On the Lorentz covariance of conservation law for moving system.

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Abstract

The generalized conservation law is given by Noether's theorem, the conservation law of momentum is given based on the symmetry of space translation, the conservation law of energy is given based on the symmetry of time translation, and the conservation law of angular momentum is given based on the isotropy of space.However, we all know that in the Lorentz transformation, the space-time between two frames of reference of relative uniform motion is not equivalent conversion, which means that the conservation law maybe not meet the precondition of symmetry of translation.In this paper, by establishing the Lorentz transformation equations of work and impulse between reference frames,I deduce some physical quantities cannot keep their symmetry in Lorentz transformation, and discuss the conditions for the law of conservation of dynamics is covariant. By applying the equations of the Lorentz transformation, a new Lorentz covariate is obtained.

1. Introduction

There are many laws of conservation of dynamics in classical mechanics, such as "law of conservation of momentum", "law of conservation of energy".They originate from Newton's law of motion. After a lot of experiments, they have been proved to be beyond classical mechanics and become the core theory of physics.In 1918,**Emmy Noether** proposed "Noether's theorem", which verified conserved quantities originate from the symmetry of translation , that is, every continuous symmetry transformation of a mechanical system has a conservation quantity corresponding to it.This also became the pillar of modern theoretical physics.

It is clear to us that the basis of dynamics is the translation and rotation of physical quantity with space-time. Because of the uniformity and isotropy of space-time, physical laws are universal. This is also the basis of the conservation law.In classical mechanics, since the Galilean transformation is independent of time and space, it assumes that space and time are constants that are independent of motion. They always remain uniform and isotropic. Therefore, the law of conservation is covariant to the Galilean transformation.However, in the theory of relativity, it can be known from the Lorentz transformation that the space-time transformation of the reference frame of relative motion and the stationary frame of reference are not equivalent, but a function related to the speed of motion.The best verification of this transformation is the "Length Contraction Effect" and "Time Dilation Effect". Therefore, when we introduce conservation laws into the Lorentz transformation, we must be extra cautious.

In the real world, this effect is so weak that it can be almost ignored, but in high-energy physics, the relativistic effect due to high-speed motion will be very remarkable.It is necessary to translate the physical quantity from the "Moving frame of reference" to the "Static frame of reference" in accordance with the Lorentz transformation.In order to explore the transformation of physical quantities between these two reference frames, we choose two reference frames S and S' that move relative to each other, and the relative velocity is $\vec{v}_0$ (As shown in Figure 1).Since the establishment of reference frames will not affect the analysis results, x , x' and $\vec{v}_0$ can be set to be parallel in the same direction to simplify the operation.

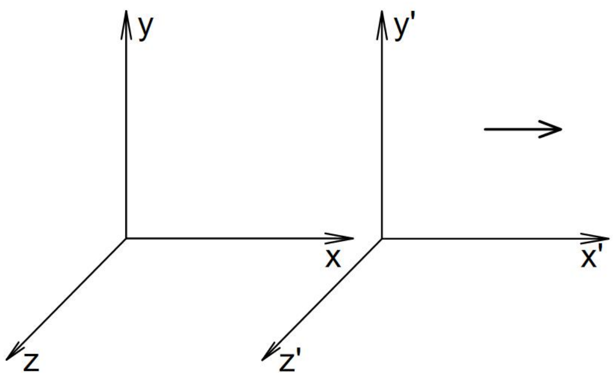

Figure 1

2.The Lorentz Transformation of work.

According to mass-velocity relation and the kinetic energy theorem, $dW = dE_k = dmc^2$ can be derived(Derivation process in Appendix).In the S and S' frame of references,the infinitely short displacement $d\vec{r}$ and $d\vec{r}'$ are taken. Since the frame of reference is unchanged, the Lorentz transformation is still true. Let the static mass of the object is m_0 .Initially,the corresponding speeds are $\vec{v}_1$ and $\vec{v}_1'$ in the two frames , and become $\vec{v}_2$ and $\vec{v}_2'$ after the object is affected by work. For the two frames of reference, there are:

$$S: dW = \vec{F}d\vec{r} = (m_2 - m_1)c^2 = dmc^2 \quad \textbf{(2.1)}$$

$$S': dW' = \vec{F}'d\vec{r}' = (m_2' - m_1')c^2 = dm'c^2 \quad \textbf{(2.2)}$$

In order to find out the relationship between dW and dW', we must first find out the relationship between dm and dm'.

