

# Motion state modification of Coulomb's law and dynamic gravitation

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## Abstract

As we know, Coulomb's law describes the interaction between static charges. In this paper, the modified formula of Coulomb's law in the state of charge motion is given. Based on this formula, Ampere's law and Lorentz's law of force are derived by pure mathematics. According to the similarity between the formula of universal gravitation and Coulomb's law, the correction of the formula of universal gravitation under the state of motion is assumed boldly, and some inferences are made on the motion law of celestial bodies.

Key words: Coulomb's law, moving charge, electromagnetism, dynamic gravitation

## 1. The expression and explanation of the interaction force between moving charges

We believe that the force between charges is not Action at a distance, so there must be a force transmitter as the medium of interaction, which is generally believed to be achieved by exchanging force photons<sup>[1]</sup>, which move at the speed of light.

We know that Coulomb's law elucidates the interaction force between stationary charges, and when facing moving charges, Coulomb's law clearly requires some modifications. I believe that the following three revisions are necessary:

1. Due to the fact that charges move and the movement of force transmitting photons takes time, when charges receive force transmitting photons, the relative positions between charges have changed, so the direction of force application needs to be corrected,
2. The intensity of the received force transfer photon should be corrected according to the Doppler effect,
3. When a charge emits a force transmitting photon, the intensity of the reaction it receives should also be corrected according to the Doppler effect.

By combining these three correction terms with Coulomb's law, a formula is generated to describe the interaction force between moving charges.

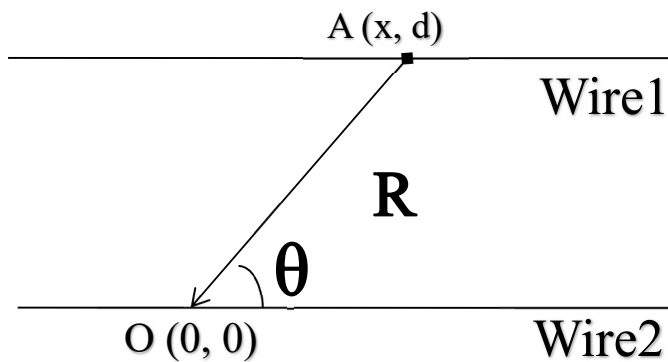
If charge  $Q_1$  and  $Q_2$ , move in a straight line or approximately in a straight line, and the velocity of the motion are  $\vec{V}_1$  and  $\vec{V}_2$ , then the force exerted on the charge  $Q_2$  can be described by the following expression:

$$\vec{F}_2 = kQ_1Q_2 \frac{\hat{R} - \vec{V}_1/c + \vec{V}_2/c}{(R - \vec{R} \cdot \vec{V}_1/c + \vec{R} \cdot \vec{V}_2/c)(R + \vec{R} \cdot \vec{V}_2/c)} \quad (1-1)$$

Where,  $\vec{R}$ : Vector from charge  $Q_1$  to  $Q_2$        $\hat{R}$ : Unit vector of  $\vec{R}$   
 $k$ : Electrostatic constant       $c$ : Velocity of light

As you see:  $\vec{F}_2$  is not equal to  $-\vec{F}_1$ , this Closed system should be composed of  $Q_1$ ,  $Q_2$  and force transmitting photons. Since the transmission of the photons takes time, the force applied to  $Q_1$  and  $Q_2$  at any time does not constitute the relationship between force and reaction force.

## 2, Derivation of Ampere's law of force



As shown in the figure above, there are two infinitely long parallel wires with a distance of  $d$ . We want to calculate the stress condition in wire 2.

### 2.1, Expression of force acting from one point in wire 1 to one point in wire 2

Take any point in wire 2 as the coordinate origin  $O(0, 0)$ , there are positive and negative charge pairs  $Q_2$  at point O in wire2, the velocity of positive charge is 0 and that of negative charge is  $\vec{V}_2 = (V_2, 0)$ .

Take any point a in wire 1 and set the coordinate as A(x, d), there are positive and negative charge pairs at point A. the velocity of positive charge is 0 and that of negative charge is  $\vec{V}_1 = (V_1, 0)$ . The angle between OA and wire 2 is  $\theta$ .

