REVISION OF KINEMATICS OF LINEAR MOTION OF A PARTICLE

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ABSTRACT. Based on plain reasoning and simple algebraic calculation from the Eqn. \( v = u + at \), all the other equations of ‘suvat’ equations are corrected when the value of \( t \) is a positive integer. Then, from this same reasoning, a general equation for displacement is calculated showing relation with all the kinematic terms that can be derived from displacement by definition.

1. suvat equations when \( t \) is positive integer valued

We know, constant acceleration is increase in velocity per unit of time and velocity is distance crossed per unit of time; this is the reasoning for doing the calculation shown below.

Consider the arithmetic progression \( u + a, u + 2a, \ldots, u + ta \) and their summation \( S_n = \frac{n(b_1 + b_n)}{2} \) where \( b_1 \) is the first and \( b_n \) is the last term. This is basically what has been shown in this section.

(In order to keep the unit of distance accurate, kinematic terms like velocity and acceleration were multiplied by 1 for necessary number of times where 1 represents 1 unit of time.)

If a point particle starts moving with constant velocity \( u \), then it will cross \( u \cdot 1 \) amount of distance in first unit of time, \( u \cdot 1 \) amount of distance in second unit of time and so on. Obviously, in \( t \) amount of time, the particle will cross \( ut \) amount of distance. The other way to get this same result is to add all distances that were crossed in each unit of time, which is \( (u \cdot 1) + (u \cdot 1) + \cdots + (u \cdot 1) \) \( t \) times \( = (u \cdot 1)t = u \cdot t \)

During the occurrence of constant acceleration \( a \), in first unit of time, the particle will cross \( u \cdot 1 \) and an additional \( (a \cdot 1) \cdot 1 \) amount of distance and their sum is \( (u + a \cdot 1) \cdot 1 \) amount of distance. So, after the first unit of time, the velocity of the particle becomes \( u + a \cdot 1 \).

In second unit of time, the particle will cross \( (u + a \cdot 1) \cdot 1 \) and an additional \( (a \cdot 1) \cdot 1 \) amount of distance and their sum is \( [u + (a \cdot 1) + (a \cdot 1)] \cdot 1 = [u + 2(a \cdot 1)] \cdot 1 = (u + a \cdot 2) \cdot 1 \) amount of distance. So, after the second unit of time, the velocity of the particle becomes \( u + 2(a \cdot 1) \equiv (u + a \cdot 2) \).

And so on for other units of time.

So, if we count the crossed distance for each unit of time, we would get the correct result. We can calculate the total amount of crossed distance from the Eqn. \( v = u + at \) following way

\[
\begin{align*}
v_1 & \cdot 1 = (u + a \cdot 1) \cdot 1 \\
v_2 & \cdot 1 = (u + a \cdot 2) \cdot 1 \\
\vdots \\
v_t & \cdot 1 = (u + a \cdot t) \cdot 1 \\
\sum_{i=1}^{t} v_i & \cdot 1 = [ut + a(1 + 2 + \cdots + t)] \cdot 1 \\
\end{align*}
\]

(Here, in \( ut \cdot 1 \) or \( (u \cdot 1)t \), numeric \( t \) multiplied by 1 unit of time would give out resultant measure as \( t \) units of time and \( 1 + 2 + \cdots + t \) units of time multiplied by 1 unit of time would give out \( (1 + 2 + \cdots + t) \) units\(^2\) of time.)

This yields

\[
\begin{align*}
s & = ut + \frac{1}{2}at(t + 1) \\
s & = \frac{1}{2}(u + a \cdot 1 + v)t \\
v^2 - u^2 & = (v - u)(a \cdot 1) - 2as = 0
\end{align*}
\]

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2. DISPLACEMENT FOR FRACTIONAL VALUED TIME DURING CONSTANT ACCELERATION

If acceleration $a$ occurs for $(t + \tau)$ amount of time where $\tau \in (0, 1)$ then,

$$s = u(t + \tau) + a(t + 1)[(t/2) + \tau]$$

Since, in the next $\tau$ amount of time after $t$, the particle will cross $(u + at)\tau$ and an additional $a \cdot 1 \cdot \tau$ amount of distance. If $v = u + a\tau$, then $s = (u + a \cdot 1)\tau$.

3. ACCOUNTING FOR ALL THE DERIVED KINEMATIC TERMS FROM DISPLACEMENT

(In this section, symbol $s$ is assigned to the kinematic term snap and displacement is represented with the symbol $r$)

Each term of the progression $u + a, u + 2a, \ldots, u + ta$ can be written with the presence of its previous term and thus we get a recurrence relation.

$$R_t = R_{t-1} + a$$

With initial condition $R_0 = u$. This relation holds even if $a$ becomes a dependent variable of $t$. By plugging in the equations $a = a_0 + jt$, $j = j_0 + st$, $s = s_0 + ct$, $c = c_0 + pt$ and so on in the recurrence relation where $t$ as a variable holds the positive integer values, we can build a sequence where the first term is $R_1 = u + a_0 + j_0 + s_0 + c_0 + p_0 + \cdots$ and the $t$-th term is $R_t = R_{t-1} + a_0 + j_0t + s_0t^2 + c_0t^3 + p_0t^4 + \cdots$. By summing all the terms of this sequence of finite length, we will get the equation for displacement with relation to all the kinematic terms derived from displacement which are velocity, acceleration, jerk, snap, crackle, pop and so on. Meaning

$$r = \sum_t R_t$$

Thus we get,

$$r = ut + a_0 \sum_{k_1=1}^{t} \sum_{i_1=1}^{k_1} i_1^0 + j_0 \sum_{k_2=1}^{t} \sum_{i_2=1}^{k_2} i_2^1 + s_0 \sum_{k_3=1}^{t} \sum_{i_3=1}^{k_3} i_3^2 + c_0 \sum_{k_4=1}^{t} \sum_{i_4=1}^{k_4} i_4^3 + p_0 \sum_{k_5=1}^{t} \sum_{i_5=1}^{k_5} i_5^4 + \cdots$$

For additional $\tau$ amount of time where $0 < \tau < 1$,

$$r = u[t + \tau] + a_0 \left[ \sum_{k_1=1}^{t} \sum_{i_1=1}^{k_1} i_1^0 + \sum_{n_1=1}^{t+1} n_1^0 \cdot \tau \right] + j_0 \left[ \sum_{k_2=1}^{t} \sum_{i_2=1}^{k_2} i_2^1 + \sum_{n_2=1}^{t+1} n_2^1 \cdot \tau \right] + s_0 \left[ \sum_{k_3=1}^{t} \sum_{i_3=1}^{k_3} i_3^2 + \sum_{n_3=1}^{t+1} n_3^2 \cdot \tau \right] + c_0 \left[ \sum_{k_4=1}^{t} \sum_{i_4=1}^{k_4} i_4^3 + \sum_{n_4=1}^{t+1} n_4^3 \cdot \tau \right] + p_0 \left[ \sum_{k_5=1}^{t} \sum_{i_5=1}^{k_5} i_5^4 + \sum_{n_5=1}^{t+1} n_5^4 \cdot \tau \right] + \cdots$$

REFERENCES