Abstract.
Any complex mathematical or physical problem can be solved easier if it is broken down into several less complex problems. A similar situation occurs when it is necessary to determine the meaning of big data or several meanings of individual data blocks. A mechanism that could combine objects of various kinds with one meaning would help solve a number of problems of sense logic and semantic analysis. In this article, we describe one of the mechanisms, sense series that allows many different and complex objects to determine the presence of a semantic connection and sense dependence between them.

1. Introduction
In traditional mathematics, functions with certain properties can be expanded into numerical or functional series. The purpose of such a decomposition, as a rule, is the transition from a complex function to a group of simple functions. In the case of the Sense Theory [1], the decomposition of a sense function into a sense series means the definition of a group, finite or infinite, properties or objects that are semantically related to a given function. This approach allows to quickly identify all possible semantic relationships between trillions of objects of different nature.
2. Problem

Traditional decomposition of a function into a numerical or functional series does not provide information about the presence of semantic relationships between the elements of this series. The main problem is \( f(x) \neq S_f(y) \), where \( x = y \) in the traditional meaning.

3. Solution

**Definition 1:** Sense Series is an endless semantic union of elements of a sense space \( S_s [2] (O^K, S_N, S_{o(N)}) \).

Such a series is called a **semantic series on union** and is indicated by:

\[
U_n = \bigcup_{i=1}^{n} a_i = a_1 \cup a_2 \cup a_3, \ldots \cup a_n
\]

where \( a_n \in S_s \).

**Definition 2:** Partial semantic union of a sense series is a semantic union of various \( K \)-elements of Sense Series with \( N \)-elements where \( K < N \):

\[
U^K_N = \bigcup_{i=1}^{K} a_i = a_1 \cup a_2 \cup a_3, \ldots \cup a_K
\]

or abbreviation

\[
U^K_N = \bigcup_{K}^{N} a
\]
\[
\begin{align*}
U_1 &= a_1 \\
U_2 &= a_1 \uplus a_2 \\
U_3 &= a_1 \uplus a_2 \uplus a_3 \\
&\vdots \\
U_N &= a_1 \uplus a_2 \uplus a_3 \ldots \uplus a_N
\end{align*}
\] (4)

**Definition 3:** Sense Series \( U_n \) is called *completely convergent* if and only if there is at least one zero-object \( O_0 \) whose properties are all elements of the given series:

\[
\forall a \in U_n (\exists O_0 | O_0 \subseteq U_n a = S_n) 
\] (5)

**Definition 4:** Sense Series \( U_n \) is called *partially convergent* if and only if there are a such sequence of \( k \)-elements belonging to \( U_n \) \((k < n)\) and at least one zero-object \( O_0 \) for which the following condition is satisfied:

\[
O_0 \subseteq U_k^n a = S_k
\] (6)

However, it does not follow from this expression that \( S_k \subseteq S_n \).

Just as the complete convergence of a sense series \( U_n \) does not imply the existence of its partial convergence.

**Definition 5:** Sense Series \( U_n \) is called *divergent* if and only if there is no object for which all elements of this series are its properties.

**Definition 6:** Sense-Function Series \( U_n^f \) is the final semantic union of sense functions defined in \( S_s \) and not semantically equal to each other:

\[
U_n^f = \biguplus_{i = 1, \ldots, n} S_{f(i)} = S_{f(1)} \biguplus S_{f(2)} \biguplus S_{f(3)} \ldots \biguplus S_{f(n)}
\] , (7)
where $S_{f(i)} \neq S_{f(j)}, i \neq j$.

**Definition 7:** Sense-Function Series $U^n_f$ is called *completely convergent* if and only if there is such a sense function $S_f$ defined on a set consisting of all the elements of sense functions included in this series:

\[ S_f \left( \bigcup_{i=1}^{n} a_i \right) = \bigcap_{a_i} \]

(8)

where $a_i = \{ S_{f(i)}, S_{f(i)}', ... S_{f(i)}^{n} \}_{i=1}^{n}$

\[ U_1^f = S_f(1) = \bigcap_{f(1)} \]
\[ U_2^f = S_f(1) \bigcup S_f(2) = S_f(k) \]
\[ U_3^f = S_f(1) \bigcup S_f(2) \bigcup S_f(3) = S_f(k+1) \]
\[ ... ... \]
\[ U_n^f = S_f(1) \bigcup S_f(2) \bigcup S_f(3) ... \bigcup S_f(n) = S_f(k+(n-2)) \]

(9)

Each of the resulting set of sense functions \{ $S_{f(k)}, S_{f(k+1)}, ... S_{f(k+(n-2))}$\} can be either definite or indefinite [3].

**Definition 8:** Sense-Function Series $U^n_f$ is called *partially convergent* if and only if there is a sense function $S_f(k)$ defined on a set consisting of elements of two or more functions included in this series, where $k \neq n$.

**Series convergence criterion:**
In order for a sequence of sense functions \{ $S_{f(i)} \}_{i=1}^{n}$ defined each separately on a sense space $S_s$ to converge in this space to a certain function $S_{f(k)}$, it *is necessary and sufficient* that the No-Sense Set of $S_{f(k)}$ is a semantic union of No-Sense Sets of $S_{f(i)}$:

\[ S_{f(k)} \subseteq \left( \bigcup_{i=1}^{n} S_{f(i)} \right) \]

(10)
**Continuity Theorem:**

“Sense-Function Series $U_k^f$ consisting of functions continuous on object $O_{N(0)}$, converges to a function $S_{f(M)}$ continuous on this object.”

