The Ticker-Tape Interpretation of Quantum Mechanics

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Abstract

In recent years there has been a move away from the Copenhagen Interpretation towards alternative interpretations of Quantum Mechanics. There has also been an acknowledgement that there is no definitive version of the Copenhagen Interpretation because the originators, Bohr, Heisenberg et. al., did not agree over all aspects of the interpretation. This paper revisits the philosophical approach taken by Bohr. The result is a new interpretation, named the Ticker-Tape Interpretation, which is closely related to Copenhagen. The formalism of Quantum Mechanics is derived from simple principles. The interpretation leads to some conjectures.

Author's Note: The author has rather playfully borrowed the title of some of Einstein's famous "Principles". Even though the principles in this paper have the same names as those found in Special or General Relativity, they have nothing to do with any specifics of Special or General Relativity; the names were chosen however because, at some level, the both versions derive from an even more general principle.

1 A Theory of Everything

Quantum Mechanics seeks to be the "Theory of Everything" - It should be consistent with modern Western Philosophy.

The problem is that much of modern theoretical Physics is divorced from these considerations. The author believes the result is much confusion and wasted effort.

The starting point of this paper is a modern presentation of the Copenhagen Interpretation. Note that the passage of time means that some aspects of Copenhagen are no longer of much interest and may be excluded from or barely mentioned in the presentation.

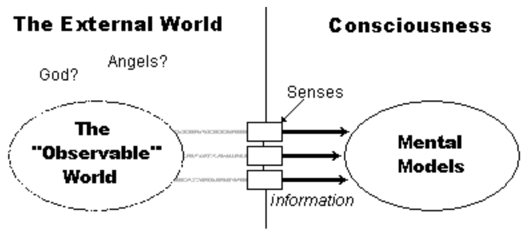

Figure 1: Katian World View (circa 1781)

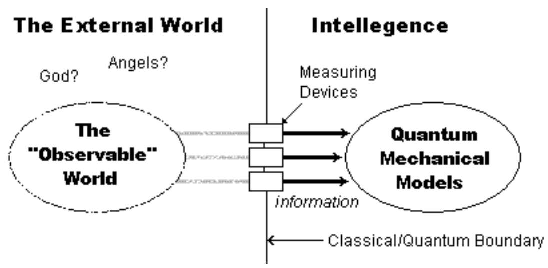

Figure 2: Copenhagen World View (circa 1928)

2 Philosophical Underpinnings

> *It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature.* — Niels Bohr, quoted in Aage Petersen, The Philosophy of Niels Bohr in Bulletin of the Atomic Scientists, 1963.

> *What might be said of things in themselves, separated from all relationship to our senses, remains for us absolutely unknown.* — Immanuel Kant, Critique of Pure Reason, 1781.

This paper adopts a world view that is essentially Kantian. The Kantian world view, shown in figure (1) consists of an external world that is perceived through the sensors of an agent; the agent builds mental models of the external world based on that sensory input but cannot know the true nature of the universe of which his sensors give but a hint.

Copenhagen is startling similar, not accidentially, to the Kantian world view. (It has been argued that the Copenhagen Interpretation is Positivist, however subtle arguments over philosophical classifications are beyond the scope of this paper) Quantum Mechanics replaces the vague idea of sensory input with that of measurement and precise mathematical description. The idea of a conscious agent is replaced by intelligence, nominally called the observer, which need not be human. It could, for example, be a robot. The Copenhagen world view is shown in figure (2).

The measuring devices represent the frontier of the interaction between the

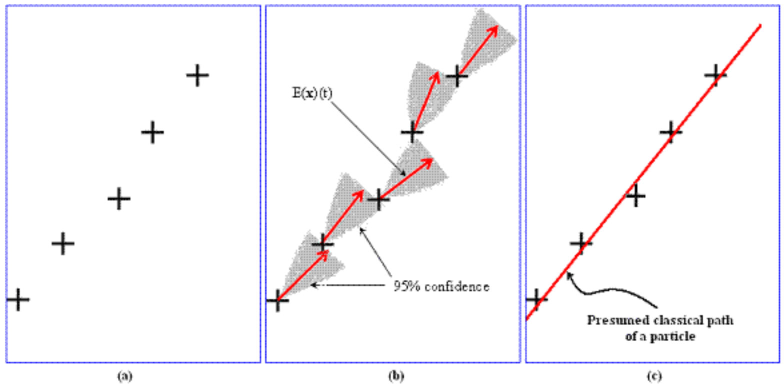

Figure 3: Measurements and Mental Models

observer and the rest of the universe; that abstract boundary is called the 'Classical/Quantum boundary' (Bohr). In Kant's world view, the corresponding boundary represents the separation of self from the Universe.