**Note: the positive charge in Q1 must exist and the velocity must be zero. See 4.1 for the reason and explanation, cutting the magnetic line of force.**

So  $\vec{R} = (-x, -d)$ ,  $\hat{R} = (-x/R, -d/R)$ .

The force at O is composed of four parts:

a, The force of  $+Q_1$  on  $+Q_2$ :

$$\vec{f}_a = kQ_1Q_2 \frac{\hat{R}}{R^2} \quad (2-1)$$

b, The force of  $-Q_1$  on  $+Q_2$ :

$$\vec{f}_b = -kQ_1Q_2 \frac{\hat{R} - \vec{V}_1/c}{R(R - \vec{R} \cdot \vec{V}_1/c)} \quad (2-2)$$

c, The force of  $+Q_1$  on  $-Q_2$ :

$$\vec{f}_c = -kQ_1Q_2 \frac{\hat{R} + \vec{V}_2/c}{(R + \vec{R} \cdot \vec{V}_2/c)^2} \quad (2-3)$$

d, The force of  $-Q_1$  on  $-Q_2$ :

$$\vec{f}_d = kQ_1Q_2 \frac{\hat{R} - \vec{V}_1/c + \vec{V}_2/c}{(R - \vec{R} \cdot \vec{V}_1/c + \vec{R} \cdot \vec{V}_2/c)(R + \vec{R} \cdot \vec{V}_2/c)} \quad (2-4)$$

In order to make this paper concise and to the point, the derivation of x component is not listed here. From the symmetry, we can know that the resultant force of x component after the integration of wire 1 is zero.

The Y component of each part is as follows:

$$f_{ay} = -kQ_1Q_2 \frac{d}{R^3} \quad (2-5)$$

$$f_{by} = kQ_1Q_2 \frac{d/R}{R(R + V_1x/c)} \quad \text{Taylor series expansion}$$

$$\approx kQ_1Q_2 \frac{d}{R^3} \left[ 1 - \frac{V_1x}{Rc} + \left( \frac{V_1x}{Rc} \right)^2 \right] \quad (2-6)$$

$$f_{cy} = kQ_1Q_2 \frac{d/R}{(R - V_2x/c)^2} \quad \text{Taylor series expansion}$$

$$\approx kQ_1Q_2 \frac{d}{R^3} \left[ 1 + 2 \frac{V_2x}{Rc} + 3 \left( \frac{V_2x}{Rc} \right)^2 \right] \quad (2-7)$$

$$\begin{aligned} f_{dy} &= -kQ_1Q_2 \frac{d/R}{(R+V_1x/c-V_2x/c)(R-V_2x/c)} \\ &\approx -kQ_1Q_2 \frac{d}{R^3} \left[ 1 - \frac{V_1x}{Rc} + \left( \frac{V_1x}{Rc} \right)^2 + 2 \frac{V_2x}{Rc} + 3 \left( \frac{V_2x}{Rc} \right)^2 - 3 \left( \frac{V_2x}{Rc} \right) \left( \frac{V_1x}{Rc} \right) \right] \end{aligned} \quad (2-8)$$

Resultant force of Y component:

$$\begin{aligned} f_y &= f_{ay} + f_{by} + f_{cy} + f_{dy} \\ &= 3kQ_1Q_2 \frac{d}{R^3} \left( \frac{V_2x}{Rc} \right) \left( \frac{V_1x}{Rc} \right) \\ &= \frac{3kQ_1V_1Q_2V_2}{d^2c^2} \text{Sin}^3\theta \cdot \text{Cos}^2\theta \end{aligned} \quad (2-9)$$

