A. Feinberg, Ph.D., DfRSoft Research, email: dfrsoft@gmail.com, ORCID: 0000-0003-4364-2460

1. Introduction

Although global warming is highly complex, often it is helpful to work with a simplified model. We create a model that uses a re-radiation factor which helps to quantify significant differences between changes in the global albedo versus greenhouse gas forcing. It takes into account what normally happens in equilibrium. This is not similar to looking at a comparison of independent feedback parameters \( \lambda_{\mathrm{GHG}}/\lambda_0 \) which provides a different kind of assessment. Here we use a re-radiation parameter obtained mainly in an equilibrium model with appropriate constraints to aid in the comparison; it is then independently found with a unique value of 0.612 (or \( \beta=0.887 \)). This is a redefined variable taken from the effective the emissivity constant of the planetary system. Then the Planck’s feedback parameter is used to verify model consistency. This model illustrates a reasonable way to view the Earth’s energy budget; it provides a number of useful insights in climatology sensitivity estimates and demonstrates the relative advantage of solar geoengineering solutions over GHG reduction in global warming mitigation [1]. Specifically, a 2.6 larger albedo advantage is found. In working the model, we also find a handy Planck-Albedo parameter that may be useful to climatologists [2] having a convenient value of 1W/m\(^2\)/K/\(\Delta \%\)albedo and this is used to help illustrates the benefits in equilibrium assessments.

2. Data and Method

In order to introduce the re-radiation surface model, it is helpful to initially look at the Planck parameter as it plays a key role in verifying modeling.

### 2.1 Overview of Planck Feedback Parameter

Estimates on Planck’s feedback parameter are varied, typically between -3.8 W/m\(^2\)/K and -3.21 W/m\(^2\)/K with some values as large as -7.1 W/m\(^2\)/K [3]. The IPCC AR4 [4] list a value of -3.21 W/m\(^2\)/K. Numerous authors have developed different expressions [3]. A typical estimate starts with

\[
F_{\text{TOA}} = (1-\alpha)S_o/4 - \sigma(\beta T_s)^4 = (1-\alpha)S_o/4 - R_{\text{LWR}}
\]

where \( S_o = 1361 \) W/m\(^2\), \( F_{\text{TOA}} \) is the radiation budget at the top of the atmosphere, \( R_{\text{LWR}} \) is the outgoing long wave radiation (a function of surface temperature and albedo), \( \sigma \) is the Stefan-Boltzmann constant and \( \beta \) is described in this section below and later will be redefined in terms of a re-radiation parameter. Then the Planck parameter \( \lambda_0 \) can be calculated as

\[
\lambda_0 = \partial F_{\text{TOA}}/\partial T_s = -\partial R_{\text{LWR}}/\partial T_s
\]

This result is

\[
\lambda_0 = -4\beta \sigma T_s^3 = -4\beta \sigma T_{\text{eq}}^3
\]

where \( \beta \) varies in the literature from 0.876 to 0.887 (averaging=0.8815) and \( T_s=288^\circ \text{K} \) [4]. This yields -3.37 W/m\(^2\)/K<\( \lambda_o < \)3.21 W/m\(^2\)/K.
However, from Eq. 3, $\beta$ is often taken as the ratio

$$\beta = \frac{T_{\text{TOA}}}{T_s} = \frac{255^\circ \text{K}}{288^\circ \text{K}} = 0.8854$$  \hspace{1cm} (4)

A common assessment uses $T_{\text{TOA}}=255^\circ \text{K}$, so that $\lambda_{\text{TOA}}=-3.33 \text{W/m}^2/\text{K}$. Another expression developed by Schlesinger [5] is dependent on the albedo and surface temperature as

$$\lambda = S_o (1 - \alpha) / T_s$$  \hspace{1cm} (5)

When $S_o=1361$, $0.294118<\alpha<0.3$, and $T_s=288^\circ \text{K}$ then $-3.308 \text{W/m}^2/\text{K}>\lambda_{\text{TOA}}>3.3358 \text{W/m}^2/\text{K}$, respectively.