Measurements are regarded as "elements of reality" (Einstein's terminology). No definition of what that means is given; they are physical interactions and they are primitive concepts. The observer is logically separate from the measurement itself. A measurement may discontinuously changes the value or probability distribution of a value associated with a measuring device. The probability distribution is said to 'collapse' to its known value.

3 The Equivalence Principle

There is no transition from the Quantum world to the Classical world. The difference between measurement in the Classical world and measurement in the Quantum world is a matter of interpretation.

Quantum Mechanics is a mental model; the waveform and any measurement operator A are mental constructions, built on top of the information gathered from "raw" measurements. Classical mechanics is also a mental model (since we know it is not "true", it cannot be otherwise); typically Classical Mechanics deals with the expected value $< A >$ and regards the difference from $< A >$ to be "error".

In the figure (3), the left pane shows the "raw" position measurements of a particle "moving" diagonally from left to right is shown. The central pane adds the intellectual machinery of Quantum Mechanics; the calculated 95% confidence intervals are shown in gray. The right pane adds the intellectual machinery of Classical Mechanics; the presumed classical path is shown in red.

In Quantum Mechanics, the uncertainty in the position of a particle created by Heisenberg Uncertainty Principle is viewed as intrinsic, and the measurement themselves are taken at face value (The Principle of Exact Measurement) since no other information is available.

In Classical Mechanics, it is the measurement that contains the "error" (uncertainty) in the position of the particle; the "error" has a multitude of sources external to the system itself, typically related to the construction of the measuring devices and lack of knowledge of initial conditions, but there is always a presumption that if these influences could be eradicated, exact measurements would be possible and that the predictions of Classical mechanics would be confirmed.

4 The Ticker-Tape Interpretation

The Copenhagen Interpretation represents a coherent merging of Kantian philosophy and quantum phenomena. Indeed it is difficult to see how to modify Copenhagen without diverging from Kant, except for a single area where the author is in dispute with Copenhagen: Bohr expressed the opinion that measuring devices are essentially classical, and that a description of reality requires a-prior understanding of classical quantities such as position and momentum [1]. Apparently a knowledge of Classical Mechanics is required before Quantum Mechanical measurements can be understood, yet Quantum Mechanics is presumed to be more fundamental theory than Classical Mechanics. It is also not easy to apply Bohr's vision of essentially classical properties to abstract concepts such as QCD colour.

Rather than follow Bohr, we take a slightly different view and create and develop a new interpretation, the Ticker-Tape Interpretation, which assumes the same Kantian world-view as Copenhagen but assumes a slightly different characterisation of measurement.

The Ticker-Tape Interpretation introduces the following:

- Alternative measurements characterisation - developed throughout this paper.
- Histories - sequences of measurements.
- A metaphorical ticker-tape that records the history of measurements made by an observer.
- Mach Devices - primitive measurement devices.

4.1 Measurement

Mach devices measure quantities that are essentially real numbers. Why? Real numbers can be put "in order" from the smallest to the largest. I.e. If M = {set of possible measured values}, then there is a strict total ordering '$<$' of M, which creates an order preserving isomorphism from M to some subset of Real numbers. I.e. It is always possible to compare measurements. If $X_a = x_1$ and , then it is possible to say $X_a > X_b$ if $x_1 > x_2$.

We shall use the notation $X = x$ to mean the device X has been used to make a measurement and result has been reported as x.

Not all measurements are single values. A measurement may only restrict the values to a specific range (e.g. $a < X < b$ or $x \in [a, b]$) or a union of ranges (e.g. $x \in (a, b) \cup [c, d]$). The notation $X = x$ can be expanded to include these examples (x can an union of intervals).

