4}, {-3, 5}, {5, 6}, {-5, 7}, {1, 8}, {-1, 9}, {9, 10}, {-9, 11}, {11, 12},
{-11, 13}, {1, 14}, {-1, 15}, {15, 16}, {-15, 17}, {17, 18}, {-17, 19}, {1, 20}, {-1, 21},
{21, 22}, {-21, 23}, {3, 24}, {-3, 25}, {1, 26}, {-1, 27}, {27, 28}, {-27, 29}, {29, 30},
{-29, 31}, {1, 32}, {-1, 33}, {3, 34}, {-3, 35}, {35, 36}, {-35, 37}, {1, 38}, {-1, 39},
{39, 40}, {-39, 41}, {41, 42}, {-41, 43}, {1, 44}, {-1, 45}, {45, 46}, {-45, 47}, {5, 48},
{-5, 49}, {1, 50}, {-1, 51}, {51, 52}, {-51, 53}, {3, 54}, {-3, 55}, {1, 56}, {-1, 57},
{57, 58}, {-57, 59}, {59, 60}, {-59, 61}, {1, 62}, {-1, 63}, {3, 64}, {-3, 65}, {65, 66},
{-65, 67}, {1, 68}, {-1, 69}, {69, 70}, {-69, 71}, {71, 72}, {-71, 73}, {1, 74}, {-1, 75},
{5, 76}, {-5, 77}, {77, 78}, {-77, 79}, {1, 80}, {-1, 81}, {81, 82}, {-81, 83}, {3, 84},
{-3, 85}, {1, 86}, {-1, 87}, {87, 88}, {-87, 89}, {5, 90}, {-5, 91}, {1, 92}, {-1, 93},
{3, 94}, {-3, 95}, {95, 96}, {-95, 97}, {1, 98}, {-1, 99}, {99, 100}, {-99, 101}, {101, 102},
{-101, 103}, {1, 104}, {-1, 105}, {105, 106}, {-105, 107}, {107, 108}, {-107, 109}, {1, 110},
{-1, 111}, {111, 112}, {-111, 113}, {3, 114}, {-3, 115}, {1, 116}, {-1, 117}, {5, 118},
{-5, 119}, {9, 120}, {-9, 121}, {1, 122}, {-1, 123}, {3, 124}, {-3, 125}, {125, 126},
{-125, 127}, {1, 128}, {-1, 129}, {129, 130}, {-129, 131}, {5, 132}, {-5, 133}, {1, 134},
{-1, 135}, {135, 136}, {-135, 137}, {137, 138}, {-137, 139}, {1, 140}, {-1, 141}, {9, 142},
{-9, 143}, {3, 144}, {-3, 145}, {1, 146}, {-1, 147}, {147, 148}, {-147, 149}, {149, 150},
{-149, 151}, {1, 152}, {-1, 153}, {3, 154}, {-3, 155}, {155, 156}, {-155, 157}, {1, 158},
{-1, 159}, {5, 160}, {-5, 161}, {161, 162}, {-161, 163}, {1, 164}, {-1, 165}, {165, 166},
{-165, 167}, {11, 168}, {-11, 169}, {1, 170}, {-1, 171}, {171, 172}, {-171, 173}, {3, 174},
{-3, 175}, {1, 176}, {-1, 177}, {177, 178}, {-177, 179}, {179, 180}, {-179, 181}, {1, 182},
{-1, 183}, {3, 184}, {-3, 185}, {9, 186}, {-9, 187}, {1, 188}, {-1, 189}, {189, 190},
{-189, 191}, {191, 192}, {-191, 193}, {1, 194}, {-1, 195}, {195, 196}, {-195, 197},
{197, 198}, {-197, 199}, {1, 200}, {-1, 201}, {5, 202}, {-5, 203}, {3, 204}, {-3, 205},
{1, 206}, {-1, 207}, {9, 208}, {-9, 209}, {209, 210}, {-209, 211}, {1, 212}, {-1, 213},
{3, 214}, {-3, 215}, {5, 216}, {-5, 217}, {1, 218}, {-1, 219}, {11, 220}, {-11, 221},
{221, 222}, {-221, 223}, {1, 224}, {-1, 225}, {225, 226}, {-225, 227}, {227, 228},
{-227, 229}, {1, 230}, {-1, 231}, {231, 232}, {-231, 233}, {3, 234}, {-3, 235}, {1, 236},
{-1, 237}, {237, 238}, {-237, 239}, {239, 240}, {-239, 241}, {1, 242}, {-1, 243}, {3, 244},
{-3, 245}, {11, 246}, {-11, 247}, {1, 248}, {-1, 249}, {249, 250}, {-249, 251}, {9, 252},
{-9, 253}, {1, 254}, {-1, 255}, {255, 256}, {-255, 257}, {5, 258}, {-5, 259}, {1, 260},
{-1, 261}, {261, 262}, {-261, 263}, {3, 264}, {-3, 265}, {1, 266}, {-1, 267}, {267, 268},
{-267, 269}, {269, 270}, {-269, 271}, {1, 272}, {-1, 273}, {3, 274}, {-3, 275}, {275, 276},
{-275, 277}, {1, 278}, {-1, 279}, {279, 280}, {-279, 281}, {281, 282}, {-281, 283},
{1, 284}, {-1, 285}, {5, 286}, {-5, 287}, {15, 288}, {-15, 289}, {1, 290}, {-1, 291},
{291, 292}, {-291, 293}, {3, 294}, {-3, 295}, {1, 296}, {-1, 297}, {11, 298}, {-11, 299}}
```

