A Proof of the Erdös-Straus Conjecture

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Abstract

In this article, we classify positive integers except for 1 into certain classes gradually, and formulate each of majority's classes after each classification to a sum of three unit fractions, until all classes.

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1. Introduction

The Erdös-Straus conjecture relates to Egyptian fractions. In 1948, Paul Erdös conjectured that for any integer n≥2, there are $\frac{4}{\mathrm{n}}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$ invariably, where X, Y and Z are positive integers.

Later, Ernst G. Straus conjectured that X, Y and Z satisfy X≠Y, Y≠Z and Z≠X, because there are the convertible formulas $\frac{1}{2\mathrm{r}}+\frac{1}{2r}=\frac{1}{r+1}+\frac{1}{r(r+1)}$ and $\frac{1}{2r+1}+\frac{1}{2r+1}=\frac{1}{r+1}+\frac{1}{(r+1)(2r+1)}$ where r≥1; [1].

Thus, the Erdös conjecture and the Straus conjecture are equivalent from each other, and they are called the Erdös-Straus conjecture collectively.

As a general rule, the Erdös-Straus conjecture states that for every integer n≥2, there are positive integers X, Y and Z, such that $\frac{4}{\mathrm{n}}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$; [2].

The significance of proving the Erdos-Straus conjecture is to study the Diophantine equation XYZ=XY+XZ+YZ, yet it remains a conjecture that has neither is proved nor disproved; [3].

2. Classify positive integers in turn and formulate each of majority's classes after each classification

The Erdös-Straus conjecture is concerned only with all positive integers except for 1, so we classify positive integers in turn and formulate each of majority's classes to a sum of three unit fractions after each classification.

First, we divide integers except for 1 into 8 kinds, i.e. 8k+1with k≥1; 8k+2, 8k+3, 8k+4, 8k+5, 8k+6, 8k+7 and 8k+8 where k≥0, as listed below.

K\n:	8k+1,	8k+2,	8k+3,	8k+4,	8k+5,	8k+6,	8k+7,	8k+8
0,	①,	2,	3,	4,	5,	6,	7,	8,
1,	9,	10,	11,	12,	13,	14,	15,	16,
2,	17,	18,	19,	20,	21,	22,	23,	24,
…,	…,	…,	…,	…,	…,	…,	…,	…,

Excepting n=8k+1, formulate each of other 7 kinds into $\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$:

(1) When n=8k+2, there are $\frac{4}{8k+2}=\frac{1}{4k+1}+\frac{1}{4k+2}+\frac{1}{(4k+1)(4k+2)}$;

(2) When n=8k+3, there are $\frac{4}{8k+3}=\frac{1}{2k+2}+\frac{1}{(2k+1)(2k+2)}+\frac{1}{(2k+1)(8k+3)}$;

(3) When n=8k+4, there are $\frac{4}{8k+4}=\frac{1}{2k+3}+\frac{1}{(2k+2)(2k+3)}+\frac{1}{(2k+1)(2k+2)}$;

(4) When n=8k+5, there are $\frac{4}{8k+5}=\frac{1}{2k+2}+\frac{1}{(8k+5)(2k+2)}+\frac{1}{(8k+5)(k+1)}$;

(5) When n=8k+6, there are $\frac{4}{8k+6}=\frac{1}{4k+3}+\frac{1}{4k+4}+\frac{1}{(4k+3)(4k+4)}$;

(6) When n=8k+7, there are $\frac{4}{8k+7}=\frac{1}{2k+3}+\frac{1}{(2k+2)(2k+3)}+\frac{1}{(2k+2)(8k+7)}$;

(7) When n=8k+8, there are $\frac{4}{8k+8}=\frac{1}{2k+4}+\frac{1}{(2k+2)(2k+3)}+\frac{1}{(2k+3)(2k+4)}$.

By this token, above 7 kinds of integers are suitable to the conjecture.

For the unsolved kind when n=8k+1 with k≥1, it divided by 3 to 3 genera:

1. the remainder is 0, when k=1+3t, where t≥0;

2. the remainder is 2, when k=2+3t, where t≥0;

3. the remainder is 1, when k=3+3t, where t≥0.