4.1.1 Quantum Measurements

Looking forward, the formalism developed here to describe quantum systems applies to systems other than quantum systems, but the only systems that naturally follows the rules are quantum systems. In line with a Kantian world view, the only source of information is those measurements.The theory does not describe non-observables like hidden particle trajectories and such constructions will not be discussed. TODO: Bells Inequality?

4.2 Histories

A history is a sequence of measurements denoted $H = (m_0, m_1, m_2, ..., m_n)$ created from a single measurement $X = x$, or from a number of meaurements conbimed using the logical connectives $\wedge$(and) or $\vee$(or).

If $H_A = (m_1, ..., m_k)$ and $H_B = (m_{k+1}, ..., m_{k+m})$ are histories, then $H_A \wedge H_B = (m_1, \ldots, m_k, m_{k+1}, ..., m_{k+m})$. The history H_A must "follow" the history H_B and not overlap in time. The operator $\wedge$ is read as "and".

Definition: Let H_A be a history, then H_B is an alternative history to H_A if H_B andH_A contain the same initial measurements m_0, and the final measurement in each history is made by the same device. I.e. m_{Afinal} is $(Y = y_1)$ and m_{Bfinal} is $(Y = y_2)$.

Definition: If $H_A = (m_0, \ldots, m_{Ai}, ..., m_{Afinal})$ and $H_B = (m_0, \ldots, m_{Bi}, ..., m_{Bfinal})$ are alternative histories, then $H_A \vee H_B$ is the history H_A, H_B. The operator $\vee$ is read as "or".

No distinction is made between alternative descriptions; various histories are equivalent if they contain the same measurements under the same conditions. E.g. $H1 \vee H1 \vee H3$ and $\{H_1, \{H_2, H_3\}\}$ and $\{\{H_1, H_2\}, H_3\}$

Histories may represent actual measurements, or they may represent hypothetical histories in some sample space.

TODO: Tidy up.

4.3 Mach Devices

A measuring device is a Mach device, or M device, with respect to an observer if the following applies:

1. The device produces a stream of measurements (Real numbers), that are recorded on a (metaphorical) ticker-tape accessible to an observer. Measurements are recorded in the order they are made.

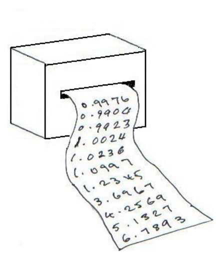

Figure 4: The output of a Mach device viewed as a "ticker tape"

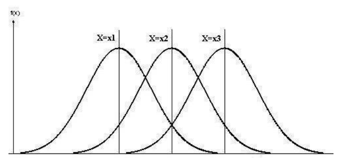

Figure 5: Probability distributions for numeric "labels"

2. The measurements are "repeatable".

3. The device's internal structure is unknown. It is a "black box". There is no a-prior information about what the numbers the device produces mean.

4. The observer does not have access to a clock.

A Mach device is the most primitive (abstract) measurement device possible.

Caveat: A "measuring device" which produces only a single number is not necessarily a Mach device. Figure 5 shows a typical set of probability distributions $P(X|X = x_i), i = 1, 2, 3$ for a classical measurement device (5). If there is overlap between the probability density functions, as is the case for the probability density functions for $X = x_1$ and $X = x_2$ in figure (5), repeated measurements of the system may result in the value x_1 or x_2. I.e. if there is any overlap in the probability density functions, the result of any measurement of X is not repeatable and so the device is not a Mach device.

5 Principle of Relativity (Mach's Principle)

How can we tell what a Mach device measures?

The output from a single Mach device is meaningless.

This is a generalisation of Mach's Principle. Mach's statement of the Principle of Relativity famously influenced Einstein but the principle itself dates back at least to Galileo. Ernst Mach argued that it would be meaningless to talk about the motion of a single particle in an empty Universe. All motion is relative. In fact, all measurement is relative. If there is no context, a measurement stream becomes a meaningless stream of numbers.

6 Mach Banishes Determinism

Suppose $X = f(t)$ is a classical quantity, and X is a Mach device with respect to an observer, then it is not possible for that observer to determine what the device measures solely from the measurement history.

Why? There is no way to calibrate the device. Suppose we construct a second device Y whose output is related to the first by $Y = \zeta(X(t))$. The second device is sealed, mixed up with the first and given to the naive observer so they both become Mach devices. Which measures the "fundamental" quantity? X and Y? X? Y? In fact, it is possible to build a device Z that returns any measurement profile we like. There is no way to chose a preferred M device.