The mass-velocity relation is $m(v) = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$, m_0 is the static mass of the object.The Lorentz transformation of velocity is also given that

$$v_x = \frac{v_x' + v_0}{1 + \frac{v_0 v_x'}{c^2}}, v_y = \frac{v_y'\sqrt{1-\frac{v_0^2}{c^2}}}{1 + \frac{v_0 v_x'}{c^2}}, v_z = \frac{v_z'\sqrt{1-\frac{v_0^2}{c^2}}}{1 + \frac{v_0 v_x'}{c^2}}$$

We also know $v^2 = v_x^2 + v_y^2 + v_z^2$,so it means

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1 + \frac{\vec{v}_0 \cdot \vec{v}'}{c^2}}{\sqrt{1-\frac{v_0^2}{c^2}} \cdot \sqrt{1-\frac{v'^2}{c^2}}} \quad \textbf{(2.3)}$$

According these we can get (The detailed derivation process is shown in the Appendix.)

$$dm' = m_2' - m_1' = m(v_2') - m(v_1') = \frac{m_0}{\sqrt{1-\frac{v_2'^2}{c^2}}} - \frac{m_0}{\sqrt{1-\frac{v_1'^2}{c^2}}}$$

$$dm = m_2 - m_1 = m(v_2) - m(v_1) = \frac{m_0}{\sqrt{1-\frac{v_2^2}{c^2}}} - \frac{m_0}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}(dm' + \frac{\vec{v}_0}{c^2} d\vec{I}')$$

So we can get the Lorentz Transformation equation of work is

$$dW = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}(dW' + \vec{v}_0 \cdot d\vec{I}') \qquad \textbf{(2.4)}$$

In classic mechanics,there is

$$dW = dW' + \vec{v}_0 \cdot d\vec{I}'$$

When $v_0 << c$,both are equivalent.

If we integrate the two sides of this equation, we have

$$\Delta W = \frac{1}{\sqrt{1-{v_0^2}/{c^2}}}(\Delta W' + \vec{v}_0 \bullet \Delta\vec{p}') \qquad \textbf{(2.5)}$$

In order to test the correctness of this equation, let's try to use it to explain the formula of photon frequency shift in different reference frames

Let a photon be generated at a certain time.The corresponding frequency is ν in the S and ν' in the S' .The angle between the photon motion direction and the relative velocity $\vec{v}_0$ direction of the reference frames are θ and θ' .By the Lorentz transformation of velocity and the principle of constant speed of light,there are

$$\cos\theta = \frac{\cos\theta' + \frac{v_0}{c}}{1 + \frac{v_0}{c}\cos\theta'} \quad \text{or} \quad \cos\theta' = \frac{\cos\theta - \frac{v_0}{c}}{1 - \frac{v_0}{c}\cos\theta}$$

In the S' ,there is $\Delta W' = h\nu' = p' \bullet c$, and p' is the magnitude of the photon's momentum in S' .It's easy to know $\Delta\vec{p}' = \vec{p}'$,so we can get

$$\vec{v}_0 \bullet \Delta\vec{p}' = p' v_0 \cos\theta' = \frac{v_0}{c} h\nu' \cos\theta'$$

In the S ,there is $\Delta W = h\nu$,by substituting equation into equation , we can get

$$\nu = \frac{1}{\sqrt{1-{v_0^2}/{c^2}}}(1 + \frac{v_0}{c}\cos\theta')\nu' \qquad \textbf{(2.6)}$$

Use the angle relation obtained before,there is

$$\nu = \frac{\sqrt{1-{v_0^2}/{c^2}}}{1-\frac{v_0}{c}\cos\theta}\nu' \qquad \textbf{(2.7)}$$

As we all know,these two equations are relativistic frequency shift equations.It means that we can get the Doppler effect of light by the transformation of work.Thus, equation 1 is objective.We can also come to the conclusion that the essence of Doppler effect is that the difference of work in reference frames results in the difference of wave energy.

