Approximation of Euler's number

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Abstract

It is commonly thought that Euler's number, denoted as 'e' is approximately equal to 2.71828. In fact this error is found everywhere and even my CASIO fx-100AU PLUS calculator claims that e = 2.718281828... The following paper will explore this misconception and analytically find another approximation for e.

Theorem 0.1.

$$e = 3$$

Proof. We start with the integral $\int_1^e \frac{1}{x} dx$. After simple rearranging

$$\int_1^e \frac{1}{x} dx = \int_1^e d\frac{x}{x}$$

Using algebaric manipulation allows us to write the integral in an alternate form

$$= \int_1^e 1d\frac{x}{x} = \int_1^e 1d1$$

This resulting integral can be easily computed

$$= \frac{1^2}{2}\Big|_1^e = \frac{1}{2}\Big|_1^e$$

$$= \frac{1}{2}(e-1)$$

Now we make use of the mathematical fact

$$\int_1^e \frac{1}{x} dx = 1$$

We may thus equate the two results

$$\frac{1}{2}(e-1) = 1$$

$$e - 1 = 2$$

Solving this equation gives the truly astounding result

$$e = 3$$

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Please forward any counterexamples to richardzhang@live.com.au