If it is known that a measuring device produces values $X = f(t)$ for some function f (X is a function of time, X is deterministic) then the output from that device is meaningless.

7 Repeatability

Measurements are expected to be repeatable. It is one of the 2 defining characteristics of a measurement. I.e. if two measurements are made, one immediately after the other, the results of the two measurements should agree. However Mach devices do not come equipped with a clock. There is no sense of time. What does "immediately after" mean? If two consecutive measurements disagree, and it is not not possible to know the time interval between the two measurements, it could be that measuring device is OK and the interval was so long that the system slowly evolved into another state. It is also possible that the device is "faulty" (not a Mach device). How is it possible to tell the two case apart?

7.1 Plan of Attack

How should we further develop the Ticket-Tape interpretation?

We adopt the following plan of attack to address the short-comings in our definitions:

1. Develop the interpretation/formalism for "stable" systems. (The next section of this paper will restrict its attention to "stable" systems).
2. Introduce a clock.
3. Remove the restrictions regarding stable systems.

Stable systems are characterised by:

- Repeated measurements by the same device produce the same result even over "long" periods of time.
- Transition probabilities do not change with time. Transition probabilities are calculated from repeated measurements using the standard formula $P(A = a|B = b) = \frac{N(A=a|B=b)}{N_{Total}}$.

7.2 Particles

It is observed (and not outlawed by the definition of repeatability) that the measurement of one system property X can randomise the value of another system property Y.

It is this property that allows use to define what we mean by a "particle" or simple quantum system. E.g. Suppose we have measuring devices A, B, C, D. Measurements of A "wipes out" the value of measurements of B, A wipes out the measurements of C but measurements of D are always unaffected. A, B, C measure properties of the same particle/system. D measures properties of another particle/system.

It turns out that repeatability is central to the definition of a particle.

Suppose that A, B, C are Mach devices and have the following relationships.

- A measurement of B produces random effect on further measurements of A. e.g. $A = a, A = a, B = b, A = z$.
- Measurements of A do not effect repeated measurements of C.e.g. $A = a, A = a, C = c, A = a$.

Devices A and B measure properties of the same particle, and device C measures properties of another separate particle.

8 The Calibration Problem

It seems that a measurement stream can only be "understood" if it contains a random component.

where the probabilities$P(X|Y)$ are derived via Bayesian analysis of a large number of previous measurements.

Fortunately early 20th Century scientists have done most of the work for us. We assert that it is possible to assign meaning to a measurement stream provided that

1. The measuring devices associated with the measurement stream are conjugate.
2. The state transition probabilities are symmetric with respect to the start and end states. i.e $P(A|B) = P(B|A)$

A physical system, measurement devices and observers that satisfy the condition above is collectively called a quantum system.

9 The Formalism of Quantum Mechanics

9.1 Feynman's Rules

We follow the argument put forward by Ariel Caticha [2]. The operators $\wedge$ and $\vee$ obey the following relations

$$a \wedge b = b \wedge a$$

$$a \vee b = b \vee a$$

$$(a \vee b) \vee c = a \vee (b \vee c)$$

$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

for histories a, b, c. In general, there is an structure preserving mapping from the space of all histories H to a field F with $\Psi : H \to F$ defined by

$$\Psi(a \vee b) = \Psi(a) + \Psi(b)$$

$$\Psi(a \wedge b) = \Psi(a).\Psi(b)$$

9.2 Quantum State

A state is any representation of the system such that there is a rule for the calculation of the probability of outcomes of future measurements from the representation.

Suppose H is the known history of a system, then minimal sub-history Ψ of H such that $P(x|\Psi) = P(x|H)$ for all x is one possible system state representation. No predictive power is lost by using this representation rather than the complete history.

The test of a good theory is whether it can make accurate predictions. In the case of Quantum Mechanics, the obvious question is: How much history is necessary before an observer can make accurate predictions? The answer cannot be that the observer must know the entire history of a system since the beginning of time since that information will never be available. So what are the alternatives? One possibility is to only count the last N measurements for some N, perhaps giving the more recent measurements more "weight". But how do we chose N? How do we assign "weights" to newer and older measurements?

So what should a suitable representation look like? What do we know about state?