3. The Lorentz transformation of impulse.

Referring to the previous content, we use the same method to explore the Lorentz Transformation of impulse:

Still in the S and S', taking infinitely short time intervals dt and dt'.Let the static mass of the object is m_0,and have corresponding speeds $\vec{v}_1$ and $\vec{v}_1'$ in the two frames of reference, and become $\vec{v}_2$ and $\vec{v}_2'$ after effected by the impulse. For both frames of reference, there are:

$$S: d\vec{I} = \vec{F}dt = m_2\vec{v}_2 - m_1\vec{v}_2 \qquad \textbf{(3.1)}$$

$$S': d\vec{I}' = \vec{F}'dt' = m_2'\vec{v}_2' - m_1'\vec{v}_2' \qquad \textbf{(3.2)}$$

Using the Lorentz Transformation of velocity and $m(v)$,there is

$$dI_x = \frac{1}{\sqrt{1-{v_0^2}/{c^2}}}(dm'v_0 + dI_x'),\ dI_y = dI_y',\ dI_z = dI_z' \qquad \textbf{(3.3)}$$

(The detailed derivation process is shown in the Appendix.)

Reduced to matrix form:

$$(dI_x, dI_y, dI_z) = (dm'v_0 + dI_x', dI_y', dI_z')\begin{bmatrix} \frac{1}{\sqrt{1-{v_0^2}/{c^2}}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \textbf{(3.4)}$$

The relationship between dm and dm' has been given before,so it is equivalent to

$$d\vec{I} = dm \cdot \vec{v}_0 + d\vec{I}' \begin{bmatrix} \sqrt{1 - {v_0^2}/{c^2}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \textbf{(3.5)}$$

We have obtained the Lorentz Transformation of work and impulse,now let's test the Lorentz covariance of these two physical quantities:

It is easy to know $dW = \vec{v} \cdot d\vec{I}$,take it to equation 2,we have

$$dW = \vec{v} \cdot d\vec{I} = \vec{v} \cdot (dm'\vec{v}_0 + d\vec{I}') \begin{bmatrix} \frac{1}{\sqrt{1 - {v_0^2}/{c^2}}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Using the Lorentz Transformation of velocity and simplify this equation to get

$$dW = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} (dW' + \vec{v}_0 \cdot d\vec{I}')$$

It can be seen that the equations corresponding to these two physical quantities are covariant to Lorentz transformation.

Now let's consider the following case:

We assume that there is a α system which consists of n objects be stationary in the S' frame of reference,and we take the resultant force in the S' on this system is 0 .That is, $\sum_{\alpha} d\vec{I}' = 0$ .In the S ,there is

$$\sum_{\alpha} d\vec{I} = \vec{v}_0 \cdot \sum_{\alpha} dm + \sum_{\alpha} d\vec{I}' \begin{bmatrix} \sqrt{1 - \frac{v_0^2}{c^2}} & & \\ & 1 & \\ & & 1 \end{bmatrix} = dm_{\alpha} \vec{v}_0 = \frac{dm_{\alpha}'}{\sqrt{1 - \frac{v_0^2}{c^2}}} \vec{v}_0 \qquad \textbf{(3.6)}$$

Where $dm_{\alpha}(dm_{\alpha}')$ is the total mass change of system ɑ in the $S(S')$.

It means even if the resultant force on a system in a reference frame is 0,the resultant force on the system in other reference frames which have relative motion with the reference frame is not 0.

The process of effect is divided into innumerable parts, each time interval is dt (dt' in the S'), and each process conforms to the above formula,and integrate the entire process change, there is:

$$\Delta\vec{P}_{\alpha} = \int_{t_1}^{t_2} \sum_{\alpha} d\vec{I} = \int_{m_1}^{m_2} dm_{\alpha}\vec{v}_0 = \Delta m_{\alpha}\vec{v}_0$$

Table 1. Description of parameter

	Time	Mass
Start of effect	t_1	m_1
Now	t_2	m_2

This is an amazing conclusion,due to the mass of the system α will change with work in the Lorentz transformation,we cannot define that the impulse of the α in any reference frame is 0.It means that in the process of introducing the law of conservation of momentum into the Lorentz transformation, we may have neglected something worthy of discussion.

4.Discussion on conservation law in the Lorentz transformation.

