Approximation of Euler’s number

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Abstract

It is commonly thought that Euler’s number, denoted as ‘e’ is approximately equal to 2.71828. In fact this error is found everywhere and even my CASIO fx-100AU PLUS calculator claims that $e = 2.718281828...$. The following paper will explore this misconception and analytically find another approximation for $e$.

Theorem 0.1.

\[ e = 3 \]

Proof. We start with the integral $\int_1^e \frac{1}{x} \, dx$. After simple rearranging

\[
\int_1^e \frac{1}{x} \, dx = \int_1^e \frac{d}{x} \, dx
\]

Using algebraic manipulation allows us to write the integral in an alternate form

\[
= \int_1^e \frac{d}{x} \, dx = \int_1^e 1 \, d1
\]

This resulting integral can be easily computed

\[
= \frac{1}{2} \bigg| \frac{e}{1} \bigg|_1^e = \frac{1}{2} \bigg|_1^e
\]

\[
= \frac{1}{2} (e - 1)
\]

Now we make use of the mathematical fact

\[
\int_1^e \frac{1}{x} \, dx = 1
\]

We may thus equate the two results

\[
\frac{1}{2} (e - 1) = 1
\]

\[
e - 1 = 2
\]

Solving this equation gives the truly astounding result

\[ e = 3 \]

Please forward any counterexamples to richardzhang@live.com.au