1. Feynman's Rules imply that state has an algebraic structure generated by the aggregation and concatenation of histories (and therefore states). In particular,
$$P(\Psi \vee \Theta|\Phi) \equiv P(\Psi + \Theta|\Phi) \tag{1}$$
and
$$P(\Phi|\Psi \vee \Theta) \equiv P(\Phi|\Psi + \Theta) \tag{2}$$

2. Probability is not linear in state. If this was so then $P(\Psi + \varphi|\Psi) = P(\Psi|\Psi) + P(\varphi|\Psi) = 1 + P(\varphi|\Psi) > 1$ which cannot be correct.

3. If Ψ_i are the states associated with distinct outcomes from a single measurement device, then
$$P(\Psi_i|\Psi_k) = \delta_{ik} \tag{3}$$

Equation (1) implies

$$P(\lambda\Psi|\Phi) = P(\Psi|\Phi) \tag{4}$$

where $\lambda\Phi = \Phi + \ldots + \Phi$ (λ times) for all λ = 1,2,3,4… It appears that in the quantum world "size doesn't matter".

The minimal path state representation is incomplete; it can express all possible experiment outcomes, both past and future, but does not incorporate the probability information present in the probability transition matrix.

We make the leap that the state space is a vector space. Feynman's rules can be used to satisfy many of the requirements, except that the defintion of $\lambda\Psi$ is not well defined,and consequentially neither is $-\lambda\Psi$. We proceed on the basis that the meaning of $\lambda\Psi$ will "come out in the wash".

Equation (4) implies the probability is a function of $\frac{\Psi}{|\Psi|}$ rather than just Ψ. I.e.

$$P(\Psi) = g\left(\frac{\Psi}{|\Psi|}\right)$$

for some positive function $g : R \to [0, 1]$ with $g(0) = 0$.

Equation (3) looks like an inner product, except that probability is always positive. It is then natural to speculate that

$$P(\Psi_i|\Psi_k) = f\left(\left\langle \frac{\Psi_i}{|\Psi_i|} | \frac{\Psi_k}{|\Psi_k|} \right\rangle\right) \tag{5}$$

where $< .|. >$ is an inner product and $f : R \to [0, 1]$ with $f(0) = 0$ and $f(x) > 0$ for $x \neq 0$.

Putting $f(x) = x^2$ yields the "correct" QM formalization.

It is well known that each device has an operator representation A associated with it such that $< A >=< \Psi|A|\Psi >$ for the normalized state Ψ.

9.3 Some Examples - Algebraic Extensions

A "wipes out" any knowledge of B, so any state vector, which encompasses our knowledge of the system, can be expressed only in terms of the probability of outcomes of measurements of A. I.e. the space is spanned by $\{\Psi_{A,\uparrow}, \Psi_{A,\downarrow}\}$ where $\Psi_{A,\uparrow}$ is the state $A =\uparrow$.

$$\Psi_{B,\uparrow} = a.\Psi_{A,\uparrow} + b.\Psi_{A,\downarrow}$$

$$\Psi_{B,\downarrow} = c.\Psi_{A,\uparrow} + d.\Psi_{A,\downarrow}$$

for some a, b, c, d.

Noting that the probability of specific outcomes... Solving yields (non-unique)

$$\Psi_{A,\uparrow} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \Psi_{A,\downarrow} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Psi_{B,\uparrow} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \Psi_{B,\downarrow} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

which leads to an obvious geometric interpretation shown in figure (**??**).

If a 3rd 2-valued conjugate device C is introduced, the relevant equations admit no real-valued solutions. The Principle of Relativity (no preferred frame of reference) however suggests that all devices and states should be treated equally; a particular choice of basis should not preclude a solution. Solutions do exist if the "coordinate" domain is expanded to included complex numbers (a standard practice in mathematics over the centuries). The resulting solution looks like:

$$\Psi_{C,\uparrow} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ i \end{bmatrix}, \Psi_{C,\downarrow} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ -i \end{bmatrix}$$

The measurement operators are found to be

$$L_x = \frac{1}{2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, L_y = \frac{1}{2} \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}, L_z = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

If we add a 4th 2-valued quantity conjugate, then re-solving to choose symmetric solutions, yields

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ \pm i \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ \pm j \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ \pm k \end{bmatrix}$$

where i, j, k are quaternions. The corresponding operators are

$$K_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, K_1 = \begin{bmatrix} 0 & -i\sigma_x \\ i\sigma_x & 0 \end{bmatrix}, K_2 = \begin{bmatrix} 0 & -i\sigma_y \\ i\sigma_y & 0 \end{bmatrix}, K_3 = \begin{bmatrix} 0 & -i\sigma_z \\ i\sigma_z & 0 \end{bmatrix}$$

which are recognizable as variants of the Dirac's matrices.