4.1The conservation law within the Lorentz transformation

Before we begin this part of the discussion, let's review the source of the conclusion that "energy is equal to mass": from the mass velocity relation $m(v) = \frac{m_0}{\sqrt{1-v^2/c^2}}$ and the kinetic energy theorem, $dE_k = dmc^2$ can be obtained by combining calculus.This is the equation for the equivalence of kinetic energy and mass of the object. We all know that in addition to kinetic energy, there is also interaction energy between the components of the system. So how to give the equation for the equivalence of interaction energy and mass?

First of all, there is the law of conservation of energy and mass for a closed system, which means that the energy and mass of the system remain the same during the process of the conversion of kinetic energy and interaction energy of the object in the system; secondly, when the kinetic energy of the objects in the system decreases(or increases) ΔE_k , that is, when the corresponding mass decreases (or increases) Δm_k , the interaction energy of the system increases(or decreases) $\Delta E_p = \Delta E_k$,and the corresponding mass also increases(or decreases) $\Delta m_p = \Delta m_k$, so there is obviously $\Delta E_p = \Delta m_p c^2$, that is, the equation that interaction energy equivalent to the mass.If we consider a system of motion, because the mass of the system is constant, we can also draw the conclusion that the conservation of momentum for this system.

So far, all the conclusions about "the interaction energy is equal to the mass" and "the law of conservation of momentum is covariant with the Lorentz transformation" are drawn by similar methods.That is," the law of conservation of energy", "the law of conservation of mass", "the law of conservation of momentum" and "the interaction energy is equal to the mass", and at least two of them as know are introduced into the Lorentz transformation to deduce other conclusion.But it involves a fatal defect. If as the known conclusion itself can not have the Lorentz covariance but is introduced into the Lorentz dynamics equations, then the conclusion derived from it which is covariant with itself can not be used as the support to satisfy the Lorentz covariance. For instance:Assuming that conclusion A is correct, we can deduce conclusion B ; assuming that conclusion B is correct, we can deduce conclusion A , but these can not be used as the basis for A and B to be independent and correct.

Conservation laws are very important conclusions in physics. They have been introduced into the theory of physics as experimental conclusions. Until 1918, based on the analysis of singular integral equation, German mathematician Amy nott obtained nott's theorem by connecting symmetry and conserved quantity.This theorem describes that for every continuous symmetric transformation of a mechanical system, there is a conserved quantity corresponding to it, which is also regarded as the theoretical basis of the conservation law. For example, the law of conservation of energy can be regarded as the inference that the relationship between work and energy change keeps the time translation unchanged.(When the ratio coefficient of work to energy increase or decrease $k = \frac{\Delta W}{\Delta E}$ is no longer a fixed value, but a function of time change, it means that when the coefficient is large, we select do positive work, energy is absorbed;and when the coefficient is small, energy is selected to be released, and the energy released at last will be greater than the energy absorbed),and the law of conservation of momentum corresponds to the symmetry of spatial translation.

However, we also realize that in the Lorentz transformation, the conversion of time and space between the reference frames that keep relative motion is not equivalent, but a function related to the relative motion speed of the reference frame. For the analysis of the same physical process, different reference frame will be different, which is significantly different from the classical mechanics. When we translate a physical law from one reference frame to another, it can not strictly meet the translational symmetry required by nott's theorem, which means that we can not directly introduce the existing conservation laws into Lorentz transformation, but should discuss the transformation equation of these physical quantities whether have the Lorentz covariance to support these conservation laws.

4.2 The principle of constant velocity of centroid.

In the Lorentz transformation of impulse, we get $\Delta\vec{P}_\alpha = \Delta m_\alpha \vec{v}_0$ .This equation shows that the momentum of the system in other reference frame may change even if the resultant force of the system in a particular reference frame is 0.This means that the law of conservation of momentum is not covariant with Lorentz transformation, and Lorentz transformation does not satisfy the symmetry of space translation of physical laws.When the mass of the system changes, the momentum will also change in the reference frame of relative motion.

Now we discuss the conserved quantity of this kind of variable mass system to the Lorentz transformation：

We assume that the α system is stationary in the S' frame of reference and that S' and S maintain relative motion with a constant speed.To simplify the analysis, we assume that there is no interaction within α , namely $m_p = 0$ (interaction energy).A set of forces whose vector sum is zero are defined in the S' ,namely $\sum_\alpha \vec{F}_i' = 0$ .Since the resultant force is zero, α remain static in S' , i.e. the velocity of the centroid of α system is 0, and the corresponding equation is $\sum_\alpha \frac{m_i'\vec{v}_i'}{m_i'} = 0$ .Since α and S' remain stationary, α moves at a constant speed in S ; however, due to the work by external forces, the mass of α will change.So the conserved quantity of α to S is not $\vec{p}_\alpha$, but the overall movement speed of the α .That is, the velocity of the centroid $\vec{v}_\alpha = \sum_\alpha \frac{m_i \vec{v}_i}{m_i}$ is constant.