**2.2, Integral along the x-axis, the y-direction force expression of wire 1 to O point can be obtained:**

Let the line charge density of wire 1 be  $q_1$ , we get  $Q_1 = q_1 dx$

$$\begin{aligned} F_y &= \int_{-\infty}^{\infty} \frac{3kq_1V_1Q_2V_2}{d^2c^2} \text{Sin}^3\theta \cdot \text{Cos}^2\theta dx \\ &= \frac{3kq_1V_1Q_2V_2}{d^2c^2} \int_{\pi}^0 \text{Sin}^3\theta \cdot \text{Cos}^2\theta \cdot \frac{-d}{\text{Sin}^2\theta} d\theta \\ &= \frac{3kq_1V_1Q_2V_2}{dc^2} \int_0^{\pi} \text{Sin}\theta \cdot \text{Cos}^2\theta \cdot d\theta \\ &= \frac{2kq_1V_1Q_2V_2}{dc^2} \end{aligned} \quad (2-10)$$

**2.3, Find the force acting on the wire 2 in length L**

Let the line charge density of wire 2 be  $q_2$ , we get  $Q_2 = q_2L$ ,

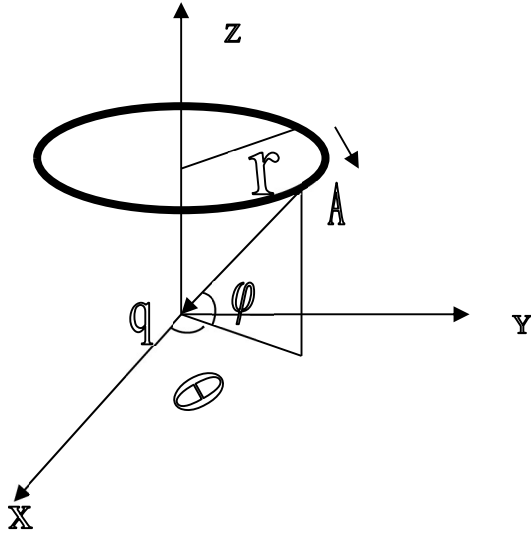
$$F = \frac{2kq_1V_1q_2V_2}{dc^2} L \quad (2-11)$$

Insert:  $I_1 = q_1V_1$ ,  $I_2 = q_2V_2$ ,  $u_0 = \frac{4\pi k}{c^2}$

$$F = \frac{u_0}{2\pi} I_1 I_2 L \quad (2-12)$$

This result is consistent with the definition of Ampere force.

### 3, Derivation of Lorentz law of force



As shown in the figure: the solenoid with radius  $R$  is arranged along the  $z$ -axis, and the charge  $q$  moves at the speed  $V_2$  along the  $x$ -axis at the origin. We need to calculate the force of the solenoid on the charge  $q$ .

3.1, Calculate the expression of the force of a point in the solenoid on the charge  $q$

Take any point  $A$  on the solenoid, and set its coordinates as  $(r\cos\theta, r\sin\theta, rtg\phi)$ , there are positive and negative charges  $Q$  at point  $A$ , the velocity of positive charge is  $0$  and that of negative charge is  $\vec{V}_1 = (-V_1\sin\theta, V_1\cos\theta, 0)$ .

**Note: the positive charge in  $Q$  must exist and the velocity must be zero. See 4.1 for the reason and explanation, cutting the magnetic line of force.**

$$\vec{R} = (-r\cos\theta, -r\sin\theta, -rtg\phi), \quad \vec{V}_2 = (V_2, 0, 0), \quad R = r/\cos\phi$$

$$\hat{R} = (-\cos\theta \cdot \cos\phi, -\sin\theta \cdot \cos\phi, -\sin\phi)$$

$$\vec{R} \cdot \vec{V}_1 = 0, \quad \vec{R} \cdot \vec{V}_2 = -V_2 r \cos\theta$$

