Many people are using the term “Assigned Value” or “Analytic Continuation” for divergent series
But this explanation is so lacking and can be replaced with a much easier and simpler term of explanation

For me (as I see it) when I am looking at the zeta function I dont see (or use) the term “Assigned Value” or “Analytic Continuation”
Instead I see “spirals” all around the grid

The simplest way is to first look at the Complex plane $\zeta(s) = \zeta(x + iy) = a + ib$ where $s > 1$ and the behavior of convergent points
The spiral swirls around inwards to an unique point which the series Converges - Same goes for the other way around!

When I look at the Complex plane $\zeta(s) = \zeta(x + iy) = a + ib$ where $s < 1$ and the behavior of divergent points
The spiral swirls around outwards but if you look closely you will notice that the spiral has a “center point” or an “origin”
and that “origin” is the “Assigned Value” everyone is talking about

when I first started to read about the zeta function I didn’t know what are those “Assigned Values” or “Analytic Continuation”
and how and why people are trying to give a value for divergent series And why that specific value and not something else?
I wanted an explanation other then “because the formula says so” and without going deeper into all the “Analytic Continuation stuff”.

Those “origin points” did the trick!

If you are assigning a value for a series that decreases to a specific value (case #1)
Then you can assigning a value for a series that increases from a specific value (case #2)

Other then those two cases there is one more
This is when the spiral at some point start to spin around a specific value with a “fixed radius”
those cases appears at the zeta function $\zeta(s) = \zeta(x + iy) = a + ib$ when $x = 1$ and the radius will be $1/y$
meaning that this is a divergent series with a “fixed radius”

This was a small intro for the eta function spirals

Its true that the zeta function spirals have 3 cases but they are all spirals with one arm
Now at the eta function the spirals have two arms (that is because of the +/- swapping) with the same 3 cases

By the way the “fixed radius” appears at the eta function $\eta(s) = \eta(x + iy) = a + ib$ when $x = 0$

$\eta(k) = 1^k - 2^k + 3^k - 4^k + ... \pm k^k$
\[ e^{\theta} = \left(1 + \frac{\theta}{n} \right)^n \]

\[ \theta = -ib \cdot \ln k \]

\[ \frac{1}{k^{a+ib}} = \frac{1}{k^a} \cdot k^{-ib} = \frac{1}{k^a} \cdot e^{\theta} = \frac{1}{k^a} \left(1 + \frac{\theta}{n} \right)^n = \frac{1}{k^a} \cdot \left(1 - i \cdot \frac{b \cdot \ln k}{n} \right)^n \]

\[ (x + y)^n = \left( \frac{n}{0} \right) x^n y^0 + \left( \frac{n}{1} \right) x^{n-1} y^1 + \left( \frac{n}{2} \right) x^{n-2} y^2 + \ldots + \left( \frac{n}{n-1} \right) x^1 y^{n-1} + \left( \frac{n}{n} \right) x^0 y^n \]

\[ (x - iy)^n = x^n - i \left( \frac{n}{1} \right) x^{n-1} y^1 - \left( \frac{n}{2} \right) x^{n-2} y^2 + i \left( \frac{n}{3} \right) x^{n-3} y^3 + \ldots + i \left( \frac{n}{n} \right) x^0 y^n \]

\[ \left( \frac{n}{k} \right) = \frac{n!}{(n-k)!k!} \]

\[ (x - iy)^n = \left[ x^n - \frac{n! x^{n-2} y^2}{(n-2)!} + \frac{n! x^{n-4} y^4}{(n-4)!} - \frac{n! x^{n-6} y^6}{(n-6)!} + \ldots \right] + i \left[ -\frac{n! x^{n-1} y^1}{(n-1)!} + \frac{n! x^{n-3} y^3}{(n-3)!} - \frac{n! x^{n-5} y^5}{(n-5)!} + \ldots \right] \]

\[ y = \frac{b \cdot \ln k}{n} \]

\[ \left( 1 - i \cdot \frac{b \cdot \ln k}{n} \right)^n = \left[ 1 - \frac{n! \left( \frac{b \cdot \ln k}{n} \right)^2}{(n-2)!} + \frac{n! \left( \frac{b \cdot \ln k}{n} \right)^4}{(n-4)!} - \frac{n! \left( \frac{b \cdot \ln k}{n} \right)^6}{(n-6)!} + \ldots \right] + i \left[ -\frac{n! \left( \frac{b \cdot \ln k}{n} \right)^1}{(n-1)!} + \frac{n! \left( \frac{b \cdot \ln k}{n} \right)^3}{(n-3)!} - \frac{n! \left( \frac{b \cdot \ln k}{n} \right)^5}{(n-5)!} + \ldots \right] \]
\[
\frac{1}{k^a} \left( 1 - i \cdot \frac{b \cdot \ln k}{n} \right)^n = \frac{1}{k^a} \left[ 1 - n! \left( \frac{b \cdot \ln k}{n} \right)^2 + \frac{n! \left( b \cdot \ln k \right)^4}{(n-2)!} - \frac{n! \left( b \cdot \ln k \right)^6}{(n-4)!} + \frac{n! \left( b \cdot \ln k \right)^8}{(n-6)!} - \cdots \right] + i \cdot \frac{1}{k^a} \left[ 1 - \frac{n! (b \cdot \ln k)^2}{n^2 (n-2)!} + \frac{n! (b \cdot \ln k)^4}{n^4 (n-4)!} - \frac{n! (b \cdot \ln k)^6}{n^6 (n-6)!} + \frac{n! (b \cdot \ln k)^8}{n^8 (n-8)!} - \cdots \right]
\]