In fact, in the formula $\Delta\vec{P}_\alpha = \Delta m_\alpha \vec{v}_0$ discussed before, such the conclusion has been included.In the S , before the work is done,the velocity of centroid of α is $\vec{v}_\alpha = \frac{\vec{p}_1}{m_1} = \vec{v}_0$; after the work is done, the momentum of α system changes to $\vec{p}_2 = \vec{p}_1 + \Delta\vec{p}_\alpha$, and the mass changes to $m_2 = m_1 + \Delta m_\alpha$, so the velocity of centroid of α after the work is $\vec{v}_\alpha = \frac{\vec{p}_1 + \Delta m_\alpha \vec{v}_0}{m_1 + \Delta m_\alpha} = \vec{v}_0$, that is, the velocity of mass center before and after the work is done will not change.

It can be seen that the resultant force of the system in a frame of reference is zero, and the momentum in other frames of reference which has relative motion with it may not be constant.But the velocity of the system centroid will not change in other frames of reference, we can call it "**The principle of constant velocity of centroid**".

In addition, even if the resultant force is zero, the impulse transformation produces an additional value in the direction of centroid velocity, which means that the angular momentum of the system is no longer conserved.

4.3Discussion on "Interaction energy is equivalent to mass".

Note that in the previous derivation, we did not distinguish whether interaction forces originate from in or out of the system.For the interaction forces outside the system, this is a good understanding. Because the external force does work on the system, it will cause the change of the system mass and the change of the system momentum;if it is the internal interaction of the system, according to the conclusion that the interaction energy is equivalent to mass, the mass of the system is constant, so the momentum of the system will not change.For the same physical system, will the interaction from within the system differ from that from outside the system?

As mentioned before, the conclusion that interaction energy is equivalent to mass can be regarded as the inference that conservation law directly introduces the Lorentz transformation.However, as can be seen from the previous inference, the Lorentz transformation can not satisfy the translation symmetry of all physical laws, so this conclusion should be doubted.(Note here that objects have excitation fields in space, but the excitation field belongs to the nature of the object itself. That is, there also will be the excitation field when a single object has no interaction. Therefore, the energy and mass of the excitation field do not belong to the interaction energy, and it cannot be proved that there is mass within the interaction energy.)

To discuss the relationship between interaction energy and mass, let's look at this case:

We assume that two boxes are fixed on the horizontal plane, and the inner wall of the box is smooth, insulated and perpendicular to the horizontal plane.There is a uniform electric field vertically upward with the horizontal plane. As shown in the figure, each inner wall of the box each has a charged of $-Q$, which can be regarded as a point charge, with a mass of m. Both are at the same horizontal height, and the distance between the charge centers is R.At first, they are stationary. Now let's analyze their acceleration.

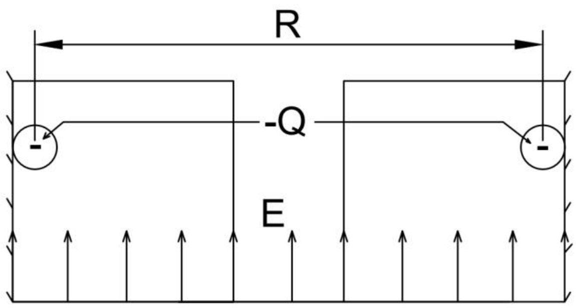

Figure 2

This is a physical model that is not difficult to analyze,we have two ways to solve it.Because of the acceleration at the initial time of analysis,the object is static at now，it means

$$\vec{F}=\frac{d(m\vec{v})}{dt}=m\frac{d\vec{v}}{dt}\Rightarrow\vec{a}=\frac{\vec{F}}{m}$$

This makes the calculation much easier.