### 2.2 Estimating Planck's Parameter with an Albedo Method

Consider a global albedo change corresponding to $1^\circ \text{K}$ rise from solar absorption. Since we are only concerned with an albedo change

$$F_{\text{TOA}} = 0 = (1 - \alpha) E_o - \sigma (T_s)^4$$  \hspace{1cm} (6)

where $E_o=S_o/4$. Then a $1^\circ \text{K}$ change is

$$\Delta T_s = T_2 - T_1 = \left( \frac{E_o}{\sigma} (1 - \alpha_1) \right)^{1/4} - \left( \frac{E_o}{\sigma} (1 - \alpha_2) \right)^{1/4} = 1^\circ \text{K}$$  \hspace{1cm} (7)

Here we will use the AR5 albedo starting value of 0.294118 [6]. We find that the corresponding albedo change is 0.28299 when $E_o=340 \text{W/m}^2$. This corresponds to an absorption of

$$\Delta E_o = E_o \{ (1 - \alpha_2) - (1 - \alpha_1) \} = E_o (\alpha_1 - \alpha_2) = 3.784 \text{W/m}^2$$  \hspace{1cm} (8)

Since this is for a $1^\circ \text{K}$ rise, then it can also be written as

$$\lambda_{1\circ \text{K}} = 3.784 \text{W/m}^2/\text{K}$$  \hspace{1cm} (9)

We note this is related to the surface value, then

$$\lambda_{1\circ \text{K}} = -4 \sigma T_s^3$$  \hspace{1cm} (10)

By comparison to above we have

$$\lambda_{1\circ \text{K}} = -4 \sigma T_s^3$$  \hspace{1cm} (11)

This is very close to the -3.3 $\text{W/m}^2/\text{K}$ value obtained in the traditional manner.

### 2.3 Top of the Atmosphere and Beta

From Eq. 1

$$R_{\text{LWR}} = \sigma (\beta T_s)^4 = \sigma (T_{\text{TOA}})^4$$  \hspace{1cm} (13)

giving

$$\beta^4 R_{\text{TOA},T_s} = R_{\text{TOA},T_{\text{TOA}}}$$  \hspace{1cm} (14)

We will need this expression later when showing model consistency with the Planck feedback parameter.

### 2.4 Re-radiation GHG GW Model

In this model we define

$$P_{\text{TOA}} = \sigma T_s^4 \text{ and } P_a = \sigma T_a^4$$  \hspace{1cm} (15)

We consider a time when there is no feedback issues. Then by conservation of energy, the equivalent power re-radiated from GHGs in this model is

$$P_{\text{GHG}} = P_f - P_a = \sigma T_s^4 - \sigma T_a^4$$  \hspace{1cm} (16)
Since typically, $T_e \approx \text{255}^\circ\text{K}$ and $T_s \approx \text{288}^\circ\text{K}$, then we note in keeping with the definition of Beta (see Eq. 4) for the moment, that $\beta \approx T_e / T_s$. This allows us to write

$$P_{GHG} = \sigma T_s^4 - \sigma T_a^4 = \frac{\sigma T_s^4}{\beta^4} - \sigma T_a^4 = \sigma T_a^4 \left( \frac{1}{\beta^4} - 1 \right)$$  \tag{17}

We note that when $\beta = 1$, there are no GHGs as required by definition of $\beta$. We now define a re-radiation parameter $f = \beta^4$. We know that some fraction of the blackbody radiation is re-radiated by the GHGs so $f$ is a re-radiation parameter. That is, the energy, $P_{GHG}$, must be some fraction $P_a$ so that

$$P_{GHG} = f P_a = f \sigma T_a^4$$ \tag{18}

However, in order for this to be true it requires

$$P_{GHG} = \sigma T_a^4 \left( \frac{1}{f} - 1 \right) = f \sigma T_a^4$$ \tag{19}

This leads us to solutions of the quadratic equation

$$f^2 + f - 1 = 0 \text{ yielding } f = 0.618034 \beta^4, \beta = (0.618)^{1/4} = 0.88664$$ \tag{20}

This is very close to the value estimated for $\beta$ and was obtained though energy balance in the planetary system providing a completely independent assessment without any approximations. In Section 2.6, we double check in another way by balancing energy in and out.