\[
\lim_{n \to \infty} \frac{n! (b \cdot \ln k)^m}{n^m (n-m)! m!} = \left( \frac{b \cdot \ln k}{m} \right)^m
\]

\[
\frac{1}{k^a} = \frac{1}{k^a} \left[ 1 - \frac{(b \cdot \ln k)^2}{2!} + \frac{(b \cdot \ln k)^4}{4!} - \frac{(b \cdot \ln k)^6}{6!} + \frac{(b \cdot \ln k)^8}{8!} - \cdots \right] + i \cdot \frac{1}{k^a} \left[ - \frac{(b \cdot \ln k)^1}{1!} + \frac{(b \cdot \ln k)^3}{3!} - \frac{(b \cdot \ln k)^5}{5!} + \frac{(b \cdot \ln k)^7}{7!} - \cdots \right]
\]

\[
\cos(x) = \frac{1}{0!} - \frac{(x)^2}{2!} + \frac{(x)^4}{4!} - \frac{(x)^6}{6!} + \frac{(x)^8}{8!} - \cdots \quad \sin(x) = \frac{(x)^1}{1!} - \frac{(x)^3}{3!} + \frac{(x)^5}{5!} - \frac{(x)^7}{7!} - \cdots
\]

\[
\frac{1}{k^a} \left( \cos(b \cdot \ln k) - i \cdot \sin(b \cdot \ln k) \right)
\]

\[
\eta(a + ib) = \frac{1}{1^{(a+ib)}} - \frac{1}{2^{(a+ib)}} + \frac{1}{3^{(a+ib)}} - \frac{1}{4^{(a+ib)}} + \cdots = \left[ 1^{a} - \frac{\cos(b \ln 1)}{1^a} + \frac{\cos(b \ln 2)}{2^a} - \frac{\cos(b \ln 3)}{3^a} + \frac{\cos(b \ln 4)}{4^a} + \cdots \right] + i \left[ \frac{\sin(b \ln 1)}{1^a} + \frac{\sin(b \ln 2)}{2^a} - \frac{\sin(b \ln 3)}{3^a} + \frac{\sin(b \ln 4)}{4^a} + \cdots \right]
\]
another way (and much more easier way) to look at this is:

\[ \eta(a + ib) = \left[ \frac{1}{1^a} \cos(-b \ln 1) - \frac{1}{2^a} \cos(-b \ln 2) + \frac{1}{3^a} \cos(-b \ln 3) - \frac{1}{4^a} \cos(-b \ln 4) + \ldots \right] + \left[ \frac{1}{1^b} \sin(-b \ln 1) - \frac{1}{2^b} \sin(-b \ln 2) + \frac{1}{3^b} \sin(-b \ln 3) - \frac{1}{4^b} \sin(-b \ln 4) + \ldots \right] \cdot i \]

\[ \vec{V}_k = \frac{1}{1^k} \quad \theta_k = -b \ln k \]

\[ \eta(a + ib) = \left[ \vec{V}_1 \cdot \cos(\theta_1) - \vec{V}_2 \cdot \cos(\theta_2) + \vec{V}_3 \cdot \cos(\theta_3) - \vec{V}_4 \cdot \cos(\theta_4) + \ldots \right] + \left[ \vec{V}_1 \cdot \sin(\theta_1) - \vec{V}_2 \cdot \sin(\theta_2) + \vec{V}_3 \cdot \sin(\theta_3) - \vec{V}_4 \cdot \sin(\theta_4) + \ldots \right] \cdot i \]

moving on the xAxis \( \vec{V}_1 \cdot \cos(\theta_1) - \vec{V}_2 \cdot \cos(\theta_2) + \vec{V}_3 \cdot \cos(\theta_3) - \vec{V}_4 \cdot \cos(\theta_4) + \ldots \)

moving on the yAxis \( \vec{V}_1 \cdot \sin(\theta_1) - \vec{V}_2 \cdot \sin(\theta_2) + \vec{V}_3 \cdot \sin(\theta_3) - \vec{V}_4 \cdot \sin(\theta_4) + \ldots \)

when xAxis=0 and yAxis=0 you are at the number zero!

this alternative series extends the zeta function from \( \text{Re}(s) > 1 \) to the larger domain \( \text{Re}(s) > 0 \)

meaning that all the points of \( \eta(a + ib) \) when \( \text{Re}(s) > 0 \) are converging!

Meaning that you don’t need to use the complex plane to solve or understand the the famous Riemann hypothesis

because \( (1 - 2^{1-s})\xi(s) = \eta(s) \) when one side is zero the other one is zero as well
the only difference between the two is that one is not a “real zero” only “Analytic Continuation” value of the center of the spiral of a divergent series while the other is a “real zero” the spiral converges to the value zero

The Riemann hypothesis equivalent to:

\[
0 = \cos(b \ln 1) - \cos(b \ln 2) + \cos(b \ln 3) - \cos(b \ln 4) + \ldots \quad \text{and} \quad 0 = \sin(b \ln 1) - \sin(b \ln 2) + \sin(b \ln 3) - \sin(b \ln 4) + \ldots
\]

where a and b are real numbers and the only solution for \( 0 < a < 1 \) is when \( a = \frac{1}{2} \)