```
In[148]:= Map [Last,Select [z1,#[ [2] ]==2-#[ [1] ]& ]  
Prime/@Range [2,62]
```

```
Out[148]= {3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101,
103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193,
197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293}
```

```
Out[149]= {3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101,
103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193,
197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293}
```

We can see that all and only the primes are present and for the odd primes all the couples are $(-p+2,p)$

```
In[150]:= k=Map [Last,Select [z1,# [ [1] ] ==1&] ]  
Differences [k]
```

```
Out[150]= {2, 8, 14, 20, 26, 32, 38, 44, 50, 56, 62, 68, 74, 80, 86, 92, 98, 104,  
110, 116, 122, 128, 134, 140, 146, 152, 158, 164, 170, 176, 182, 188, 194, 200,  
206, 212, 218, 224, 230, 236, 242, 248, 254, 260, 266, 272, 278, 284, 290, 296}
```

```
Out[151]= {6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6,  
6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6}
```

```
In[152]:= k=Map [Last,Select [z1,# [ [1] ] ==3&] ]  
Differences [k]
```

```
Out[152]= {4, 24, 34, 54, 64, 84, 94, 114, 124, 144, 154, 174, 184, 204, 214, 234, 244, 264, 274, 294}
```

```
Out[153]= {20, 10, 20, 10, 20, 10, 20, 10, 20, 10, 20, 10, 20, 10, 20, 10, 20, 10, 20}
```

```
In[154]:= k=Map [Last,Select [z1,# [ [1] ] ==5&] ]  
Differences [k]
```

```
Out[154]= {6, 48, 76, 90, 118, 132, 160, 202, 216, 258, 286}
```

```
Out[155]= {42, 28, 14, 28, 14, 28, 42, 14, 42, 28}
```

```
In[156]:= k=Map [Last,Select [z1,# [ [1] ] ==9&] ]  
Differences [k]
```

```
Out[156]= {10, 120, 142, 186, 208, 252}
```

```
Out[157]= {110, 22, 44, 22, 44}
```

```
In[158]:= k=Map [Last,Select [z1,# [ [1] ] ==11&] ]  
Differences [k]
```

```
Out[158]= {12, 168, 220, 246, 298}
```

```
Out[159]= {156, 52, 26, 52}
```

So the gap between equal values follows a rule: if n is the number (always odd) the gap is a multiple of $2(n+2)$.

Main Results

*Each subsequence length corresponds to a gap between primes in each sequence.

*With minrad each subsequence has other properties:

+every first item of a subsequence is equal to last item of the preceding subsequence but with changed sign

+excluding first and last item of a subsequence, other items are in couples $(n,-n)$