### 2.5 Re-radiation Model Applied to Two Different Time Periods

Global warming can be modeled by looking at two different time periods. We can model the radiation for 1950 as due to blackbody radiation with the addition of GHG re-radiation where in this time period

- we will assume no feedback issues causing a warming trend so that

$$P_{total,1950} = P_a + P_{GHG} = P_a + f_i P_a$$ \tag{21}

where $P_a = S_e \{0.25x(1 - \text{Albedo})\}$ and $S_e = 1361 \text{W/m}^2$. The equilibrium model is constrained by energy balance discussed in Section 2.4 and 2.6. In 2019 due to global warming trends, this model is more complex and harder to separate out terms. However, it can still be done looking at a snapshot point in time using equilibrium theory, so

$$P_{total,2019} = P_{a'} + P_{GHG'+Feedback} = P_{a'} + f_i P_{a'}$$ \tag{22}

Here $P_{GHG'+Feedback}$ includes GHGs and its increase comprising also of water-vapor increase, lapse rate feedback and other effects such as an increase in snow-ice albedo changes that are hard to separate out. That is, some of this feedback is related to GHG increases and some is related to albedo change. $P_{a'}$ represents any albedo change due to UHI absorption increases, cloud absorption change, ice and snow melting and so forth that can be discerned. We note that $f$, a measure of the emissivity, is not constant but must change since the amount of GHGs change.

However, the re-radiation still must connect the absorption to re-radiation. We have used a linear $f$ parameter that indicates the fraction of $P_a$ power that must be re-radiated back to obtain the observed temperature. To be clear, $f$ is just a fractional parameter related to the emissivity. In 1950 it is some function of the GHGs (with no feedbacks). In 2019 it is more complex. The model is also constrained relative to $f_i$ as described in Section 2.6. However, it is primarily related to GHGs re-radiation since $P_{GHG} \approx P_{GHG'+Feedback}$.

### 2.6 Balancing $P_{out}$ and $P_{in}$

Although Eq. 15 is reasonably simple, it turns out that $f_i$ has a uniquely defined value obtained when balancing the energy.
2.6.1 Balancing $P_{out}$ and $P_{in}$ in 1950

In order to balance the energy in with the energy out in 1950 with no global warming imbalance we can still start with Eq. 21. In equilibrium the radiation that leaves must balance what comes in $P_a$ so that

$$\text{Energy}_{\text{out}} = (1-f_i)P_a + (1-f_i)P = (1-f_i)P_a + (1-f_i)\{P_a + f_iP_a\} = (1-f_i)\{2P_a + f_iP_a\} = 2P_a - f_iP_a - f_i^2P_a = \text{Energy}_{\text{in}} = P_a$$

(23)

In 1950 the value of $f$ solves the quadratic equation

$$f_i^2 + f_i - 1 = 0 \text{ yielding } f_i = 0.618$$

(24)

Interestingly, this also says that

$$P_a = f_iP_{\text{Total, 1950}} \text{ or } P_a = f_i(P_a + f_iP_a) \text{ or } 1 = f_i(1 + f_i)$$

(25)

The RHS of Eq. 25 is Eq. 24 and Eq. 20. This is why $f_i$ is unique. It is the fractional amount of total radiation that is in equilibrium. As a final check, results will show in Section 3 and Table 1, that the value $f_i$ provides reasonable results.