10 The Principle of Scale

How do we know what we are measuring? The commutator relationships between measurement operators define "what we are measuring". The proposition has several advantages:

1. The algebraic relationships between operators can be extracted (with some caveats) experimentally from a measurement stream, even though this may be very computationally expensive.
2. Commutators can define a "scale" and provide a natural mechanism for the introduction of constants.

$$[X_i, X_j] = c_{ijk} X_k$$

If the devices,X_i, are rescaled $X_i \rightarrow f_i(X_i)$ then, depending on the algebra, the re-scaling may be a detectable as a change in the measured commutator relations.

$$[f_i(X_i), f_j(X_j)] = c_{ijk} f_k(X_k)$$

A set of devices $X_i, \ldots X_n$ can be assigned meaning only if any rescaling (recalibration) of the devices is detectable. The commuatator relations provide a method of ensuring different separated measuring devices use the same units.

10.1 Example 1: Angular Momentum

Three devices are related by the commutator relationship

$$[L_x, L_y] = i\hbar L_z$$

$$[L_y, L_z] = i\hbar L_x$$

$$[L_z, L_x] = i\hbar L_y$$

Rescale $L_x \rightarrow L_{x'} = \lambda L_x$ with $\lambda \neq 0, \lambda \neq 1$ then

$$[L_{x'}, L_y] = i\lambda\hbar L_z$$

$$[L_y, L_z] = i\left(\frac{1}{\lambda}\right)\hbar L_z$$

$$[L_z, L_{x'}] = i\lambda\hbar L_y$$

The rescaling of L_x can be partially hidden by the simultaneous rescaling L_y or L_z. If $L_z \rightarrow L_{z'} = \lambda L_z$., then

$$[Lx', Ly] = i\hbar L_z$$

$$[L_y, L_{z'}] = i\hbar L_x$$

$$[L_{z'}, L_{x'}] = i\lambda 2\hbar L_y$$

However no further scaling $Ly \to Ly' = \lambda Ly$, with $\lambda \neq 0, \lambda \neq 1$ (below) can disguise the original change of scale.

$$[L_{x'}, L_{y'}] = i\lambda\hbar L_z, [L_{y'}, L_{z'}] = i\lambda\hbar L_x, [L_{z'}, L_{x'}] = i\lambda 2\hbar L_{y'}$$

Conclusion: A set of Mach devices described by angular momentum operators L_x, L_y, L_z satisfies the requirements of the Principle of Scale and can therefore be regarded as producing meaningful measurements.

10.2 Example 2 : Position and Momentum

Two devices are related by commutator relationship $[x, p] = \hbar$. Rescale $x \to x' = \lambda x$ (change in the choice of units), then $[x', p] = [\lambda x, p] = \lambda\hbar$. The original scaling however can be hidden by rescaling p. I.e. $p \to \left(\frac{1}{\lambda}\right) p$.

Conclusion: A set of Mach devices described by angular momentum operators x, y, z, p_y, p_y, p_y do not satisfies the requirements of the Principle of Scale and so the set of operators cannot be regarded as producing meaningful measurements.