First of all, we consider two spheres as a system: The kinetic energy of the system is 0, and the interaction energy is $E_p=k\frac{Q^2}{R}$ (k is the Coulomb constant),So according to the traditional theory, the inertia mass of the system is $m_\alpha=2m+\frac{E_p}{c^2}$.The resultant force of the system is $\vec{F} = -2Q\vec{E}$, so the acceleration of the system as a whole,also the acceleration of each ball is $\vec{a} = \frac{\vec{F}}{m} = -\frac{Q\vec{E}}{m + \frac{E_p}{2c^2}}$.

Then take a single object as the analysis object: It is easy to know that for a single ball, its mass has been given, and the resultant force is $\vec{F} = -Q\vec{E}$,So there is $\vec{a} = \frac{\vec{F}}{m} = -\frac{Q\vec{E}}{m}$.

It can be seen here that when the interaction energy is calculated as the inertial mass of the system, there will be differences between the analysis results of the whole system and the single object in the system, and the part that produces the difference is the part that the interaction energy is calculated as the mass of the system.In the above example, if we increase the electric charge at the same time, according to the method of taking the system as a whole, since the denominator is the quadratic power of Q , the rapid increase of interaction energy leads to the rapid increase of system mass.When the acceleration increases to a certain extent, it will decrease with the increase of Q .But according to the individual analysis in the system, the acceleration will continue to increase without decrease. The two correct analysis methods will produce inconsistent results, which is puzzling.

In this thought experiment, we see that if the interaction energy is equivalent to the mass, the existing mechanical system will appear the situation of no self consistent.In fact, similar methods have been used to correct the examples of mechanical systems at that time:In order to negate the conclusion that "heavy objects fall faster than light objects", **Galileo** assume that the weight when objects are treated as a system increase,so they should fall faster. But for each object, heavy objects would be interfered by light objects and fall slower.This means that the conclusion that "heavy objects will fall faster than light objects" is wrong. In order to eliminate this contradiction, they should the same as each other.In ***On the Electrodynamics of Moving Bodies***, **Einstein** assumed that the force and acceleration in Lorentz transformation maintained the form of $\vec{F} = m\vec{a}$,It is concluded that the "transverse mass" of the electron in motion is

$$m = \frac{m_0}{1 - \frac{v^2}{c^2}}$$

but the "longitudinal mass" is

$$m = \frac{m_0}{(1 - \frac{v^2}{c^2})^{3/2}}$$

But as the property of matter itself, mass should keep the isotropy of space.Therefore, the force and acceleration in Lorentz transformation can not keep the form of $\vec{F} = m\vec{a}$ .These are the embodiment that the logic of mechanical system should keep self consistency.

We think that a complete mechanical system, including the theory and concept.On this basis of the analysis, even if the analysis of different ways of the final results should be consistent.When different analysis methods will produce different or even contrary results, it shows that some concepts and theories in this mechanical system are defective.In order to make the mechanical system self consistent, we must modify the existing concepts and theories.

4.4 Dynamic equations of compound object.

By comparing the equations of acceleration obtained by two different analysis methods, we can see that they are different only in terms of the mass which corresponding to the interaction energy. When this term is removed, the acceleration obtained by the two analysis methods is the same.In order to satisfy the self consistency of mechanical analysis, we give the judgment: the interaction energy is not equivalent to the mass, and the total mass of the system is the algebraic sum of the component masses. Namely:

$$m_\alpha = \sum_\alpha m_i \tag{4.1}$$

The significance of this equation is significant.As mentioned before, we do not distinguish the source of interaction in the Lorentz transformation of impulse.It is easy to understand for the change of system mass caused by external forces.Now for the interaction within the system, since the interaction energy is not equivalent to the system mass, it means that the internal force of the system will also cause the change of the system mass, thus causing the change of momentum in the relative motion reference frame.In this way, we unify the interaction within and outside the system, both of which follow the "**The principle of constant velocity of centroid**".Therefore, we conclude that if the resultant force of the system is 0 in a certain reference frame, then the velocity of the centroid of system is constant in all the reference frame which maintain constant velocity with it.That is,

$$\frac{\sum_{\alpha} m_i \vec{v}_i}{\sum_{\alpha} m_i} = \frac{\vec{p}_\alpha}{m_\alpha} = \vec{C} \qquad \textbf{(4.2)}$$

As we all know, the objects in reality is not a single structure, but composed of its corresponding internal particles. Now let's explore the dynamic equations of these composite objects.To think about such a problem: All objects have their internal structure, they are static at the macroscopical level, but has relative motion at the microscopic level.Even if an object remains stationary in the frame of reference, its interior still maintains complex relative motion. It means that all objects in realities,their static mass is actually only the sum of the dynamic masses of the microscopic movements of its various components.The so-called stationary object is essentially a system whose centroid is static .

