PROOF OF NOVEL LIMIT IDEA

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$$\lim_{x \to h} \frac{ab + d + c}{ab + d + c}$$

How to use- One may multiply the coefficients above as shown. One must always factor before use. This works for $\frac{0}{0}$ *and* $\frac{\infty}{\infty}$.

a- This is the coefficient of any function inside the fraction.
b- This is any coefficient inside the function
c- Constant

I wanted to find a postulate that was easier and faster than L'Hospital's rule for solving limits because it is often extremely challenging to solve these limits with existing ideas. This is a postulate that succeeds in making it easier to solve these limits. I wanted to prove this postulate so that it could become a recognized part of Mathematics as we understand it. This can not only be useful to anyone who needs to deal with limits (think electricians with AC current) but also useful to those who will prove new concepts in Calculus with this postulate. I am using the restrictions of L'Hospital's rule. This postulate should be used because it is easier and faster to use than L'Hospital's rule and Trigonometric identities. It also lends itself to proving other arguments very well. It is also faster than existing methods and mathematically sound, as I will prove. This proof has explanations at each step.

L'Hospital's rule: $\frac{\frac{d}{dx}(abx+dx+c)}{\frac{d}{dx}(abx+dx+c)}$

$$\lim_{x \to 0} \frac{\frac{d}{dx}(trig\ function)}{\frac{d}{dx}(trig\ function)}$$

$$\lim_{x \to 0} \frac{opposite\ of\ trig\ function\ or\ derivative\ of\ x}{opposite\ of\ trig\ function\ or\ derivative\ of\ x}$$

$$\lim_{x \to 0} \frac{ab + d + c}{ab + d + c}$$

Why is this necessary?

This idea is necessary because it allows the user to solve limits at greater ease than L'Hospital's rule does. It works for the same type of cases and is extremely simple. It takes much less time and has a smaller margin of error as it is much simpler. This means that one is less likely to make a careless error using this idea than using L'Hospital's idea.

How is this new?

This is new because it takes L'Hospital's a step further. Not only that, but it combines methods previously used on their own with this new idea. This is allowed because many methods do this, such any method of u-substitution and even the Order of Operations.

NOTE:

If the limit can be factored and it is something like (x)(x)(x), add up the x's. Also, if it is a trigonometric function approaching a multiple of pi, it will be negative on odd multiples and positive on even ones.

One may use the way in which L'Hospital's takes the derivatives of an expression for functions. For a function approaching 0 or a function like a trigonometric equation or a logarithmic function, it always ends up multiplying the coefficients inside the function and the outside to create the limit. I noticed this and thought that this alone may make an easier way to solve the limits.

Solving expressions approaching 0

Idea:

$$\lim_{x \to 0} \frac{\sin(5x)}{\tan(3x)}$$

$$\lim_{x \to 0} \frac{5x}{3x}$$

$$\frac{5}{3}$$

I used the coefficients in my idea to divide the 5 inside the sine by the 3 inside the tangent.

L'Hospitals

$$\lim_{x \to 0} \frac{\sin(5x)}{\tan(3x)}$$

$$\lim_{x \to 0} \frac{5\cos(5x)}{3sec^2(3x)}$$

$$\lim_{x \to 0} \frac{5\cos(0)}{3sec^2(0)}$$

$$\frac{5}{3}$$

One may use the way in which L'Hospital's takes the derivatives of an expression for functions. For a function approaching ∞, L'Hospital's always does the equivalent of dividing by the highest power in the denominator. I noticed this and thought that dividing my functions so that they do the same thing may make an easier way to solve the limits.

Solving limits at infinity

This is not an original idea, but is incorporated into my original idea to make my idea applicable to all limits and to form an alternative to L'Hospital's.

Idea

$$\lim_{x\to\infty} \frac{\sqrt{x}}{\text{x}}$$

$$\frac{1}{\infty}$$

$$0$$

I divided a number that was less than its denominator. In this case, we would put it as zero as the numerator is approaching infinity at a lesser rate than the denominator.

L'Hospital's

$$\lim_{x\to\infty} \frac{\sqrt{x}}{\text{x}}$$

$$\lim_{x\to\infty} \frac{1}{2x^{\frac{1}{2}}}$$

$$0$$

For a function approaching a number other than 0 and infinity, L'Hospital's always does the equivalent of factoring. I noticed this and thought that factoring first so that they do the same thing may make an easier way to solve the limits.

Solving simple equations

This is not an original idea, but is incorporated into my original idea to make my idea applicable to all limits and to form an alternative to L'Hospital's.

Idea

Idea:

$$\lim_{x\to 5} \frac{x^2-4x-5}{x^3-125}$$

$$\lim_{x\to 5} \frac{(x-5)(x+1)}{(x^2+5x+25)(x-5)}$$

$$\lim_{x\to 5} \frac{x+1}{x^2+5x+25}$$

$$\frac{6}{25+25+25}$$

$$\frac{6}{75}$$

I factored and got the answer. Don't overthink my idea!

L'Hospital's:

$$\lim_{x\to 5} \frac{x^2-4x-5}{x^3-125}$$

$$\lim_{x\to 5} \frac{2x-4}{3x^2}$$

$$\lim_{x\to 5} \frac{6}{75}$$

As you can see, the answers are the same as the conventional formulas for all of these. This idea is better than L'Hospitals in that it is faster and easier to perform. This same idea works for all equations of limits.