2.6.2 Warming Imbalance in 2019

The re-radiation parameters $f_1$ and $f_2$ are connected and from Eq. 21 and 22 we have

$$f_2 = f_1 + (\frac{P_{2019}}{P_a'} - \frac{P_{1950}}{P_a'}) = f_1 + \Delta f$$

(26)

In this way $f_2$ is a function of $f_1$ = 0.618 and the differences in the global warming residuals that is defined in Eq. 26 as $\Delta f$.

3.0 Results and Discussion

Since the re-radiation parameter $f_1$ = 0.618, in order to obtain $T_{1950}$ = 13.89°C (287.038°K), the only adjustable parameter in our simple model is the Earth’s albedo. This value requires an albedo value of 0.3008 (see Table 1) to obtain the correct value $T_{1950}$. This is a reasonable and similar to values cited in the literature [11].

In 2019, the average temperature of the Earth is $T_{2019}$ = 14.84°C (287.99°K). Here we are not sure of the albedo since it likely changed due to UHI increase, snow and ice melting and cloud coverage changes. The IPCC value in AR5 [6] is 0.294118. However, this would represent a 3% change since 1950 which may be an overestimation. In our assessment, we will assume a 1% change. Then the $f_2$ parameter is adjusted to 0.6124 in order to obtain $T_{2019}$. Results are provided in the Table 1. The results yields $P_{\text{Total, 1950}}$ = 384.918 W/m$^2$ and $P_{\text{Total, 2019}}$ = 390.024 W/m$^2$. We find that

$$\Delta P_{\text{Total}} = P_{2019} - P_{1950} = 5.097 W/m^2$$

(27)

and

$$\Delta T_{\text{Total}} = T_{2019} - T_{1950} = 0.95°C$$

(28)

which is the observed surface temperature increase since 1950.

<table>
<thead>
<tr>
<th>Year</th>
<th>$T^o$K</th>
<th>$T_\alpha^o$K</th>
<th>$f_1, f_2$</th>
<th>$\alpha, \alpha'$</th>
<th>$P_a, P_{a'}$ (W/m$^2$)</th>
<th>$P_{\text{GHG}}$ (W/m$^2$)</th>
<th>$P_{\text{Total}}$ (W/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2019</td>
<td>287.991</td>
<td>254.78</td>
<td>0.63253</td>
<td>29.779</td>
<td>238.927</td>
<td>151.128</td>
<td>390.055</td>
</tr>
<tr>
<td>1950</td>
<td>287.041</td>
<td>254.51</td>
<td>0.6180</td>
<td>30.08</td>
<td>237.903</td>
<td>147.032</td>
<td>384.935</td>
</tr>
<tr>
<td>$\Delta 2020-1950$</td>
<td>$0.95$</td>
<td>0.27</td>
<td>1.45%</td>
<td>-0.3</td>
<td>1.024</td>
<td>4.096</td>
<td>5.121</td>
</tr>
</tbody>
</table>

(1%)

The table below summarizes model results for the specified albedos and setting the model to the observed Earth’s surface temperatures.

To show model consistency, the forcing change 5.121 W/m$^2$ resulting in a 0.95°C rise, should agree with what is expected from Planck’s feedback parameter. From Eq. 14 it is evident that
\[ \beta^4 \Delta R_{TOA} = 5.121 \times \beta^4 = 3.1W/m^2 \]

This illustrates the consistency of the simple re-radiation model. Then Planck’s feedback parameter (3.3 W/m²/°K) temperature rise is in reasonable agreement with what is observed by equilibrium modeling

\[ 3.165W/m^2 \times (1/3.3)°K/W/m^2 = 0.959°K \]

### 3.1 Why the Re-radiation Parameter is Significant

In Table 1, the measure of \( \Delta f = 1.45\% \) fractional increase is due to re-radiation change. This is significant. From Eq. 21, 22 and 26 we can illustrate this key characteristic of the climate change

\[ \Delta f = (P_{GW} + P_{FGF}) - (P_{GW} + fP_{GW}) \approx (P_{GW} + fP_{GW}) - P_{GW} \]

Therefore \( f \) is an estimate of climate re-radiation and \( \Delta f \) an estimate of climate emissivity change. It is a measure of GHG increase and the feedback relative to the initial radiation, and is generally helpful in looking at how our climate is working. Furthermore, we can deduce an albedo advantage.

