Global Warming Re-Radiation Model with New Albedo-Planck Parameter
Alec Feinberg

Key Words: Re-Radiation Model, Global Warming Modeling, Plank Parameter, Planck-Albedo Parameter

Abstract In this paper, we show how global warming can be modeled using a re-radiation factor and use the Planck’s feedback parameter to verifying consistency. The re-radiation factor is important in quantifying the fact that albedo versus a greenhouse gas change will have more impact on global warming. In our simple model we also found an alternate way to assess the Planck parameter. As well we define a handy Planck-Albedo feedback parameter that has a convenient value of 1 W/m²/%albedo/K.

1 Introduction

Although global warming models are highly complex, often it is helpful to use a simple model and relate it to the top of the atmosphere using the plank feedback parameter. The model uses a re-radiation factor that helps to quantify significance in albedo versus greenhouse gas changes. In working with the model we find a handy Planck-albedo parameter that may be useful to climatologists [1]. This model illustrates a reasonable way to view the Earth’s energy budget, it is likely useful as a teaching aid, and provides a number of useful insights in climatology sensitivity estimates and for alternate albedo solutions to global warming [2].

2. Data and Method

In order to introduce the re-radiation surface model, it is helpful to look at the Planck feedback parameter as it plays a key role in verifying modeling.

2.1 Overview of Planck Feedback Parameter

Estimates on Planck’s feedback parameter are varied typically between 3.21 to 3.8 with some values as high as 7.1 [3]. The IPCC AR4 [4] list a value of 3.21. Numerous authors have developed different expressions [3]. A typical estimate uses

\[
F_{TOA} = (1 - \alpha)S_o / 4 - \sigma(\beta T_s)^4 = (1 - \alpha)S_o / 4 - R_{LWR}
\]

where \(S_o = 1361\, \text{W/m}^2\), \(F_{TOA}\) is the radiation budget at the top of the atmosphere, \(R_{LWR}\) is the outgoing long wave radiation (a function of surface temperature and albedo), \(\sigma\) is the Stefan-Boltzmann constant and \(\beta\) is described below. Then the Planck parameter \(\lambda_o\) can be calculated as

\[
\lambda_o = \partial F_{TOA} / \partial T_s = -\partial R_{LWR} / \partial T_s = -4\sigma(\beta T_s)^4
\]

This result is

\[
\lambda_o = -4\beta^4\sigma T_s^3 = -4\beta \sigma T_s^3
\]

where \(\beta\) varies from 0.876 to 0.887 (average 0.8815) and \(T_s=288^\circ\text{K}\) [4]. This yields 3.21<\(\lambda_o<3.37\). However, from Eq. 3 \(\beta\) is taken as the ratio

\[
\beta = T_{atm} / T_s = 255^\circ\text{K} / 288^\circ\text{K} = 0.8854
\]

Where we take \(T_{TOA}=255^\circ\text{K}\), so that \(\lambda_o =3.33\). Another expression developed by Schlesinger [5] dependent on the albedo and surface temperature, given by

\[
\lambda_o = S_o (1 - \alpha) / T_s
\]

When \(S_o=1361,\ 0.294118<\alpha<0.3,\ \text{and}\ T_s=288^\circ\text{K}\) then 3.3358 >\(\lambda_o>3.308\) respectively.

2.2 Estimating Planck’s Parameter with an Albedo Method

Consider a global albedo change corresponding to 1°K rise from solar absorption. Since we are only concerned with an albedo change that corresponds to a surface temperature change we can write
\[ F_{TOA} = 0 = (1 - \alpha)E_o - \sigma(T_o)^4 \]  
(6)

where \( E_o = \frac{S_o}{4} \). Then a 1\(^\circ\)K change is

\[ \Delta T_o = T_2 - T_1 = \left( \frac{E_o}{\sigma} (1 - \alpha_t) \right)^{1/4} \left( \frac{E_o}{\sigma} (1 - \alpha_t) \right)^{1/4} = 1^\circ K \]  
(7)

Here we will use the AR5 albedo starting value of 0.29411.\(^6\). We find that the corresponding albedo change is

\[ \Delta E_o = E_o \{ (1 - \alpha_t) - (1 - \alpha_t) \} = E_o (\alpha_t - \alpha_t) = 3.784 W/ m^2 \]  
(8)

Since this is for a 1\(^\circ\)K rise then let

\[ \lambda_{1K} = 3.784 W/ m^2 K \]  
(9)

We note this is related to the surface value, then

\[ \lambda_{1K} = 4\sigma T_1^3 \]  
(10)

By comparison to above we have

\[ \lambda_{1K} \beta = \lambda_o = 3.784 W/ m^2 K = 3.349 W/ m^2 K \]  
(11)

This is very close to the 3.33 W/m\(^2\)/\(^\circ\)K value obtained in the traditional manner.