These 3 genera of odd numbers and the remainders of them divided by 3 are listed below:

k:	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,	11,	12,	13,	14,	15, ...
8k+1:	9,	17,	25,	33,	41,	49,	57,	65,	73,	81,	89,	97,	105,	113,	121, ...
the remainder:	0,	2,	1,	0,	2,	1,	0,	2,	1,	0,	2,	1,	0,	2,	1, ...

Excepting the genus 3, we formulate other 2 genera as follows:

(8) Where the remainder of $\frac{8k+1}{3}$ is equal to 0, there are $\frac{4}{8k+1}=\frac{1}{\frac{8k+1}{3}}+\frac{1}{8k+2}+\frac{1}{(8k+1)(8k+2)}$. Due to k=1+3t and t≥0, then there are $\frac{8k+1}{3}=8t+3$, so we confirm that $\frac{8k+1}{3}$ in the above equation is an integer.

(9) Where the remainder of $\frac{8k+1}{3}$ is equal to 2, there are $\frac{4}{8k+1}=\frac{1}{\frac{8k+2}{3}}+\frac{1}{8k+1}+\frac{1}{\frac{(8k+1)(8k+2)}{3}}$. Due to k=2+3t and t≥0,then there are $\frac{8k+2}{3}=8t+6$, so we confirm that $\frac{8k+2}{3}$ and $\frac{(8k+1)(8k+2)}{3}$ in the above equation are two integers.

For the unsolved genus $\frac{8k+1}{3}$ where the remainder is equal to 1, and k=3+3t and t≥0, as listed above 8k+1=25, 49, 73, 97, 121 etc.

So divide the genus into 5 sorts: 25+120c, 49+120c, 73+120c, 97+120c and 121+120c where c≥0, as listed below.

C\n:	25+120c,	49+120c,	73+120c,	97+120c,	121+120c,
0,	25,	49,	73,	97,	121,
1,	145,	169,	193,	217,	241,

2, 265, 289, 313, 337, 361,

…, …, …, …, …, …,

Excepting n=49+120c and n=121+120c, formulate other 3 sorts, they are:

(10) When n=25+120c, there are $\frac{4}{25+120c}=\frac{1}{25+120c}+\frac{1}{50+240c}+\frac{1}{10+48c}$;

(11) When n=73+120c, there are $\frac{4}{73+12oc}=\frac{1}{(73+120c)(10+15c)}+\frac{1}{20+30c}+\frac{1}{(73+120c)(4+6c)}$;

(12) When n=97+120c, there are $\frac{4}{97+12oc}=\frac{1}{25+30c}+\frac{1}{(97+120c)(50+60c)}+\frac{1}{(97+120c)(10+12c)}$.

For each of preceding 12 equations which express $\frac{4}{\mathrm{n}}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$, please each reader self to make a check respectively.

3. Prove the sort $\frac{4}{49+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$

For a proof of the sort $\frac{4}{49+120c}$, it means that when c is equal to each of positive integers and 0, there always are $\frac{4}{49+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$.

Since the fraction $\frac{4}{49+120c}$ can be substituted by each of infinitely many a sum of an unit fraction plus a proper fraction.

In addition, if c is given every value of it, then each such sum contains infinitely many a sum of a concrete unit fraction plus a concrete proper fraction, and that there is no a repetition in these fractions, as listed below:

$$\frac{4}{49+120c}$$

$$=\frac{1}{13+30\mathrm{c}}+\frac{3}{(13+30c)(49+120c)}$$

$$=\frac{1}{14+30c}+\frac{7}{(14+30c)(49+120c)}$$

$$=\frac{1}{15+30c}+\frac{11}{(15+30c)(49+120c)}$$

…

$$=\frac{1}{13+\alpha+30c}+\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$$, where α and c ≥ 0.

…

Thus it can be seen that $\frac{1}{13+\alpha+30\mathrm{c}}$ is an unit fraction obviously .

Besides, if $\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$ can be expressed as an unit fraction $\frac{1}{W}$, then we regard c in $\frac{4}{49+120c}=\frac{1}{13+\alpha+30c}+\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$ as c_1, where W is an integer > 1, and c_1 ∈ positive integers and 0.

In this way, there are $\frac{4}{49+120c_1}=\frac{1}{13+\alpha+30c_1}+\frac{1}{W}$, and let $\frac{1}{13+\alpha+30c_1}$ or $\frac{1}{W}$ to equal a sum of two identical unit fractions, then follow the formula $\frac{1}{2\mathrm{r}}+\frac{1}{2r}=\frac{1}{r+1}+\frac{1}{r(r+1)}$ or $\frac{1}{2r+1}+\frac{1}{2r+1}=\frac{1}{r+1}+\frac{1}{(r+1)(2r+1)}$ to transform a sun of these two identical unit fractions to the sun of two each other's-distinct unit fractions, such that $\frac{4}{49+120c_1}$ is equal to $\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$.