10.3 Example 3: Position, Momentum and Angular Momentum

There are nine devices in all. The non-zero commutator relations are shown below.

$$[L_x, L_y] = i\hbar L_z, [L_y, L_z] = i\hbar, L_x[L_z, L_x] = i\hbar$$

$$[x, p_x] = \hbar, [y, p_y] = \hbar, [z, p_z] = \hbar$$

$$[x, p_x] = \hbar, [y, p_y] = \hbar, [z, p_z] = \hbar$$

$$[L_x, p_y] = i\hbar p_z [L_y, p_z] = i\hbar p_x [L_z, p_x] = i\hbar p_y$$

Any rescaling of the angular momentum devices is immediately detectable. The position and momentum operators can be undetectably rescaled in any direction provided that if $x \to x' = \lambda x$, then $p_x \to \left(\frac{1}{\lambda}\right) p_x$. However a simple rescaling in 1 dimension can be detected. The angular momentum commutator relationships provide a mechanism for comparing the scaling of position and momentum devices in different "orientations". For example, rescale $x \to x' = \lambda x$, then

$$[L_z, x'] = i\hbar y$$

becomes

$$[L_z, \lambda x] = i\lambda\hbar y$$

which is detectable. If however all three position operators are rescaled ($x \to x' = \lambda x$, $y \to y' = \lambda y$ and $z \to z' = \lambda z$) then the rescaling is not detectable.

Conclusion: The set of Mach devices described by angular momentum, position and momentum operators do not satisfy the requirement of the Principle of State and position and momentum operators therefore cannot be regarded as producing meaningful result.

The result is not a surprise. Einstein Theory of Relativity makes it clear there is a missing component: the speed of light.

11 Time

Naive observers, equipped only with Mach devices, do not have access to a clock. How would such an observer measure time?

11.1 Pauli's Theorem

Pauli's Theorem famously states that if the Hamiltonian $\boldsymbol{H}$ is bounded from below, then there is no (time) operator T which obeys the expected commutator relation $[\boldsymbol{H}, T] = i\hbar$, and has unbounded eigenvalues.

Pauli's Theorem suggests that there is no such thing as a "forever" time operator in Quantum Mechanics. We accept this at face value.

11.2 Different concepts of Time

At least 3 different 'roles' for time have been identified by various writers (names vary). E.g.[5].

- External Time - The time is supplied from an uncoupled external source and appears as a 'parameter' in the equations of motion. If $H = (m_0, m_1, m_2, ..., m_n)$ is a history, then an increasing numeric time value is assigned to each measurement so that the history becomes $H = ((m_0, t_0), (m_1, t_1), (m_2, t_2), ..., (m_n, t_n))$ The enhanced elements of the history are called events. As the time values have an external source, there is no quantum uncertainty.
- Clock Time - a specific observable of the system under investigation may "mirror the passing of time". The evolution of that quantity (e.g. the changing position of a second hand on a clock) is used as a proxy for the passage of time.
- Intrinsic Time - Time is a measure of the "aging" of the Universe.

11.3 Do fundamental particles have wrinkles?

A muon decays with a mean lifetime of 2.2 µs. What is different between a newly created muon and the same particle 2.2 µs later? What changes within the particle as it ages? Do sub-atomic particles get wrinkles?

The decay follows a Poisson distribution which means that the probability of decay dp is directly proportional to the infinitesimal amount of time dt passed, and not dependent on the time itself.

$$dp = \lambda dt$$

How is the muon aware of amount of time dt passed?

Electrons, photons and gluons do not decay. In that sense, they do not seem to experience time at all.

Conjecture: Perhaps elementary particles do not experience time - they do not decay, and the majority of the particles of the standard model are not elementary, rather they are compound quantum systems like protons. If so, the concept of intrinsic time can be dispensed with, and the lack of a Time operator is not disturbing since there is no intrinsic time to measure. Clocks are used to assign the time (an numerically increasing real number) to measurements in the tickertape.

Irrespective of the any concept of the aging of fundamental particles, we shall develop Quantum Mechanics without any concept of Intrinsics Time, and instead focus entirely on clocks and external time.

12 The Transition to Dynamic Probabilities

Our original treatment of QM was somewhat artifical. It assumed stable systems for which the probability transitions from one state to another did not change. Once we have a clock, that can change.

The definition of "repeatable" can be expanded so that measurements need only be repeatable "in a small period of time dt". The Hamiltonian $\boldsymbol{H}$ can then be introduced to describe the evolution of quantum systems with dynamic transition probabilities. Let

$$\boldsymbol{H}\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

then

$$\Psi(t + \Delta t) = [1 - \frac{i}{\hbar}H.\Delta t]\Psi(t)$$

13 Clocks

Clocks have the following characteristics:

1. Clocks don't affect the one another. If one clock runs fast or slows down, it does not affect any other clock.
2. Clock time flows one way; it never flows backwards.
3. Clocks should not "speed up" or "slow down". Time is, by definition, homogenous.