**2.3 Top of the Atmosphere and Beta**

At the top of the atmosphere we obtain

\[ T_{TOA} = \beta T_o \]  
(12)

and

\[ F_{TOA} = S_o (1 - \alpha_t) - \sigma T_o^4 \]  
(13)

giving

\[ \beta^4 F_{TOA,T} = F_{TOA,T_{Total}} \]  
(14)

We will need this expression later when showing model consistency with the Planck feedback parameter.

**2.4 Re-radiation GW Model**

Global warming can be modeled by looking at two different time periods. We assume no global warming in 1950 compared to 2019 as

\[ P_{Total,1950} = P_{\alpha} + P_{GHG} \quad \text{and} \quad P_{Total,2019} = P_{\alpha} + P_{GHG+Feedback} \]  
(15)

where

\[ P_{\alpha} = S_o \{ 0.25 \times (1 - \text{Albedo}) \} \]  
(16)

where \( S_o = 1361 \text{ W/m}^2 \). Note that the 2019 model has a Feedback added factor due to forcing and \( \alpha' \) indicates that warming is also occurring due to albedo change from urbanization and ice and snow melting.

our re-radiation model is simply

\[ P_{GHG} = f \ P_{\alpha} \quad \text{and} \quad P_{GHG+Feedback} = f \ P_{\alpha'} \]  
(17)

We then write

\[ P_{Total} = \sigma T^4 \quad \text{and} \quad P_{\alpha} = \sigma T_o^4 \]  
(18)

**3.0 Results and Discussion**

The re-radiation parameter \( f \) is adjustable and is set so that \( T_{1950} = 13.89^\circ C \) (287.038\(^\circ\)K) and \( T_{2019} = 14.84^\circ C \).

Consider now a small change of 0.2% albedo change from 1950 to 2019, related to events on Earth such as increases in UHI and ice and snow melting so we set

\[ \text{Albedo}_{1950} = 29.6118 \quad \text{and} \quad \text{Albedo}_{2019} = 29.4118 \]
We then note if we set the re-radiation parameters for 1950 and 2019 to 

\( f_{1950} = 0.6072 \) and \( f_{2019} = 0.624 \)

The results yield 

\[ P_{\text{Total } 1950} = 384.9177 \text{ W/m}^2 \text{ and } P_{\text{Total } 2019} = 390.0464 \text{ W/m}^2 \]

We find that 

\[ \Delta P_{\text{Total}} = P_{2019} - P_{1950} = 5.13 \text{ W/m}^2 \]  \hspace{1cm} (19)

and 

\[ \Delta T_{\text{Total}} = T_{2019} - T_{1950} = 0.95^\circ C \] \hspace{1cm} (20)

The table below summarizes the model results for the specified albedos and setting the temperatures to those observed at the surface.

<table>
<thead>
<tr>
<th>Year</th>
<th>( T(\text{K}) )</th>
<th>( T^0(\text{K}) )</th>
<th>( f )</th>
<th>( \alpha, \alpha' )</th>
<th>( P_{\alpha}, P_{\alpha'} )</th>
<th>( P_{\text{GHG}, P_{\text{GHG, feedback}}} )</th>
<th>( P_{\text{Total}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2020</td>
<td>288.0389</td>
<td>255.11</td>
<td>0.62512</td>
<td>29.4118</td>
<td>240.176</td>
<td>150.139</td>
<td>390.315</td>
</tr>
<tr>
<td>1950</td>
<td>287.0388</td>
<td>254.93</td>
<td>0.60722</td>
<td>29.6118</td>
<td>239.496</td>
<td>145.427</td>
<td>384.92</td>
</tr>
<tr>
<td>( \Delta 2020-1950 )</td>
<td>1.00</td>
<td>0.18</td>
<td>1.68</td>
<td>-0.2 (0.68%)</td>
<td>0.681</td>
<td>4.712</td>
<td>5.39</td>
</tr>
</tbody>
</table>

“What If - 1K From Albedo Change”

<table>
<thead>
<tr>
<th>Year</th>
<th>( T(\text{K}) )</th>
<th>( T^0(\text{K}) )</th>
<th>( f )</th>
<th>( \alpha, \alpha' )</th>
<th>( P_{\alpha}, P_{\alpha'} )</th>
<th>( P_{\text{GHG}, P_{\text{GHG, feedback}}} )</th>
<th>( P_{\text{Total}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2020</td>
<td>288.039</td>
<td>255.11</td>
<td>0.62512</td>
<td>29.4118</td>
<td>240.176</td>
<td>150.139</td>
<td>390.315</td>
</tr>
<tr>
<td>1950</td>
<td>287.0391</td>
<td>254.11</td>
<td>0.62835</td>
<td>30.5248</td>
<td>236.389</td>
<td>148.535</td>
<td>384.925</td>
</tr>
<tr>
<td>( \Delta 2020-1950 )</td>
<td>1.00</td>
<td>1.00</td>
<td>-0.323</td>
<td>-1.113 (3.65%)</td>
<td>3.787</td>
<td>1.6</td>
<td>5.39</td>
</tr>
</tbody>
</table>