For example, when α=1 and c_1=0, $\frac{1}{49+120c_1}=\frac{4}{49}$, $\frac{1}{13+\alpha+30c_1}=\frac{1}{14}$, and

$$\frac{4\alpha+3}{(13+\alpha+30c_1)(49+120c_1)}=\frac{1}{2\times49}=\frac{1}{2(2\times49)}+\frac{1}{2(2\times49)}=\frac{1}{2\times49+1}+\frac{1}{2\times49(2\times49+1)},$$

then we get $\frac{4}{49}=\frac{1}{14}+\frac{1}{2\times49+1}+\frac{1}{2\times49(2\times49+1)}$.

For another example, when α=1 and c_1=7, $\frac{4}{49+120c_1}=\frac{4}{889}$, $\frac{1}{13+\alpha+30c_1}=$ $\frac{1}{2\times 224}+\frac{1}{2\times 224}=\frac{1}{224+1}+\frac{1}{224(224+1)}$ and $\frac{4\alpha+3}{(13+\alpha+30c_1)(49+120c_1)}=\frac{1}{224\times 127}$,

then we get $\frac{4}{889}=\frac{1}{224+1}+\frac{1}{224(224+1)}+\frac{1}{224\times 127}$.

If $\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$ cannot be expressed as an unit fraction $\frac{1}{W}$, then we regard c in $\frac{4}{49+120c}=\frac{1}{13+\alpha+30c}+\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$ as c_2, where $c_2\in$ positive integers and 0, and c_1 with c_2 express conjointly all positive integers and 0.

In this way, there are $\frac{4}{49+120c_2}=\frac{1}{13+\alpha+30c_2}+\frac{4\alpha+3}{(13+\alpha+30c_2)(49+120c_2)}$.

Also, there are $\frac{4\alpha+3}{(13+\alpha+30c_2)(49+120c_2)}=\frac{1}{(13+\alpha+30c_2)(49+120c_2)}+$ $\frac{4\alpha+2}{(13+\alpha+30c_2)(49+120c_2)}$.

Of course, $\frac{1}{(13+\alpha+30c_2)(49+120c_2)}$ in above equation is an unit fraction.

Thus, we only need to prove that $\frac{4\alpha+2}{(13+\alpha+30c_2)(49+120c_2)}$ can be converted identically to an unit fraction, *ut infra*.

Proof: First, let us compare the size of the numerator 4α+2 and the denominator $(13+\alpha+30c_2)(49+120c_2)$.

Due to $c_2\in$positive integers and 0, $13+\alpha+30c_2$ can always be greater than 4α+2. And then, we just take $13+\alpha+30c_2$ as the denominator temporarily, while reserve $49+120c_2$ for later.

In the fraction $\frac{4\alpha+2}{13+\alpha+30c_2}$, since the numerator 4α+2 is an even number, in addition, the reserved $49+120c_2$ is an odd number, so the denominator

$13+\alpha+30c_2$ must be an even numbers. Only in this case, it can reduce the fraction to become possibly an unit fraction, so α in the denominator $13+\alpha+30c_2$ is a positive odd number, accordingly α of 4α+2 is too.

After α is assigned to odd numbers 1, 3, 5 and otherwise, the numerator and the denominator of the fraction $\frac{4\alpha+2}{13+\alpha+30c_2}$ divided by 2, then the fraction $\frac{4\alpha+2}{13+\alpha+30c_2}$ is turned into the fraction $\frac{3+4k}{7+k+15c_2}$ where $c_2 \in$ positive integers and 0, α=2k+1 and k≥1.

If let k=0, then the numerator is 3 and the denominator is $7+15c_2$, since $15c_2$ are integral multiples of 3, yet 7 is not, then $\frac{3}{7+15c_2}$ cannot become an unit fraction, so we abandon k=0, and get derivable α≠1.