The force on charge  $q$  consists of two parts:

a, The force of  $+Q$  on moving charge  $q$ :

$$\vec{f}_a = kqQ \frac{\hat{R} + \vec{V}_2/c}{(R - V_2 r \cos\theta/c)^2} \quad (3-1a)$$

b, The force of -Q on moving charge q:

$$\vec{f}_b = -kqQ \frac{\hat{R} - \vec{V}_1/c + \vec{V}_2/c}{(R - V_2 r \cos\theta/c)^2} \quad (3-1b)$$

$$\vec{f} = \vec{f}_a + \vec{f}_b = kqQ \frac{\vec{V}_1/c}{(R - V_2 r \cos\theta/c)^2} \quad (3-2)$$

x component:

$$f_x = kqQ \frac{-V_1 \sin\theta/c}{(r/\cos\varphi - V_2 r \cos\theta/c)^2} \quad (3-3a)$$

y component:

$$f_y = kqQ \frac{V_1 \cos\theta/c}{(r/\cos\varphi - V_2 r \cos\theta/c)^2} \quad (3-3b)$$

z component:

$$f_z = 0$$

3.2, The expression of the force of a single coil solenoid on the charge q is obtained by circle integration

Let the line density of charge in the solenoid be  $q_1$ , then  $Q = q_1 r d\theta$  .

For x component integration:

$$F_x = \int_0^{2\pi} kq q_1 \frac{-V_1 \sin\theta/c}{(r/\cos\varphi - V_2 r \cos\theta/c)^2} r d\theta = 0 \quad (3-4)$$

For y component integration:

$$\begin{aligned} F_y &= \int_0^{2\pi} kq q_1 \frac{V_1 \cos\theta/c}{(r/\cos\varphi - V_2 r \cos\theta/c)^2} r d\theta \\ &= \frac{kq q_1 V_1 \cos^2\varphi}{rc} \int_0^{2\pi} \frac{\cos\theta}{(1 - V_2 \cos\theta \cdot \cos\varphi/c)^2} d\theta \\ &\approx \frac{kq q_1 V_1 \cos^2\varphi}{rc} \int_0^{2\pi} \cos\theta + 2V_2 \cos^2\theta \cdot \cos\varphi/c d\theta \\ &= \frac{2\pi kq q_1 V_1 V_2 \cos^3\varphi}{rc^2} \end{aligned} \quad (3-5)$$

Insert  $I_1 = q_1 V_1$ ,  $u_0 = \frac{4\pi k}{c^2}$ , let  $V = V_2$  the force of one single coil solenoid on the moving charge q is obtained:

$$F_y = \frac{u_0 I_1}{2r} q V \cos^3\varphi \quad (3-6)$$

### 3.3, Integrating the z-axis

Let the number of turns per unit solenoid be  $n$ ,

$$\begin{aligned} F &= \int_{-\infty}^{\infty} \frac{u_0 I_1}{2r} qV \cos^3 \varphi \cdot n dz \\ &= \int_{-\pi/2}^{\pi/2} \frac{u_0 I_1}{2r} qV \cos^3 \varphi \cdot n \cdot \frac{r}{\cos^2 \varphi} d\varphi \\ &= qV \frac{u_0 n I_1}{2} \sin \varphi \Big|_{-\pi/2}^{\pi/2} \end{aligned} \quad (3-7)$$

$$= qV u_0 n I_1 \quad (3-8)$$

As we know the magnetic induction in the center of an infinitely long solenoid:

$$B = u_0 n I_1,$$

Therefore, the result of equation (3-8) can be written as:

$$F = qVB \quad (3-9)$$

## 4, Confusing frame of reference

### 4.1, Cutting magnetic lines

We know that the electric charge must cut the magnetic line in the magnetic field to sense the effect of magnetic force. The magnetic line of force is an imaginary curve to study the magnetic field vividly, but it is not the real curve objectively existing in the magnetic field.

The main body of magnetic field can be a magnet or an electrified solenoid. If we go into microcosmic research on the behavior of cutting magnetic line of force, it is the movement of electric charge relative to the magnet or solenoid. At this time, we don't need to care about the motion mode of the main body of the magnetic field, whether it is acceleration or inertial motion, as long as the movement of the charge relative to the magnetic field body does not change, the magnetic force induced by the magnetic field does not change.

Of course, the motion of electric charge relative to the main body of magnetic field has different magnetic force due to different relative motion direction.