To show model consistency, we need to see how the 5.39 W/m², resulted from a 1K change, agrees with what is expected from Planck’s feedback parameter. We recall that 

\[ \beta^4 \Delta F_{\text{TOA}} = 5.25 x \beta^4 = 3.2 \text{ W/m}^2 \] \hspace{1cm} (21)

This illustrates the consistency of the simple re-radiation model. Then Planck’s feedback temperature rise finds an increase of 

\[ 3.2 \text{ W/m}^2 x (1/3.3)^4 \text{K/W/m}^2 = 0.95^\circ \text{K at } T_s \] \hspace{1cm} (22)

### 3.1 Why is the Re-radiation Parameter Significant?

First we see from Table 1 that a 1.8\% change in re-radiation increase is occurring. This provides an estimate of the climate change from a different perspective and can be helpful in looking at how our climate is working. We note that the re-radiation parameter is about 60\%. This shows significance in climatology as well. It indicates how much of the black body portion is re-radiated back to Earth. We note in the chain of events, prior to GHG re-radiation, black body absorption must occur. This indicates that an albedo change corresponds to about 160\% impact on global warming. However, a 100\% GHG change only impacts global warming by about 60\%. Therefore, one would conclude an albedo solution to global warming has a larger impact and it more advantageous. As well, an albedo solution has other major advantages, as it can reverse global warming and possibly preventing a tipping point from occurring.

### 3.2 Planck-Albedo Feedback Parameter

There are two albedo changes in Table 1, they are: \( \Delta \alpha = -0.2 \) or \( \% \Delta \alpha = 0.68\% \) and \( \Delta \alpha = 0.1113 \) or \( \% \Delta \alpha = 3.42\% \).

The albedo power changes \( \Delta P_{\alpha} \) in Table 1 are 0.681 W/m² and 3.787 W/m², respectively.

We note that we can define a unique Planck-albedo parameter as \( \lambda_{\% \Delta \alpha} = \Delta P_{\alpha} / \% \Delta \text{albedo} \). To illustrate from Table 1 

\[ \lambda_{\% \Delta \alpha} = 1 \text{ W/m}^2 / \% \text{albedo} = 0.681 / 0.68\% \text{ and } 1.04 \text{ W/m}^2 / \% \text{albedo} = 3.767 / 3.65 \] \hspace{1cm} (23)

This parameter can also be expressed per degree since in both case we have a 1K change, then
\[ \lambda_{%\Delta \text{albedo}} \approx 1 \text{W/m}^2 / \Delta \% \text{albedo} / ^\circ K \]  

The parameter was first noted in Feinberg 2020 [1] but is featured here as a modeling tool. We term it the Plank-albedo parameter, since it relates to black body \( P_\alpha \) absorption. This interesting parameter arises from the basic assessment

\[ \lambda_{%\Delta \text{albedo}} = \frac{\Delta E_\alpha}{\alpha_i - \alpha_1} \approx \frac{E_\alpha (\alpha_i - \alpha_1)}{100} = \frac{E_\alpha}{\alpha_i} \frac{\alpha_1 - \alpha_i}{100} = 1 \text{W/m}^2 / \Delta \% \text{albedo} \]  

where \( E_\alpha = 340 \text{ W/m}^2 \) and we see the closer that \( \alpha_1 \) is to 29.4118\%, the nearer a value of 1W/m\(^2\)/\%albedo is obtained. We note the value 29.4118\% (100/340) is given in AR5 [6]. We note the parameter’s relationship to

\[ \lambda_\alpha = \lambda_{%\Delta \text{albedo}} \times \% \Delta \alpha \]  

4.0 Conclusion

In this paper we provided a simple re-radiation model. The model shows consistency with the Planck parameter. We noted that the re-radiation parameter increased by about 1.8\% illustrating the warming from a different perspective. The re-radiation parameter was quantified showing about 60\% of black-body radiation is re-emitted to Earth. One can conclude that an albedo change in global warming has a 160\% impact. Furthermore, one can conclude that GHG change impact is only 60\% by comparison. We also found a handy parameter that we termed the Planck-albedo parameter which is about \( \lambda_{%\Delta \text{albedo}} \approx 1 \text{W/m}^2 / \Delta \% \text{albedo} / ^\circ K \) and can be helpful in estimating \( \lambda_\alpha \).

References

2. Feinberg, A., The Alternate Solution to Global Warming, Vixra