An isolated clock is not of much use since it is not possible to determine whether it is running true by comparing it to other clocks.

Multiple clocks need to run at the same rate, or at least in concert, to be able make time-senstive predictions about systems which are potentially separated from each other across time and space.

Each system in a set of $n > 1$ quantum systems $\{C_i\}$with observable T_i is a clock if

1. Each observable T_i is independent of each other observable T_j.
2. Each observable T_i is strictly monotonic increasing. i.e. If separate measurements t_1 and t_2 are the result of measurements of T_i and t_2 is made "after" t_1, then $t_1 < t_2$.
3. Each pair of clocks C_i and C_j are in concert. I.e. the following applies: $T_i = \kappa.(T_j + \Delta t)$ where Δt is the difference in clock time assigned to the same event in the tickertape, and κ is an arbitrary constant.

13.1 An Ensemble Clocks (Quantum Egg-Timers) - An Example

The diagram below shows a ensemble quantum clock frequently described in the literature. (Type "ensemble quantum clock" into Google to see this) The device consists of a large number of subsystems ("particles") that are initially prepared so that they are in identical states, denoted $\uparrow$. The "particles" spontaneously change state to a second state, denoted $\downarrow$. They cannot revert to their previous state or the clock only counts the initial transition. The transitions are not under the control of an observer, except perhaps that the observer may be able to switch the whole process off or on. The clock *may* also "reverse" so that the particles move back to the original state $\uparrow$, and the whole cycle start again (Flipping the Egg-Timer).

A statistical estimate of the time past since the assembly of particles was prepared ($t = 0$) and the clock started can be made by counting the number of particles that have changed state ($t > 0$). A clock can be designed with arbitrarily high confidence by increasing the number of particles in the ensemble.

If $E_t(\boldsymbol{N})$ is the expected proportion of "particles" in the $\uparrow$ state at time t, then

$$E_t(\boldsymbol{N}) = < \boldsymbol{N} >_t = e^{-\lambda t}$$

Solving for the time T:

$$T = \frac{-ln(N)}{\lambda}$$

Flipping the Egg-Timer also provides us with ability to accumulate statistics and check the operation of the clock. The individual particles that make up the clock are independent of each other. I.e. $(P(AB) = P(A)P(B)$ where A and

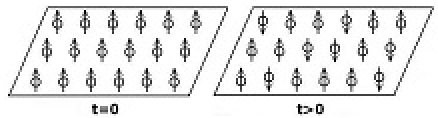

Figure 6: A Ensemble Clock

B represent specific particle state transitions). Each particle should be equally likely to make a transition "at any time".

The opertor t defined above for an esemble clock satisfies the requirement for a clock time since:

1. Each subsystem transition ("decay" from $\uparrow$ to $\downarrow$) is statistically independent from any other.
2. The proportion of decayed particles always increases, (T is monotonic)
3. The clock actually consists of multiple sub-clocks. A second clock can be constructed from the main clock simply by selecting a subset of the particles in the main clock; the proportion of particles in the decayed state will be the same in both clocks. Both clocks will be synchronised.

Both clocks are of the same construction and $\kappa = 1$.

14 Comments

The Kantian world view leads to a calculus that is recognisably Quantum Mechanics. Paradoxes such as the Measurement Problem simply do not exist; they are the artifical creation of moving away from Copenhagen's Kantian roots.

The current situation with interpretations is not good. Von Neumann has managed to spread his "waveform" confusion to several generations of Physicists and spawn a small industry producing quantum interpretations. The Copenhagen Interpretation meanwhile has become so polluted as a brand that it is now virtually impossible to determine what it is without inordinate research. This confusion is unlikely to get better in the near future as unfortunately there is a distaste for or lack of knowledge of philosophy amonst practical engineers who would prefer to crunch numbers and build things. Many feel philosophy and interpretations add nothing to the Physics. String theoriest, in particular, have a habit of making glib comments such as "Everything is a wave". Of course, waveforms feel comfortable to those who like their quantum mechanics with a dash of the macroscopic familiar, but that's not the real world. In fact, waveforms cannot be fundamental since the quantum formalism does not even require space-time, a prerequisite for any waveform.

Bohr got it essentially right.

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