**Proof.**
The proof of the theorem follows from the Axiom of Sense Conformity and semantic equality:

$$S_f(k) = S_f(n) \left( \bigcup_{i=1}^{n} S_{f(i)} \right)$$

(11)

**Differentiation Theorem:**

“Sense-Function Series $diff(U_k^f)$ consisting of derivatives on property $p_i$ functions, converges to a function $S_{f(M)}$ that has the derivative on this property [4].”

**Proof.**
The proof of the theorem follows from the Theorem of Existence of Subsets [1] and the Axiom of Semantic Union [1].

$$diff(U_k^f) = diff(p_1)[S_{f(1)}] \bigcup diff(p_1)[S_{f(2)}] \bigcup diff(p_1)[S_{f(3)}] ... \bigcup diff(p_1)[S_{f(K)}] = diff(p_1)[S_{f(M)}]$$

(12)

**Integration Theorem:**

“Sense-Function Series $\int U_k^f$ consisting of functions defined and continuous on No-Sense Set $\mathcal{S}_K$ on the set of zero-objects converges to the integral of a function $S_{f(M)}$ defined and continuous on $\mathcal{S}_M$ on the set of these objects.”

$$\int U_k^f = \int S_{f(1)} \bigcup \int S_{f(2)} \bigcup \int S_{f(3)} ... \bigcup \int S_{f(K)} = S_{f(M)}$$

(13)
Proof.
The proof of the theorem follows from the Theorem of Existence of Integral Set).

Any function $S_{f(K)}$ defined on the set $\mathcal{G}_k$ and differentiable by an object $O_0$ L-times on this set can be represented as a differential sense series on the object $O_{N(0)}$:

$$S_{f(K)} = S_{f(K)} \bigcup \text{diff}(O_{N(0)})[S_{f(K+1)}] \bigcup \text{diff}(O_{N(0)})[S_{f(K+2)}] \ldots$$

$$\ldots \bigcup \text{diff}(O_{N(0)})[S_{f(K+L)}] = S_{f(M)}$$  \hspace{1cm} (14)

where $M = K + L, S_{f(K)} \overset{\mathcal{G}_k}{=} S_{f(M)}$, or

$$S_{f(K)} = S_{f(K)} \bigcup \{ \bigcup \text{diff}(O_{N(0)})[S_{f(K+i)}] \}_{i=1, \ldots, L}$$  \hspace{1cm} (15)

The expressions on the right side of (14) and (15) are denoted as $\text{Diff}_S(O_0)$.

In this case, the function $S_{f(K)}$ is said to have an $(+L)$-order semantic shell:

$$S_{f(K)} = S_{f(K)} \bigcup S_f^{\text{diff}(+L)}(O_N)$$  \hspace{1cm} (16)

Any function $S_{f(K)}$ defined on the set $\mathcal{G}_k$ and differentiable by a property $p$ L-times on this set can be represented as a differential sense series on the property $p$:

$$S_{f(K)} = S_{f(K)} \bigcup \text{diff}(p)[S_{f(K+1)}] \bigcup \text{diff}(p)[S_{f(K+2)}] \ldots$$

$$\ldots \bigcup \text{diff}(p)[S_{f(K+L)}] = S_{f(M)}$$  \hspace{1cm} (17)

or
\[ S_f(K) = S_f(K) \cup \bigcup_{i = 1}^{L} \text{diff}(p)[S_f(K+i)] \]

The expressions on the right side of (17) and (18) are denoted as \( Dif f_S(p) \).

Any function \( S_f(K) \) defined and continuous on the set \( \{\bigcirc\}_i \) on the set of zero-objects can be represented as an integral sense series on \( \{\bigcirc\}_i \):

\[ S_f(K) = S_f(K) \bigcup_{\bigcirc} [S_f(K)]_1 \bigcup_{\bigcirc} [S_f(K)]_2 \bigcup_{\bigcirc} [S_f(K)]_3 \ldots \bigcup_{\bigcirc} [S_f(K)]_L \]

or

\[ S_f(K) = S_f(K) \bigcup ( \bigcup_{j = 1}^{L} [S_f(K)]_j ) \]

The expressions on the right side of (19) and (20) are denoted as \( Int_S(\bigcirc) \).

The following semantic set differences play an important role in determining the dimension \( D_S \) of the sense space \( S_s \) on which the considered function \( S_f \) is defined:

1. \( S_f(K) \bigcup Diff_S(0_0) = \bigcirc \), “functional complement on object” to \( S_f(K) \) which determines the new properties (new knowledge) of the object.
2. \( S_f(K) \cup \text{Diff}_s(p) = \tilde{F}_p \), “functional complement on property” to \( S_f(K) \) which determines the new semantic connections with other objects (Sense Sets) of the sense space \( S_s \).

\[ S_f(K) \cup \text{Int}_s(\{\circ\}_i) = \tilde{F}_{\{\circ\}} \]

3. “functional complement on object set” to \( S_f(K) \) which determines the set of adjacent properties for the object set.

4. **Conclusion**

In this article, we presented the mechanism for identification of new semantic connections between objects, Sense Series. It allows finding semantic connections between trillions of objects of different nature.

We hope that our decent work will help other AI researchers in their life endeavors.

**To be continued.**
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