### 3.2 The Albedo Advantage

We can look at an important ratio, the power created by the albedo effect compared to GHGs in 1950. The initial radiation is \( P_a \) which heats the Earth to 254.51°K then according to Eq. 21 and Table 1, the \( P_{GW} \) energy originates from a fraction of this original heating due to re-radiation as \( fP_a \)

\[ \frac{P_a + P_{GW}}{P_{GW}} = \frac{P_a + fP_a}{P_{GW}} = 1 + f \approx 1.62 \]

In general, this also means that albedo change has a higher impact factor in climate forcing, 2.6 times larger than \( \Delta P_{GW} \) as well, that is a change, \( \Delta P_a \) compared with a change in \( \Delta P_{GW} \) would yield the same impact factor

\[ d(P_a + P_{GW}) = 2.62 \times d(P_{GW}) \] or assuming \( \Delta f \ll 1 \)

\[ \frac{\Delta P_a + \Delta P_{GW}}{\Delta P_{GW}} \approx \frac{\Delta P_a + f \Delta P_a}{f \Delta P_a} \approx \frac{1 + f}{f} = 1.62 \]

This is a key reason that UHIs, cloud coverage, snow and ice melting, can create significant climate effects. Appendix A puts this important impact factor in layman’s terms. We see this is a different kind of comparison then \( \lambda_{GW}/\lambda_{FGF} \). It uses a re-radiation emissivity parameter obtained mainly from the equilibrium model.

In this view, an albedo solution is advantageous having significant potential for reversing global warming or ignoring it, as in UHIs and roads, likely can create serious issues. Therefore, trying to control global warming by reducing GHGs is important. However, certainly an albedo approach is more advantageous. It reduces both initial absorption and its potential for re-radiation. Its impact rating can be taken as 162% compared to re-radiation \( f \) with a 62% impact by comparison according to Eq. 32 and 33, yielding a 2.6 times higher advantage. It is important to realize that because the albedo solution can highly impact GW and reverse trends, it is also vital in preventing a tipping point from occurring.

### 3.3 Planck-Albedo Feedback Parameter

The albedo and \( \Delta P_a \) change in Table 1, is: \( %\Delta \alpha = 1\% \) and 1.024W/m², respectively. We note this defines a unique Planck-Albedo parameter \( \lambda_{a,\Delta \alpha} = \Delta P_a / \%\alpha_{albedo} \). To illustrate from Table 1

\[ \lambda_{a,\Delta \alpha} = 1.024 \text{ W/m}^2 / \%\alpha_{albedo} = 1.024 / 1\% \]

This parameter can also be expressed per degree (noting the 0.95°K change in Table 1)

\[ \lambda_{%\Delta \alpha\Delta T} \approx 1 \text{ W/m}^2 / \Delta%\alpha_{albedo} / ^\circ K \]

The parameter was first noted in Feinberg 2020 [2] but is featured here as a modeling tool. We term it the Planck-Albedo parameter, since it relates to blackbody \( (P_a) \) absorption. A simple numeric example is given in the
conclusion to illustrate how it can provide helpful estimates. This interesting parameter arises from the basic
assessment of the two equilibrium time periods

\[
\lambda_{\%albedo} = \frac{\Delta E_{\text{in}}}{\alpha_1-\alpha_2} = \frac{E_{\text{albedo}}}{\alpha_1/100} = 1W/m^2/%\text{albedo}
\]  

