Angle dependencey of special and general theory of relativity and the relation to classical potential and kinetic energy in a gravitational potential

Azam Javid[*]

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At the beginning of the 20th century Albert Einstein developed the special and general theory of relativity and thus solved contradictions in classical physics. For example that the speed of light is finite[1]. The theory of relativity is widely accepted. In the following, the length contraction and time dilation of special relativity are defined angle dependent with the angle. Moreover the gravitational time dilation and the gravitational length contradiction of general relativity are rewritten with an angle. The angles are associated in a fixed circle with the kinetic and potential energy of an object in a gravitational potential.

I. SPECIAL THEORY OF RELATIVITY WITH ANGULAR DEPENDENCE

The special theory of relativity states that the duration of an event and the length in the direction of the motion of a moving object change relative to a resting object. From the perspective of the resting object with time t_2 between two events and length l_1, the time t_1 passes in the moving inertial system and the moving object has the length l_2. The velocity v of a moving inertial system is always smaller than the speed of light c: $0 \leq v < c$ [3].

The equations from special relativity are rewritten as angle-dependent.

$$t_2 = t_1 \cdot \sqrt{1 - \frac{v^2}{c^2}} \longleftrightarrow t_2 = t_1 \cos\theta \qquad (1)$$

$$l_2 = l_1 \cdot \sqrt{1 - \frac{v^2}{c^2}} \longleftrightarrow l_2 = l_1 \cos\theta \qquad (2)$$

[3]
Here $0 \leq \theta < \pi/2$ is defined as:

$$\cos\theta = \sqrt{1 - \frac{v^2}{c^2}} \qquad (3)$$

$$\sin\theta = \frac{v}{c} \qquad (4)$$

II. GENERAL THEORY OF RELATIVITY

The general theory of relativity describes gravitational time dilation and gravitational length contraction.

Time passes more slowly for an object the closer it is to the center of gravity of an object with high mass. t_4 is the time for an event that an observer far away from the center of gravity perceives. t_3 is the time that an object at distance R from the center of gravity needs for the event to occur for the far away observer [1, pp. 228].

* azam.javid@edu.azw.ac.at

A small radial distance difference is r_1 for an observer far away from the center of gravity. The observer far away perceives the same radial difference as r_2 when it is at distance R from the center of gravity [1, pp. 206].

M is the mass of the massive object, R is the distance of the object from the center of gravity, G is the gravitational constant and c is the speed of light. m is the mass of the object on the massive object [1, pp. 206].

The equations are rewritten. The escape velocity $v_e = 2GM/R$ is always smaller than c: $0 \leq v_e < c$

$$t_4 = t_3 \cdot \sqrt{1 - \frac{2GM}{Rc^2}} \longleftrightarrow t_4 = t_3 \cos\alpha \qquad (5)$$

$$r_2 = r_1 \cdot \sqrt{1 - \frac{2GM}{Rc^2}} \longleftrightarrow r_2 = r_1 \cos\alpha \qquad (6)$$

[1, pp. 228] [1, pp. 206]
Here $0 \leq \alpha < \pi/2$ is defined as

$$\cos\alpha = \sqrt{1 - \frac{2GM}{Rc^2}} \qquad (7)$$

$$\sin\alpha = \sqrt{\frac{2GM}{Rc^2}} \qquad (8)$$

III. ANGLE θ AND α

In figure 1 the angle θ is illustrated. It shows an object in the (x, y) plane that moves in the x direction. The object is shortened by the factor $\cos\theta$ in the direction of movement. If $\theta = 0$ then $v = 0$ and thus the object does not move and does not undergo a length contraction. If $\theta \to \pi/2$ then v goes to c and thus the length l_2 goes to 0 [3]. A new way of interpreting the angle is to see it as a new dimension that is made from the angle. In our 3 dimensional world that would be a 4th spacial dimension. In the perspective of the 4th dimension nothing changes for a moving object. From the view of the 3 dimensions we see the contracted object that observes a higher contradiction the higher the angle is. The same interpretation holds true for angle α. It is the smaller the smaller $2GM/R$ becomes. Therefore, in the case of $R \to \infty$ then α goes to 0.

