A speedy new proof of the Riemann's hypothesis

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Abstract: Riemann's hypothesis ([1],[2],[3],[6]) , formulated in 1859, concerns the location of the zeros of Riemann's Zeta function. The history of the Riemann Hypothesis is well known. In 1859, the German mathematician B. Riemann presented a paper to the Berlin Academy of Mathematic. In that paper, he proposed that this function, called Riemann-zeta function takes values 0 on the complex plane when s=0.5+it . This hypothesis has great significance for the world of mathematics and physics.([4]) This solutions would lead to innumerable completions of theorems that rely upon its truth. Over a billion zeros of the function have been calculated by computers and shown that all are on this line s = 0.5+it. In this paper we show that Riemann's function (xi) ξ, involving the Riemann's (zeta) ζ function, is holomorphic and is expressed as an infinite polynom product in relation to their zeros and their conjugates.([5],[7]) By applying the functional equation of symmetry
$\xi(1-s) = \xi(s)$, we deduce a relation between each zero of the function ξ and its conjugate. We obtain the searched result: the real part of all zeros is equal to 1/2.

Riemann's Hypothesis is expressed as following:

All non-trivial zeros of the function $\zeta(s)$ are located on the complex line $\Re(s) = \frac{1}{2}$

Introduction - The Riemann's functional equation

The zeta function satisfies the functional equation was established by Riemann in 1859 .

For all complex numbers except 0 and 1

$$\zeta(s) = 2^s \pi^{s-1} sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s) \tag{1}$$

Riemann also found a symmetric version of the functional equation applying to the ξ function:

$$\xi(s) = \frac{1}{2}\pi^{-\frac{s}{2}}s(s-1)\Gamma\left(\frac{s}{2}\right)\zeta(s) = s(s-1)\int_1^{\infty}\left(u^{\frac{s}{2}-1} + u^{\frac{-s-1}{2}}\right)\psi(u)du + 1 \qquad (2)$$

With

$$\psi(u) = \sum_{n=1}^{\infty} e^{-\pi u n^2}$$

This function satisfies

$$\xi(1-s) = \xi(s)$$

Also,

ξ is an holomorphic function on $\mathbb{C}$ because of expression (2), then $\overline{\xi(s)} = \xi(\overline{s})$

If s_k is a zero of ξ, and $\overline{\xi(s_k)} = \xi(\overline{s_k})$ then $\overline{s_k}$ is a zero of ξ.

All holomorphic functions can be represented as an infinite product involving its zeroes [7]

$$\xi(s) = A\big(s(1-s)\big)\prod_k^{\infty}\left(1-\frac{s}{s_k}\right)\left(1-\frac{s}{\overline{s_k}}\right) = A\big(s(1-s)\big)\prod_k^{\infty}\left(1 - s\left(\frac{s_k+\overline{s_k}-s}{s_k\overline{s_k}}\right)\right)$$

$\xi(1-s) = \xi(s)$ then $\xi(1-s_k) = \xi(s_k) = 0$

A is an holomorphic function whose zeros are not s_k or $\overline{s_k}$

$$\prod_k^{\infty}\left(1-(1-s)\left(\frac{s_k+\overline{s_k}-(1-s)}{s_k\overline{s_k}}\right)\right) = \prod_k^{\infty}\left(1 - s\left(\frac{s_k+\overline{s_k}-s}{s_k\overline{s_k}}\right)\right)$$

As these are infinite products of polynomials, by recurrence we show that the equality of the product implies the equality of the polynomials two by two.

$$\forall\, k \in \mathbb{N}$$

$$\left(1-(1-s)\left(\frac{s_k+\overline{s_k}-(1-s)}{s_k\overline{s_k}}\right)\right) = \left(1 - s\left(\frac{s_k+\overline{s_k}-s}{s_k\overline{s_k}}\right)\right)$$

i.e

$$\left(1-\frac{s_k+\overline{s_k}-1}{s_k\overline{s_k}} - s\left(\frac{-s_k-\overline{s_k}+2-s}{s_k\overline{s_k}}\right)\right) = \left(1 - s\left(\frac{s_k+\overline{s_k}-s}{s_k\overline{s_k}}\right)\right)$$

This equality is true if and only if

$$s_k + \overline{s_k} - 1 = 0 \qquad (3)$$

i.e

$$\Re(s_k) = \frac{1}{2}$$

Conclusion

We have demonstrated:

that the holomorphic function $\xi(s)$ had the same zeros as the function $\zeta(s)$ which is an functional equation $\xi(s) = \frac{1}{2}\pi^{-\frac{s}{2}}s(s-1)\Gamma\left(\frac{s}{2}\right)\zeta(s)$

We used the Weierstrass's factorization theorem([5],[7]) of holomorphic functions for $\xi(s)$ involving its zeros and apply functional relationship of symmetry, $\xi(1-s) = \xi(s)$, to demonstrate all non-trivial zeros s_k of the function ζ have their real part equal to ½.([8])

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