(36)

where \( E_{\text{in}} = 340 \) W/m² and when \( \alpha_1 \) is 29.4118%, the value of 1.000W/m²/Δ%albedo is obtained. We note the value
29.4118% (100/340) is given in AR5 [6]. The parameter’s relationship to \( \lambda_{\%albedo} \) is

\[
\lambda_{\%albedo} = \lambda_{\%albedo} x \%\Delta \alpha
\]

(37)

and the feedback parameter including \( f \) re-radiation is in 2019

\[
\lambda^f = \lambda_{\%albedo} x \%\Delta \alpha x 1.618
\]

(38)

4.0 Conclusion

In this paper we provided a simple re-radiation global warming model. The model shows consistency with the
Planck parameter. We noted that the re-radiation parameter increased by about 1.45% due to global warming from
1950 to 2019, illustrating the warming from a different perspective. From the model, the albedo effect was
quantified having an impact rating of 162% compared to GHGs with 62%. The albedo effect then yields a 2.6 times
higher advantage upon comparison. These results strongly support moving forward with solar geoengineering
solutions [2, 7-9].

We also found a handy parameter that we termed the Planck-Albedo parameter which is about
\( \lambda_{\%albedo} \approx 1W/m^2/%\text{albedo}/\degree \text{K} \). This can be helpful in quickly estimating the effect of an albedo change on global
warming and in assessing \( \lambda_{\%albedo} \). For example, Feinberg 2020 [1] suggested a goal of 1.5% geoengineering albedo
change. Using this parameter, an impact of 1.5 Watts/m² warming reduction should result. Given a 1.62 remission
factor (Eq. 32), this is 2.4W/m² improvement. With a reduction in water-vapor feedback, often estimated by a factor
of 2 [10], provides an overall resulting effect that could be as high as 4.8W/m². Feasibility is discussed in more
detail in Feinberg’s 2020 paper [1] and other solutions have been proposed [6-9].

Appendix A: Quantifying the Albedo Advantage in Layman’s Terms

It may be helpful for the reader to have a layman’s view of the 2.62 factor. Consider the Earth with a roof. The roof
represents the GHGs over the Earth and only allows 40% of any energy to leave with the rest returning to Earth.
Sunlight comes in and is absorbed and heats the Earth’s floor to 255°C (-2.3°F very cold). Let’s say it takes
100 units of energy. The heat rises but only 40 units of energy can leave from the roof, so 60 units comes back and
warms the Earth’s floor some more to 288°F (57°F average temp of Earth). On average the Earth’s floor is warmed
a total of 160 units. The Sun keeps warming the Earth’s floor at 100 units on average and the roof keeps sending
back 60. So the roof is responsible for 60 units on average and the Earth’s floor is warmed up to 160 units
on average. We can write this as

Energy units: 160=100+60=100+100x0.6

We see the 100 units is in two places in the equation due to the floor and roof, while the 60 is only in one place. That
is, without the floor absorption first, the roof cannot keep the Earth warm. Therefore, the heat coming from the
Earth’s floor results in160 units and the roof is only 60 units by comparison. The impact factor is

- 160/60=2.66, that is the heat from the Earth’s floor has this much larger impact.

Alternately, for every unit of energy given off by the Earth’s floor after absorption, it is equivalent to causing 1.6
units of heating while the roof (GHG) is only responsible for 0.6.

How much heat leaves in equilibrium? There was the initial 40 leaving of the 100 units of energy absorbed and
radiated. As well the Earth’s floor received a total of 160 units but the roof only let 40% leave that is another 64
(=0.4 x 160) units of energy leaving. The total leaving is 104 units in equilibrium so roughly 100 units comes in and
almost same goes out.

This can be refined to 61.8% (Eq. 20). Then 100 units is absorbed and radiated, then 38.2 units initially leave, and
61.8 units is radiated so the Earth’s floor is heated to 161.8 units of energy. From this 0.382 x 161.8 leaves=61.8
units or energy. The total is 61.8+38.2=100 units of energy leaves and another 100 units comes and equilibrium is
established. Any difference causes global warming.
References