Abstract

The reason for this article is to give an account of the numerical relationship between the gravitational potential energy and the quantum energy of einstein-planck. By means of discrete numerical quantities and by quantifying the harmonic oscillators involved by means of a special parameter we have found that the ratio of both energies is equal to the value of the fine structure constant \( \alpha \).

Keywords. Fine structure constant, gravitational potential energy, quantum energy, discrete numbers.

Introduction

In the early 20th century the physicist Arnold Sommerfeld [1] introduced several corrections to Bohr's atomic model. He applied the Einstein’s special theory of relativity to the deviations observed in the spectral emission lines of the hydrogen atom. This is why one of the first physical interpretation consisted of measure the ratio of two speeds: the relativistic speed of the electron in the first circular orbital of the Bohr atom with respect to the speed of light in a vacuum. Sommerfeld calculated how much the deviation of the spectral lines of the hydrogen atom would be. There are different expressions to define the fine
structure constant $\alpha$ [2] One of the simplest ones defines it as the quotient of the elementary charge with the planck charge, squared.

$$\alpha = \frac{q^2}{q_P^2}$$ (1)

There is another physical interpretation that defines $\alpha$ as the quotient of two energies: the energy resulting from approximating two charges of the same sign, for instance two electrons, up to a distance $x$ and the energy of a photon whose wavelength is equal to $2\pi x$.

At this point, someone might ask: what does a Newtonian gravitational potential energy have to do with fine structure constant? As for the fine structure constant, his knowledge comes from the analysis of the spectral lines of the hydrogen atom and one set of interactions of the electron, mediated both by quantum mechanics and by electric charge and electromagnetism.

What we are going to explore is a numerical derivation of the fine structure constant from the ratio of two energies: gravitational potential energy and quantum energy. Using the international system of units (SI) [3] in which the meter (m), kilogram (kg) and second (s) are included as units of the dimensions length (L) mass (M) and time (T).

**Method and results**

A classical formulation for the gravitational potential energy [4] can be written as

$$U = G\frac{M^2}{R}$$ (2)

we omit the minus sign that is usually placed in the equation. Of first approximation we'll use the gravitational potential energy that exist between two masses, 1 kg each. Unit of mass in the SI system of units, separated by a distance $R$ of 1 m, unit of length in the SI.
\[ G = 6.6739 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} \] refers to Newtonian constant of gravitation[5]

The concise quantum energy formula described by means of Planck-Einstein relation [6] reads

\[ E = \hbar \nu \quad (3) \]

\[ \hbar = 1.054572 \times 10^{-34} \text{ Js} \] refers to Planck’s constant over \(2\pi\) [7] and \(\nu\) symbolizes the frequency of a given electromagnetic wave.

Let’s define now a useful parameter with which we can operate with sets of huge orders of magnitude

\[ (N...) = \frac{a_0}{eL_p} = 1.2045 \times 10^{24} \quad (4) \]

\[ a_0 = 5.291772 \times 10^{-11} \text{ m} \] refers to Bohr’s radius [8].

\[ L_p = 1.6162 \times 10^{-35} \text{ m} \] refers to Planck length [9]

\[ e = 2.71828... \] a mathematical constant known as Euler’s constant. Base of natural logarithms [10].

Readers will have already noticed that the numerical value of parameter \((N...)\) is two times the value of Avogadro’s number [11]. Notwithstanding which, both have different units. Our parameter \((N...)\) has been defined using physical length units only and therefore is a dimensionless parameter, while the units of the Avogadro number are given in \(\text{mol}^{-1}\).

Now, looking back to the equation (3), we could consider a huge collection of harmonic oscillators, \(1.2045 \times 10^{24}\) harmonic oscillators, so that
where \((N...\)) symbolizes \((N...\)) harmonic oscillators and \(\nu\) symbolizes the fundamental frequency mode of vibration. An initial allowed value, for our purposes, must be

\[
\nu = 72 \, s^{-1}
\]

because we want to achieve that the ratio, or relationship, between the gravitational potential energy and the energy of a quantum system of \((N...\)) harmonic oscillators equals \(\alpha\). Therefore the equation that relates (2) and (5) reads

\[
\frac{U}{E} = \alpha
\]

\[
\alpha = 0.007297353... \quad \text{the current accepted value for the fine structure constant.}
\]

Discussion

Typed point by point the formula (6)

\[
\frac{G \frac{1\, kg^2}{1\, m}}{\frac{1}{(N...)^l \nu}} = \alpha
\]

we also can write the above equation as

\[
\frac{G \frac{(X\, kg)^2}{(Y\, m)^2}}{\frac{1}{(N...)^l K \nu}} = \alpha
\]
if we want assign, *ad libitum*, different discrete values to

\( X, Y \) and \( K \)

*if and only if* discrete values are involved, so that

\[
\frac{X}{Y} = \frac{1}{2^2} = \frac{1}{4}
\]

for example, you can consider the case of two masses of \( 8 \times 10^{-8} \text{kg} \) each, separated by a distance of \( 2 \times 10^{-16} \text{m} \), thus the fundamental frequency must be multiplied by 32, thus \( \nu = 2304 \text{ s}^{-1} \) therefore

\[
\frac{G \left( 8 \times 10^{-8} \text{kg} \right)^2}{2 \times 10^{-16} \text{m}} \frac{1}{\hbar (N\ldots)x (2304 \text{ s}^{-1})} = \alpha
\]  

(10)

in any other case the numerical equality of (7) is not satisfied and

\[
\frac{U}{E} \neq \alpha
\]  

(11)

as long as the ratio between both energies is given by the fine structure constant \( \alpha = 0.007297353\ldots \)

(A concise conceptual depiction of the matter is in the figure 1.)
Conclusion

We’ve explored of a numerical relationship of the classical gravitational potential energy that exists between two masses and the fine structure constant by means of Planck-Einstein quantum energy. A numerical discretization is necessary to reach such a result. In scientific literature there are a multitude of numerical derivations for the fine structure constant. Here we have presented one more in which gravitation has a role.
Bibliography