After assigning values of k from small to large to the fraction $\frac{3+4k}{7+k+15c_2}$, there are $\frac{3+4k}{7+k+15c_2}=\frac{7}{8+15c_2}$, $\frac{11}{9+15c_2}$, $\frac{15}{10+15c_2}$, …

Such being the case, letting the numerator and the denominator of the fraction $\frac{3+4k}{7+k+15c_2}$ divided by 3+4k, then we get an indeterminate unit fraction, and its denominator is $\frac{7+k+15c_2}{3+4k}$, and its numerator is 1.

Thus, we are necessary to prove that the fraction $\frac{7+k+15c_2}{3+4k}$ as the denominator contains a number of positive integers or infinitely many positive integers, where k≥1 and $c_2 \in$ positive integers and 0.

After k is assigned values from small to large, $\frac{7+k+15c_2}{3+4k}$ is equal to $\frac{8+15c_2}{7}$, $\frac{9+15c_2}{11}$, $\frac{10+15c_2}{15}$, …

As listed above, it can be seen that each positive odd number as the denominator can match infinite more numerators if $c_2 \geq 1$, but $\frac{7+k+15c_2}{3+4k}$ as positive integers are merely a part in them due to $c_2 \in$ positive integers and 0, such as $\frac{7+k+15c_2}{3+4k}=\frac{7+2+15\times 6}{3+4\times 2}=\frac{99}{11}=9$, where k=2 and c_2=6.

After k is given a value and c_2 is given a kind of values, it also enables $\frac{7+k+15c_2}{3+4k}$ to become at least one kind of positive integers, and vice versa.

For example, when k=1, there are $\frac{7+k+15c_2}{3+4k}=\frac{8+15c_2}{7}=14+15s$ where c_2=6+7s, and s is equal to each of positive integers and 0.

For another example, when k=8, there are $\frac{7+k+15c_2}{3+4k}=\frac{15+15c_2}{35}=3+3s$ where c_2=6+7s, and s is equal to each of positive integers and 0.

From above two examples, it can be seen that s is equal to each of positive integers and 0, then $\frac{7+k+15c_2}{3+4k}$ contains infinite more positive integers, so we use the symbol μ to represent each and every such positive integer.

Of course, $\frac{3+4k}{7+k+15c_2}$ expresses also infinite more unit fractions of $\frac{1}{\mu}$.

After that, we multiply the denominator of $\frac{1}{\mu}$ by $49+120c_2$ reserved to get the unit fraction $\frac{1}{\mu(49+120c_2)}$.

Or rather, there doubtlessly are $\frac{4\alpha+2}{(13+\alpha+30c_2)(49+120c_2)}=\frac{1}{\mu(49+120c_2)}$.

To sum up, we have proved $\frac{4}{49+120c_2}=\frac{1}{\mu(49+120c_2)}+\frac{1}{13+\alpha+30c_2}+\frac{1}{(13+\alpha+30c_2)(49+120c_2)}$ where μ expresses each and every positive integer of $\frac{7+k+15c_2}{3+4k}$, k≥1, α=2k+1, and c_2∈positive integers and 0.

Enable $\frac{4}{49+120c}$ to become the sum of two terms which consist of positive integers, there are only two cases, namely two terms are unit fractions and only one term is an unit fraction.

Since c_1 with c_2 in these two cases express conjointly c, namely c_1 with c_2 express conjointly all positive integers and 0, and that we have proved that $\frac{4}{49+120c_1}$ and $\frac{4}{49+120c_2}$ are expressed as the sum of 3 unit fractions, therefore, we have proved $\frac{4}{49+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$ where c≥0.

4. Prove the sort $\frac{4}{121+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$

The proof in this section is exactly similar to that in the section 5. Namely, for a proof of the sort $\frac{4}{121+120c}$, it means that when c is equal to each of positive integers and 0, there always are $\frac{4}{121+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$.

Since the fraction $\frac{4}{121+120c}$ can be substituted by each of infinitely many a sum of an unit fraction plus a proper fraction.

In addition, if c is given every value of it, then each such sum contains infinitely many a sum of a concrete unit fraction plus a concrete proper fraction, and that there is no a repetition in these fractions, as listed below:

$$\frac{4}{121+120c}$$

$$=\frac{1}{31+30c}+\frac{3}{(31+30c)(121+120c)}$$

$$=\frac{1}{32+30c}+\frac{7}{(32+30c)(121+120c)}$$

$$=\frac{1}{33+30c}+\frac{11}{(33+30c)(121+120c)}$$

…

$$=\frac{1}{31+\alpha+30c}+\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$$, where α and c ≥ 0.