To explore the relationship between them, suppose that an object (system) α is composed of objects ranging from 1 to N , which are the simplest and purest substances (the definition is to ensure that these objects do not have more basic composition ,so it does not involve mass changes due to their internal motion). And that their **absolute static mass** (the mass of when these objects are relatively stationary with the frame of reference) is $m_i^0 (i \in 1,2,\cdots,N)$.

Let α be static in S' frame of reference, because of internal motion, the objects constituting α will not remain relatively static with S', but have relative velocity, which is recorded as $\vec{v}_i'$ and the corresponding mass is m_i' .Let $m' = \sum_{\alpha} m_i'$, m' be called **relative static mass** (The sum of the dynamic mass of each part of the system, which is static at the macroscopical level, but has relative motion at the microscopic level). α and S' keep relative motion with S frame of reference, and the relative velocity is $\vec{v}_0$.

The corresponding mass of the objects constituting α in the S is m_i, total mass $m = \sum_{\alpha} m_i$. It can be obtained by combining the mass-velocity relationship with the Lorentz velocity transformation that

$$m=\sum_{\alpha} m_i(v_i)=\sum_{\alpha}\frac{m_i^0}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{\sum_{\alpha} m_i'+\frac{\vec{v}_0}{c^2}\cdot\sum_{\alpha} m_i'\vec{v}_i'}{\sqrt{1-\frac{v_0^2}{c^2}}}$$

Since α is defined as static in S', it can be seen from the "The principle of constant velocity of centroid" that $\sum_{\alpha} m_i'\vec{v}_i'=0$, so there is

$$m=\frac{m'}{\sqrt{1-\frac{v_0^2}{c^2}}} \quad \textbf{(4.3)}$$

It can be found out by similar methods that

$$\vec{P}=\sum_{\alpha} m_i(v_i)\vec{v}_i=\frac{m'}{\sqrt{1-\frac{v_0^2}{c^2}}}\vec{v}_0 \quad \textbf{(4.4)}$$

$$E_k=\sum_{\alpha}(m_i-m_i')c^2=(\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}-1)m'c^2 \quad \textbf{(4.5)}$$

m' represents the mass of the compound object at rest in the frame of reference, as can be seen that in terms of the dynamic properties of mass, momentum and kinetic energy, a relative static mass and an absolute stationary mass keep the same form.

It can be seen here that the absolute static mass and the relative static mass are equivalent in the dynamic equations. In reality, the form of objects' mass is relative static mass. Relative static mass depends on the speed of internal motion. The internal kinetic energy and interaction energy of the object are changing all the time, which means that the relative static mass of the object is a floating value.

At a certain time, the mass of a compound object cannot be predicted accurately. We can't know its exact mass,and can only give its mass density function $f(m)$ and the probability $f(m)dm$ corresponding to the current mass.For $f(m)$, the main parameters affecting it are the total energy of the system and the proportion of the interaction energy for the system.The higher the proportion of interaction energy is, the larger the relative floating range of mass is.For many objects in reality, because they are composed of a large number of atoms with the interaction energy (electromagnetic interaction) is low proportion, it can be seen from the nature of variance that

$$D_\alpha = \frac{D}{N} \to 0 \qquad \textbf{(4.6)}$$

This shows that for macroscopic objects, the mass is stable.

5.Discussion on conservation law in the Lorentz transformation.

For the system α at rest in the S' reference frame, if the mass changes $\Delta m'$ due to the work done, the mass changes Δm in the reference frame S which moves relative to the S' reference frame. Because of the mass-kinetic energy relation, the Lorentz transformation of the work for the system is

$$\Delta W = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} \Delta W' \qquad \textbf{(5.1)}$$

Note that since the system is stationary in S', the resultant force of it is 0. Therefore, this equation can also be obtained by summing up equation **(2.5)**.

This equation shows that for the same work process, the work done at high speed will be amplified, and the magnification is determined by the speed, which has never appeared in the existing theory.

