Refutation of orthomodular quantum logic

Abstract: We evaluate four equations, two for distributivity in Hilbert space and two for Boolean models of ZFC set theory with all as not tautologous. This refutes orthomodular quantum logic, to form a non tautologous fragment of the universal logic VL4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬; + Or, ∨, ∪; - Not Or; & And, ∧, ∩, ·, ⊓; \ Not And;
> Imply, greater than, →, ⇒, ⊃, ⊨; < Not Imply, less than, ∈, ⊂, ⊄, ⊬, ⊣,
= Equivalent, ≡, :=, ↔, ≜, ≈; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ∃!, ∇, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, T, ordinal 3; (z@z) F as contradiction, Ø, Null, ⊥, zero;
(%z>=%z) N as non-contingency, Δ, ordinal 1; (%z<=%z) C as contingency, V, ordinal 2;
~( y < x) ( x ≤ y), ( x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

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2 Orthomodular quantum logic

Given a quantum system S, the space of states is always assumed to be a Hilbert space H, and physical quantities (e.g. angular momentum, position, mass etc) are represented not as real valued functions on H, but rather as self-adjoint operators on H. The spectral theorem for self-adjoint operators tells us that physical propositions (statements about the values of physical quantities) are in bijective correspondence up to logical equivalence with the closed subspaces of H or, equivalently, with projection operators onto the closed subspaces of H. Thus the logical structure of the physical propositions associated with a quantum system is given by the lattice of projection operators on the corresponding Hilbert space. But since closed subspaces are not closed under unions, this is not a simple subset algebra. In particular, although we can take the meet.. of two subspaces to be the intersection, we need to define the join of two subspaces as the closed linear sum, not the union. And as Birkhoff and von Neumann pointed out .. , these two operations do not satisfy the distributive laws, i.e.

\[ a \lor (b \land c) = (a \lor b) \land (a \lor c), \]
\[ a \land (b \lor c) = (a \land b) \lor (a \land c), \]

are not guaranteed to hold where a, b, c are subspaces of a Hilbert space H and \( \land, \lor \) are the lattice operations given by intersection and closed linear sum, respectively. …

Remark 2.1: Eqs. 2.1.1 and 2.2.1 are classical tautologies which do not hold as distributions in Hilbert spaces. This proves Hilbert spaces are not bivalent and exact but rather vector spaces and probabilistic.

3 QST: the basics
3.1 Boolean valued models
Boolean valued models of set theory were originally developed in the 1960’s by Scott, Solovay and Vopenka, as a way of providing a novel perspective on the independence proofs of Paul Cohen. Intuitively, they can be thought of as generalizations of the usual universe $V$ (the ‘ground model’) of classical set theory (ZFC).

So it only remains to define Boolean truth values for atomic B-sentences. For technical reasons, it turns out that the following simultaneous definition is best.

\[(ix) \lbrack u \in v \rbrack = \lbrack \exists y \in v (u = y) \rbrack \quad (3.1.9.1)\]

\[
\text{LET } p, q, r, s: u, v, x, y.
\]

\[(p=q) = ((%s<q) \& (q\&(p=s))) ; \ \text{FTTF FTTF FTTF FTTF} \quad (3.1.9.2)\]

\[(x) \lbrack u = v \rbrack = \lbrack \forall x \in u (x \in v) \rbrack \land \lbrack \forall y \in v (y \in u) \rbrack \quad (3.1.10.1)\]

\[(p=q) = (((#r<p)\& (r<q))\&((#s<q)\&(s<p))) ; \ \text{FTTF FTTF FTTF NTTF} \quad (3.1.10.2)\]

**Remark 3.1:** Eqs.3.1.9.2 and 3.1.10.2 are *not* tautologous. This refutes Dana Scott, Robert M. Solovay, and Petr Vopěnka, after Paul Cohen, in mapping Boolean valued models of ZFC set theory.