IV. SPECIAL AND GENERAL THEORY OF RELATIVITY IN A FIXED CIRCLE AND THE MECHANICAL ENERGY IN A GRAVITATIONAL FIELD

As in figure 2 a triangle is drawn in a circle with the angles θ and α. If you leave chord $\overline{BC}$ fixed and select any other point on the circle, the inscribed angle theorem states that the angle at this any point is always θ. The point D is obtained by selecting the triangle so that the chord $\overline{BD}$ passes through the center of the circle. Thales's theorem states that there is a right angle at point C. Thus it is easy to recognize that $\overline{BC} = \sin\theta c$. If instead of BC the chord AC is left fixed, it can easily be shown that $\overline{AC} = \sin\alpha c$. The diameter of this circle must be the speed of light c to fulfill the definitions of the angles in equation 3 and 7.

Since with a fixed chord $\overline{AB}$ the angle at point C always remains the same according to the circumferential angle theorem, the sum of α and θ is also the same [2]. The positions of points A and B are determined by the total energy of the system. The points must lie in the upper half of the circle, since $0 \le \theta < \pi/2$ and $0 \le \alpha < \pi/2$. If A or B were located in the lower half of the circle and C were close to A or B, that would make θ or α go above $\pi/2$.

With the classical definition of the kinetic energy of an object in a gravitational field, the length of chord $\overline{AB}$ is obtained. If α goes towards zero, the potential energy goes towards zero and $E = E_{\text{kin}} = mv^2/2$ and also $v = \sqrt{2E/m}$ apply. Since the lenght of chord $\overline{BC} = v$ goes towards the length of chord $\overline{AB}$ it holds that $\overline{AB} = v = \sqrt{2E/m}$.

With the cosine theorem the total energy can be related to the potential (E_{pot}) and kinetic energy (E_{pot}) of the system [4].

$$\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2 - 2\overline{BC} \cdot \overline{AC}\cos\left(\pi - (\theta + \alpha)\right) \quad (9)$$

$$\sqrt{\frac{2E}{m}}^2 = (\sin\alpha c)^2 + (\sin\theta c)^2 - 2\sin\alpha\sin\theta c^2(-1)\cos(\theta + \alpha) \quad (10)$$

$$E = \frac{mc^2}{2}(\sin\alpha^2 + \sin\theta^2 + 2\sin\alpha\sin\theta\cos(\alpha + \theta)) \quad (11)$$

$$E = E_{\text{pot}} + E_{\text{kin}} + mc^2\sin\alpha\sin\theta\cos(\alpha + \theta) \quad (12)$$

The fixed circle in figure 2 illustrates the connection between the special and general theory of relativity and

the classical consideration of the energy $E_{\text{kin}} = mv^2/2$ and $E_{\text{pot}} = GMm/R$. If the point C is moved on the arc, E_{kin} and E_{pot} are changed. If the point C moves against the point A, θ becomes maximum and α goes towards 0. The length of chord $\overline{BC}$ goes towards $\sqrt{2E/m}$ and the length of chord $\overline{AC}$ goes towards 0. That is, E_{kin} becomes maximum and E_{pot} goes towards 0. If the point C goes against the point B, θ goes towards 0 and α becomes maximum. Therefore E_{kin} goes towards 0 and E_{pot} becomes maximum. $\overline{AB}$ remains fixed with the value $\sqrt{2E/m}$. If chord $\overline{AB}$ passes the center of the circle, equation 12 is simplified to the well-known form $E = E_{\text{kin}} + E_{\text{pot}}$.
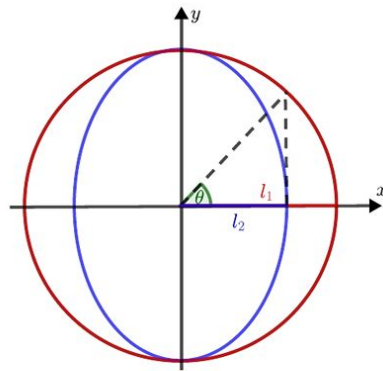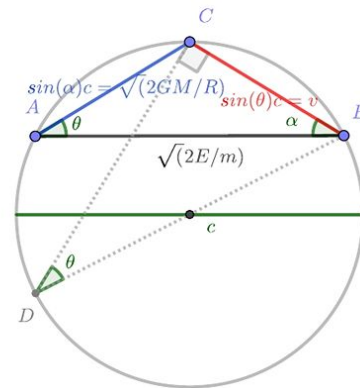

Figure 1. Length contraction visualized with θ.

Figure 2. α and θ in a fixed circle with diameter c.

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