Five families and SUSY in SO(16)

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Abstract

We present here the barebones of a SUSY realisation of SO(16) comprising the simple product of the conventional SO(10) description of the gauge interactions of one fermion family with a SO(6) that labels spin and family. In addition to the three known lepton families we find two sleptonic ones. SUSY appears to be broken (only) by the same type of operator that gives the lepton families widely different mass scales.

1 Motivation

It has been known since the 1970s that the fermions of one family fit perfectly into the 16 $\oplus$ $\overline{16}$ spinorial representation of SO(10) and that the symmetry of this group can be broken first to SU(5) $\times$ U(1)$_X$ and subsequently to SU(3)$_C$ $\times$ SU(2)$_L$ $\times$ U(1)$_L$ by means of scalar fields having masses in the region $10^{15} - 10^{19}$ GeV to reproduce the known gauge boson spectrum. Attempts to use a larger orthogonal group that would contain representations of (just) the three known lepton families were thought not to be viable, because they seem to predict equal numbers of $V$ $+$ $A$ and $V$ $-$ $A$ families [1]. There also did not seem to be any way to incorporate SUSY partners, because particle spin played no role in the algebra of SO(10).

2 Notation

We will be using the following shorthand notation for elements of the Clifford algebra:

\begin{align}
\tau_u &\equiv \frac{1}{2}(\tau_0 + \tau_3) \\
\tau_d &\equiv \frac{1}{2}(\tau_0 - \tau_3) \\
\tau_{\pm} &\equiv \frac{1}{2}(\tau_1 \pm i\tau_2) \\
T_{ABCDEFGH} &\equiv \tau_A \times \tau_B \times \tau_C \times \tau_D \times \tau_E \times \tau_F \times \tau_G \times \tau_H \\
T_{FGH} &\equiv \tau_0 \times \tau_0 \times \tau_0 \times \tau_0 \times \tau_0 \times \tau_F \times \tau_G \times \tau_H
\end{align}

where \{A, B, C, D, E, F, G, H\} = \{0, 1, 2, 3, u, d, +, -\}

3 Model

- We suppose that the $\mathbf{128} \oplus \mathbf{128}$ of SO(16) contains eight copies of the SO(10) fermions, additionally labelled by spin and family index.
- We further suppose that in addition to the three ($e, \mu, \tau$) leptonic families there are two (possibly identical) sleptonic families $\chi, \chi'$.

With the following choice of spin and family assignments in the $\mathbf{4} \oplus \mathbf{4}$ of SO(6):

\begin{equation}
\Psi = \begin{pmatrix}
|e^+_u\rangle = |+ - -\rangle \\
|\mu^+_u\rangle = |- + -\rangle \\
|\tau^+_u\rangle = | - - +\rangle \\
|e^-_d\rangle = |- + +\rangle \\
|\mu^-_d\rangle = |+ + -\rangle \\
|\tau^-_d\rangle = |+ + +\rangle \\
|\chi\rangle = |+ + +\rangle \\
|\chi'\rangle = |- - -\rangle
\end{pmatrix}
\end{equation}
...we have

\[
\begin{align*}
\sigma_+ &= \frac{1}{2} \left( \tau_{++} + \tau_{+-} + \tau_{-+} + \tau_{--} \right) \\
\sigma_- &= \frac{1}{2} \left( \tau_{--} + \tau_{+-} + \tau_{-+} + \tau_{++} \right) \\
\sigma_3 &\equiv \sigma_+ \sigma_- - \sigma_+ \sigma_- = \frac{1}{4} \left[ 3 \tau_{333} + \tau_{000} + \tau_{030} + \tau_{003} \right] \\
\tau_e &= \frac{1}{4} \left( \tau_{000} + \tau_{033} + \tau_{003} + \tau_{303} \right) \\
\tau_\mu &= \frac{1}{4} \left( \tau_{000} - \tau_{033} + \tau_{003} - \tau_{303} \right) \\
\tau_\tau &= \frac{1}{4} \left( \tau_{000} - \tau_{033} - \tau_{003} + \tau_{303} \right) \\
\tau_\chi &= \frac{1}{4} \left( \tau_{000} + \tau_{033} + \tau_{003} + \tau_{303} \right) \\
\tau_\chi' &= \frac{1}{4} \left( \tau_{000} - \tau_{033} - \tau_{003} - \tau_{303} \right)
\end{align*}
\]

(3)

4 Predictions of the model

- Gauginos have the same masses as the corresponding gauge boson and are to be thought of as spinorial polarisation modes additional to the familiar \{+, -, 3\} polarisations.
- SUSY is only broken by a term in the lagrangian that renders the 2 slepton families very heavy relative to the 3 lepton families. The failure to observe any SUSY particles at the LHC and elsewhere is solely due to the heaviness of the slepton families effectively forbidding all vertices involving a gaugino.
- The fixed gauge potentials \(A^a_\mu\) are related to rotations in the \(SU(5)_{GUT} \otimes SU(2)_{\text{spin}}\) space of the form \(e^{i\Theta}\) where \(\Theta = \theta^a j \tau^a \otimes \sigma_j\) according to:

\[
\begin{align*}
A^a_0 &= \partial_0 \theta^a_j \\
A^a_j &= \partial_0 \theta^a_j + i \epsilon_{ijk} \partial_i \Theta^k
\end{align*}
\]

(4)

- There is no \(M^2_{GUT}\) contribution to the Higgs mass because the gauge and gaugino loop effects exactly cancel.
- There are exactly three lepton families and exactly two, possibly identical, slepton families that have hitherto escaped detection because their particle members all have masses \(\gg 1\) TeV.

References