…

Thus it can be seen that $\frac{1}{31+\alpha+30c}$ is an unit fraction obviously.

Besides, if $\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$ can be expressed as an unit fraction $\frac{1}{V}$,

then we regard c in $\frac{4}{121+120c}=\frac{1}{31+\alpha+30c}+\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$ as c_1,

where V is an integer>1, and c_1∈positive integers and 0.

In this way, there are $\frac{4}{121+120c_1}=\frac{1}{31+\alpha+30c_1}+\frac{1}{V}$, and let $\frac{1}{31+\alpha+30c_1}$ or $\frac{1}{V}$ to equal a sum of two identical unit fractions, then follow the formula $\frac{1}{2r}+\frac{1}{2r}=\frac{1}{r+1}+\frac{1}{r(r+1)}$ or $\frac{1}{2r+1}+\frac{1}{2r+1}=\frac{1}{r+1}+\frac{1}{(r+1)(2r+1)}$ to transform a sun of these two identical unit fractions to the sun of two each other's-distinct unit fractions, such that $\frac{4}{121+120c}$ is equal to $\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$.

For example, when α=2 and c_1=0, $\frac{4}{121+120c_1}=\frac{4}{121}$, $\frac{1}{31+\alpha+30c_1}=\frac{1}{33}$ and

$$\frac{4\alpha+3}{(31+\alpha+30c_1)(121+120c_1)}=\frac{1}{3\times121}=\frac{1}{2(3\times121)}+\frac{1}{2(3\times121)}=\frac{1}{3\times121+1}+\frac{1}{3\times121(3\times121+1)}$$

then, we get $\frac{4}{121}=\frac{1}{33}+\frac{1}{3\times121+1}+\frac{1}{3\times121(3\times121+1)}$. For another example,

when α=1 and c_1=5, there are $\frac{4}{121+120c_1}=\frac{4}{721}$, $\frac{1}{31+\alpha+30c_1}=\frac{1}{182}$ and

$$\frac{4\alpha+3}{(31+\alpha+30c_1)(121+120c_1)}=\frac{1}{182\times103}=\frac{1}{2\times18746}+\frac{1}{2\times18746}=\frac{1}{18746+1}+\frac{1}{18746(18746+1)}$$

then, we get $\frac{4}{721}=\frac{1}{182}+\frac{1}{18746+1}+\frac{1}{18746(18746+1)}$.

If $\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$ cannot be expressed as an unit fraction $\frac{1}{V}$, then we regard c in $\frac{4}{121+120c}=\frac{1}{31+\alpha+30c}+\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$ as c_2, where c_2 ∈positive integers and 0, and c_1 with c_2 express conjointly all positive integers and 0.

In this way, there are $\frac{4}{121+120c_2}=\frac{1}{31+\alpha+30c_2}+\frac{4\alpha+3}{(31+\alpha+30c_2)(121+120c_2)}$.

Also, there are $\frac{4\alpha+3}{(31+\alpha+30c_2)(121+120c_2)}=\frac{1}{(31+\alpha+30c_2)(121+120c_2)}+\frac{4\alpha+2}{(31+\alpha+30c_2)(121+120c_2)}$

Of course, $\frac{1}{(31+\alpha+30c_2)(121+120c_2)}$ in above equation is an unit fraction.

Thus, we only need to prove that $\frac{4\alpha+2}{(31+\alpha+30c_2)(121+120c_2)}$ can be converted identically to an unit fraction, *ut infra*.

Proof: First, let us compare the size of the numerator 4α+2 and the denominator $(31+\alpha+30c_2)(121+120c_2)$.

Due to c_2 ∈positive integers and 0, $31+\alpha+30c_2$ can always be greater than 4α+2. And then, we just take $31+\alpha+30c_2$ as the denominator temporarily, while reserve $121+120c_2$ for later.

In the fraction $\frac{4\alpha+2}{31+\alpha+30c_2}$, since the numerator 4α+2 is an even number, in addition, the reserved $121+120c_2$ is an odd number, so the denominator $31+\alpha+30c_2$ must be an even numbers. Only in this case, it can reduce the fraction to become possibly an unit fraction, so α in the denominator $31+\alpha+30c_2$ is a positive odd number, accordingly α of 4α+2 is too.

