A speedy new proof of the Riemann's hypothesis

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Abstract: Riemann's hypothesis ([1],[2],[3],[6]) , formulated in 1859, concerns the location of the zeros of Riemann's Zeta function. The history of the Riemann Hypothesis is well known. In 1859, the German mathematician B. Riemann presented a paper to the Berlin Academy of Mathematic. In that paper, he proposed that this function, called Riemann-zeta function takes values 0 on the complex plane when $s=0.5+it$. This hypothesis has great significance for the world of mathematics and physics.([4]) This solutions would lead to innumerable completions of theorems that rely upon its truth. Over a billion zeros of the function have been calculated by computers and shown that all are on this line $s = 0.5+it$. In this paper we show that Riemann's function $(\zeta)$, involving the Riemann’s (zeta) $\zeta$ function, is holomorphic and is expressed as an infinite polynomial product in relation to their zeros and their conjugates.([5],[7])

By applying the functional equation of symmetry $\zeta(1-s) = \zeta(s)$, we deduce a relation between each zero of the function $\zeta$ and its conjugate. We obtain the searched result: the real part of all zeros is equal to $1/2$.

Riemann's Hypothesis is expressed as following:

All non-trivial zeros of the function $\zeta(s)$ are located on the complex line $\Re(s) = \frac{1}{2}$

Introduction - The Riemann’s functional equation

The zeta function satisfies the functional equation was established by Riemann in 1859.

For all complex numbers except 0 and 1

$$\zeta(s) = 2^s\pi^{s-1}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)$$  \hspace{1cm} (1)

Riemann also found a symmetric version of the functional equation applying to the $\xi$ function:
\[ \xi(s) = \frac{1}{2} \pi^{-s/2} s(s-1) \Gamma \left( \frac{s}{2} \right) \zeta(s) = s(s-1) \int_{1}^{\infty} \left( u^{s-1} + u^{-s-1} \right) \psi(u) du + 1 \]

With

\[ \psi(u) = \sum_{n=1}^{\infty} e^{-\pi n^2 u} \]

This function satisfies

\[ \xi(1-s) = \xi(s) \]

Also,

\[ \xi \text{ is an holomorphic function on } \mathbb{C} \text{ because of expression (2), then } \overline{\xi(s)} = \xi(\overline{s}) \]

If \( s_k \) is a zero of \( \xi \), and \( \overline{\xi(s_k)} = \xi(\overline{s_k}) \) then \( \overline{s_k} \) is a zero of \( \xi \).

All holomorphic functions can be represented as an infinite product involving its zeroes [7]

\[ \xi(s) = A(s(1-s)) \prod_{k} \left( 1 - \frac{s}{s_k} \right) \left( 1 - \frac{s}{\overline{s_k}} \right) = A(s(1-s)) \prod_{k} \left( 1 - s \left( \frac{s_k + \overline{s_k} - s}{s_k \overline{s_k}} \right) \right) \]

\[ \xi(1-s) = \xi(s) \text{ then } \xi(1-s_k) = \xi(s_k) = 0 \]

\( A \) is an holomorphic function whose zeros are not \( s_k \) or \( \overline{s_k} \)

\[ \prod_{k} \left( 1 - (1-s) \left( \frac{s_k + \overline{s_k} - (1-s)}{s_k \overline{s_k}} \right) \right) = \prod_{k} \left( 1 - s \left( \frac{s_k + \overline{s_k} - s}{s_k \overline{s_k}} \right) \right) \]

As these are infinite products of polynomials, by recurrence we show that the equality of the product implies the equality of the polynomials two by two.

\[ \forall k \in \mathbb{N} \]

\[ \left( 1 - (1-s) \left( \frac{s_k + \overline{s_k} - (1-s)}{s_k \overline{s_k}} \right) \right) = \left( 1 - s \left( \frac{s_k + \overline{s_k} - s}{s_k \overline{s_k}} \right) \right) \]

i.e

\[ \left( 1 - \frac{s_k + \overline{s_k} - 1}{s_k \overline{s_k}} - s \left( \frac{-s_k - \overline{s_k} + 2 - s}{s_k \overline{s_k}} \right) \right) = \left( 1 - s \left( \frac{s_k + \overline{s_k} - s}{s_k \overline{s_k}} \right) \right) \]

This equality is true if and only if
\[ s_k + \overline{s_k} - 1 = 0 \quad (3) \]

i.e

\[
\Re(s_k) = \frac{1}{2}
\]

**Conclusion**

We have demonstrated:

that the holomorphic function \( \xi(s) \) had the same zeros as the function \( \zeta(s) \) which is an functional equation

\[
\xi(s) = \frac{1}{2} \pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) \zeta(s)
\]

We used the Weierstrass’s factorization theorem([5],[7]) of holomorphic functions for \( \xi(s) \) involving its zeros and apply functional relationship of symmetry, \( \xi(1-s) = \xi(s) \), to demonstrate all non-trivial zeros \( s_k \) of the function \( \zeta \) have their real part equal to \( \frac{1}{2} \).(8)

**References**