If we go further into the micro level, the so-called motion of the main body of the magnetic field is the motion of the atoms in the magnet or solenoid, or the movement of the nucleus in it, that is, the motion of the positive charge. Then, in the process of studying magnetic force, based on the known experience of "magnetic line cutting", the positive charge of the main body of the magnetic field will be regarded as the reference frame of the whole motion system, that is to say, the velocity of the positive

charge in the main body of the magnetic field must be assumed to be zero.

## 4.2, Running, on two relatively static electronic sides

So, when there are only two relatively stationary electrons in a system, will the force between them conform to Coulomb's law? Are they still static charges if I run past them, or if my thoughts run around them irregularly? If it is a moving charge and there is no positive charge, how to determine the reference frame of the whole motion system?

This is an interesting question, and it is on this issue that I started to study these issues.

There is no doubt that logically, no matter how I think, no matter what angle I look at and how I look at these two electrons, the force between them is certain.

According to the known physical research, the force between the charges is not over distance interaction. We assume that the transmission of the force depends on a kind of photon (or electromagnetic wave). Let's go back to the scene with a positive charge, that is, a scene with a complete array of atoms, in which photons (or electromagnetic waves) are transported. When the motion state of the medium changes, the movement of the cutting magnetic line will change, and the interaction force between the two electrons will change. Then we can infer that the propagation path and velocity of photons have changed, that is to say, **the propagation velocity vector of photons is affected by the medium motion vector**, which is another problem worthy of study.

Here we take such a judgment as a conclusion: when calculating the interaction force between two moving charges, **the medium in which the force transfer photon is located is taken as the reference frame**.

But what if it's in a vacuum? Do you need an absolute coordinate system? This is a problem. Fortunately, in the general case of magnetic field research, we can avoid it.

## 5, Conjecture of universal gravitation in motion

### 5.1, A bold assumption of the law of universal gravitation

All along, we have noticed that Coulomb's law and the law of universal gravitation have the same structural form, and try to connect them.

In this paper, I have given the expression of Coulomb's law in motion. We may as well make a deduction that the law of universal gravitation should also have a corresponding description and revision in the state of motion.

If two objects with mass  $m_1$  and  $m_2$ , move in a straight line or approximately in a straight line, and the velocity of the motion are  $\vec{V}_1$  and  $\vec{V}_2$ , then the gravitational force on object of mass  $m_2$  can be described by the following expression:



$$\vec{F}_2 = -\frac{Gm_1m_2}{R^2}(\hat{R} - A(x)\vec{V}_1 + B(x)\vec{V}_2) \quad (5-1)$$

Where  $A(x)$  and  $B(x)$  are unknown functions of  $\vec{V}_1, \vec{V}_2, R$ , mass, speed of light, etc.,  $\hat{R}$  is unit vector of  $m_1$  pointing to  $m_2$ , The negative sign in the front represents the direction of gravity.

After removing the static component, the moving state component of gravity can be expressed as follows:

$$\vec{F}_g = \frac{Gm_1m_2}{R^2}(A(x)\vec{V}_1 - B(x)\vec{V}_2) \quad (5-2)$$

From formula (5-2), we can see that the velocity of two objects under the action of gravity in the moving state has the effect of convergence, which I call "velocity convergence".

## 5.2, Characteristics of gravitation in motion

According to the "velocity convergence", we can infer some characteristics of gravitation in motion

- a. Due to the influence of the rotation of the planet, the planetary ring will be strictly parallel to the equator and move in the same direction with the equator. <sup>[2]</sup>
- b. All galaxies are flat, and are getting flatter.
- c. Due to the influence of the rotation of the parent star, the rotation of the planets close to the stars and the satellites close to the planets will be weakened, and the rotation modes are obviously different from those of other planets in the system. <sup>[3][4][5]</sup>

Of course, these characteristics are basically consistent with natural phenomena.

## 6, Postscript

Nature is not designed by human beings, so any laws and formulas that we put forward are just the summary and approximation of natural phenomena. The exact

formula of "rational" does not exist. Errors are inevitable.

I think the formula provided in this article is also an approximation when the velocity of the charge is moving well below the speed of light, and when the velocity of the charge is moving extremely fast, the three correction terms should not be accurate enough.

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