After α is assigned to odd numbers 1, 3, 5 and otherwise, the numerator and the denominator of the fraction $\frac{4\alpha+2}{31+\alpha+30c_2}$ divided by 2, then the fraction $\frac{4\alpha+2}{31+\alpha+30c_2}$ is turned to the fraction $\frac{3+4k}{16+k+15c_2}$, where c_2∈positive integers and 0, k≥1, and α=2k+1.

If let k=0, then the denominator is 16+15c_2 and the numerator is 3, since 15c_2 are integral multiples of 3, yet 16 is not, so $\frac{3}{16+15c_2}$ cannot become an unit fraction, so we abandon k=0, and get derivable α≠1.

After assigning k values from small to large to the fraction $\frac{3+4k}{16+k+15c_2}$, there are $\frac{3+4k}{16+k+15c_2}=\frac{7}{17+15c_2}$,$\frac{11}{18+15c_2}$,$\frac{15}{19+15c_2}$ ...

Such being the case, letting the numerator and the denominator of the fraction $\frac{3+4k}{16+k+15c_2}$ divided by 3+4k, then we get an indeterminate unit fraction, and its denominator is $\frac{16+k+15c_2}{3+4k}$, and its numerator is 1.

Thus, we are necessary to prove that the denominator $\frac{16+k+15c_2}{3+4k}$ contains infinitely many positive integers in the case where k≥1 and c_2 ∈positive integers and 0.

After k is assigned values from small to large, $\frac{16+k+15c_2}{3+4k}$ is equal to $\frac{17+15c_2}{7}$, $\frac{18+15c_2}{11}$, $\frac{19+15c_2}{15}$, ...

As listed above, it can be seen that each positive odd number as the denominator can match infinite more numerators if c_2≥1, but $\frac{16+k+15c_2}{3+4k}$ as positive integers are merely a part in them due to c_2 ∈positive integers and 0, such as $\frac{16+k+15c_2}{3+4k}=\frac{16+1+15\times4}{3+4\times1}=\frac{77}{7}=11$, where k=1 and c_2=4.

After k is given a value and c_2 is given a kind of values, it also enables $\frac{16+k+15c_2}{3+4k}$ to become at least one kind of positive integers,and vice versa. For example, when k=2, there are $\frac{16+k+15c_2}{3+4k}=\frac{18+15c_2}{11}=3+15s$, where c_2=1+11s, and s is equal to each of positive integers and 0. For another example, when k=4, there are $\frac{16+k+15c_2}{3+4k}=\frac{95+15\times 38s}{19}=5+30s$, where c_2=5+38s, and s is equal to each of positive integers and 0.

From above two examples, it can be seen that s is equal to each of positive integers and 0, then $\frac{16+k+15c_2}{3+4k}$ contains infinite more positive integers, so we use the symbol λ to represent each and every such positive integer.

Of course, $\frac{3+4k}{16+k+15c_2}$ expresses also infinite more unit fractions of $\frac{1}{\lambda}$.

After that, we multiply the denominator of $\frac{1}{\lambda}$ by 121+120c_2 reserved to get the unit fraction $\frac{1}{\lambda(121+120c_2)}$.

Or rather, there doubtlessly are $\frac{4\alpha+2}{(31+\alpha+30c_2)(121+120c_2)}=\frac{1}{\lambda(121+120c_2)}$.

To sum up, we have proved $\frac{4}{121+120c_2}=\frac{1}{\lambda(121+120c_2)}+\frac{1}{31+\alpha+30c_2}+\frac{1}{(31+\alpha+30c_2)(121+120c_2)}$ where λ expresses each and every positive integer of $\frac{16+k+15c_2}{3+4k}$, k≥1, α=2k+1, and c_2∈positive integers and 0.

Enable $\frac{4}{121+120c}$ to become the sum of two terms which consist of positive integers, there are only two cases, namely two terms are unit fractions and only one term is an unit fraction.

Since c_1 with c_2 in these two cases express conjointly c, namely c_1 with c_2 express conjointly all positive integers and 0, and that we have proved that $\frac{4}{121+120c_1}$ and $\frac{4}{121+120c_2}$ can be expressed as the sum of 3 unit fractions, therefore, we have proved $\frac{4}{121+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$ where $c \geq 0$. The proof was thus brought to a close. As a consequence, the Erdös-Straus conjecture is tenable.

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