Let's use the principle of mass center to explain this phenomenon:

We assume that the system α and the reference frame maintain a constant relative velocity, which means that the resultant force of α is 0.When work is done on α, the relative static mass of α increases.From the previous derivation, it can be seen that the relative static mass and the static mass are equivalent. Therefore, due to the "center of mass principle", the increase relative static mass will also maintain the speed of the system's centroid. That is to say, for the reference frame, the system increases not only the energy of the system itself, but also the kinetic energy of the increased mass relative to the reference frame.These kinetic energy are due to the amplification of work caused by the asymmetry of space-time translation after relative motion. Therefore, the conservation of energy is not covariant with the Lorentz transformation.

Using this phenomenon, we can design a device:First, we accelerate a pair of objects which have interaction energy, such as two protons close enough; Secondly,when they reach a certain speed, they release the interaction energy of each other. From equation 5.1, the energy they increase relative to the reference frame will be greater than their the interaction energy;Third, let them release kinetic energy and do external work;Finally, Return the two protons to their initial state, store the interaction energy and prepare for the next cycle.When we choose a system with a high proportion of interaction energy, the additional energy we can obtain is considerable.When the additional energy obtained is greater than the energy loss to maintain the system operation, it means that the device can achieve external energy output.

References

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Appendix

1. Derivation of mass-kinetic energy equation.

$$dE_k = \vec{F}\cdot d\vec{r} = \vec{F}\cdot\vec{v}dt = \vec{v}\cdot d\vec{p} = \vec{v}\cdot(md\vec{v}+vdm)$$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \Leftrightarrow \frac{\vec{v}^2}{c^2} = 1-\frac{m_0^2}{m^2} \Leftrightarrow \vec{v}\cdot d\vec{v} = \frac{m_0^2 c^2}{m^3}dm$$

$$dE_k = \vec{v}\cdot(md\vec{v}+vdm)m = \frac{m_0^2c^2}{m^2}dm + c^2(1-\frac{m_0^2}{m^2})\ dm = dmc^2$$

2. The relationship between dm and dm'.

$$dm = m_2 - m_1 = m(v_2) - m(v_1)$$

$$m(v_2) = \frac{m_0}{\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{m_0(1+\frac{\vec{v}_0\cdot\vec{v}_2'}{c^2})}{\sqrt{1-\frac{v_0^2}{c^2}}\cdot\sqrt{1-\frac{v_2'^2}{c^2}}}$$

$$m(v_1) = \frac{m_0}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{m_0(1+\frac{\vec{v}_0\cdot\vec{v}_1'}{c^2})}{\sqrt{1-\frac{v_0^2}{c^2}}\cdot\sqrt{1-\frac{v_1'^2}{c^2}}}$$

$$dm = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}[(\frac{m_0}{\sqrt{1-\frac{v_2'^2}{c^2}}} - \frac{m_0}{\sqrt{1-\frac{v_1'^2}{c^2}}}) + \frac{\vec{v}_0}{c^2}(m_2'\vec{v}_2' - m_1'\vec{v}_1')] = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}(dm' + \frac{\vec{v}_0}{c^2}d\vec{I}')$$

3. The Lorentz transformation of impulse.

$$dI_x' = m_2'v_{2x}' - m_1'v_{1x}'$$

$$dI_x = m_2v_{2x} - m_1v_{1x} = \frac{m_0(1+\frac{v_0v_{2x}'}{c^2})}{\sqrt{1-v_0^2/c^2}\bullet\sqrt{1-v_2'^2/c^2}}\cdot\frac{v_0+v_{2x}'}{1+\frac{v_0v_{2x}'}{c^2}} - \frac{m_0(1+\frac{v_0v_{1x}'}{c^2})}{\sqrt{1-v_0^2/c^2}\bullet\sqrt{1-v_1'^2/c^2}}\cdot\frac{v_0+v_{1x}'}{1+\frac{v_0v_{1x}'}{c^2}}$$

$$dI_x = \frac{1}{\sqrt{1-v_0^2/c^2}}[m_2'(v_0+v_{2x}') - m_1'(v_0+v_{1x}')] = \frac{1}{\sqrt{1-v_0^2/c^2}}(dm' + dI_x')$$

In the same way, we can get

$$dI_y = dI_y'$$

$$dI_z = dI_z'$$