Real Domain Transforms for Special Relativity

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Abstract

The mathematical elegance and simplicity of Lorentz transform to support four-vector-based covariant formulation comes from its operation in the Minkowski domain that maps the position and calculates the time accordingly. But the same also makes it difficult to interpret. Transforming it back to the real domain that maps the time and calculates the position accordingly eases their interpretation, which leads to real domain transforms, a tool to discern the real effects from mere mathematical ones. Many new phenomena like relativity of spatial concurrence and relativistic non-localization, so far hidden under the elegance of the former, are brought to light. The real domain exhibits relativistic time-dilation, length-contraction, velocity-addition, clock compatible interval and phase relationship, and Doppler effect, but the non-simultaneity disappears. New transform is reducible to Lorentz transform establishing their equivalence.

1. Introduction

The mathematical elegance and simplicity of Lorentz transform (LT) in (1), which began the era of four vector-based covariant formulations, is unmatched [1-5].

$$X' = \gamma(x-vt),\ T' = \gamma(t-vx/c2),\ Y' = y,\ Z' = z, \qquad (1)$$

where $\gamma = 1/\sqrt{1-v^2/c^2}$, v is the relative velocity between frames, and c is lightspeed. The scheme behind this success is that LT maps the position x in one frame to an overlapped position X' in the other and recomputes new time T' accordingly, which lies in the split time domain as the unique time t of the rest frame is split due to x-dependence. This scheme enables LT to operate in the Minkowski or split domain where spacetime is mixed, rendering benefits of four-vector covariant formulation, but the same makes them difficult to interpret and discern which effects of split domain translate to the real domain, which maps the clock time of one frame to the other and then calculates the positions accordingly. The real domain view facilitates the interpretation of LT. Consider an array of infinite atoms spread all over the axial dimension in the moving frame (MF), doing cyclic motion around their fixed axial positions there. The instant of formation of a particular pattern, say a dash or a wave, in the rest frame (RF) splits into infinite instants of the time in the MF in the Minkowski domain as it maps the atom's positions of the pattern in the RF to the MF and computes time for each particle to be there according to the second postulate, resulting in the splitting of time and hence the name split domain. But, an equally valid approach is to ask when the atoms formed the dash in the RF, what pattern their locations formed in the MF. Thus, in real domain transform (RDT) the clock time t of the RF is mapped to the clock-time t' in the MF, and then the positions (x',y',z') of the atoms are computed at t' in the MF based on the second postulate, resulting in the real domain (x',y',z',t'). As another example, consider two photons emitted in the MF at the common origin at $t=t'=0$ being detected at x and $-x$ at time t in the RF. Now following the LT, one can map the positions x and $-x$ to $\gamma(x-vt)$ and $-\gamma(x+vt)$ and compute the times to occupy those positions in the MF, or following RDT, one can map RF-clock time t to the clock-time t' in the MF and compute the positions of the photons at t'. Thus, both real and split domains are equivalent views, reducible to each other.

The main purpose of RDT is to facilitate a correct interpretation of LT, and so they are derived here from scratch so that a correct relationship between LT and RDT is established. RDT in [6] is shown to exhibit relativistic time dilation, length contraction, velocity addition, preserving light-sphere, clock compatible phase and interval relationships, and Doppler effect in the real domain. However, the effects like synchronization term and clock incompatible interval and phase invariance of the Minkowski domain disappear in the real domain [7]. Also, some new phenomena such as particles' existence at different positions in different frames (DPDF), the relativity of spatial concurrence (RSC), anisotropic spatial warping (ASW), and relativistic non-localization (RNL) that were hidden under the mathematical elegance of LT are brought to light by RDT [8-12]. The RDT however is mathematically too cumbersome to support covariant formulation, but they readily reduce to LT to take advantage of four-vector formulation.

2. Derivation of new transforms

Define the RF time t to which all the clocks of the RF are synchronized and the MF time t' to which all the clocks of the MF are synchronized under the postulate of constancy and isotropicity of lightspeed c in agreement with [1]. Next, define two terms, momentum-potential as v or v/c and motion-energy-potential as v^2 or v^2/c^2. These potentials need not be confused with momentum and energy, and they are merely mathematical constructs. The two inertial frames see the empty space of the other at a relative non-zero momentum potential v/c and a non-zero motion energy-potential v^2/c^2, the former has got a sense of directionality but the latter has none. Thus, the relativity of motion between two frames has two aspects, relativity of v/c and relativity of v^2/c^2. Both of these aspects contribute to the relativity of spacetime. Further, let the influence of relativity of v/c be represented by a scaling factor 'm' and that of the relativity of v^2/c^2 by a scaling factor 'e' in the coordinate transforms. Factor e is a function of even-orders only with no odd-order dependence in v/c, while m factors may contain linear or odd-order dependence in v/c besides others.

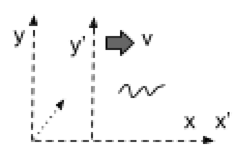

Fig 1. The primed frame moving with velocity v w.r.t. unprimed frame in +x.

Further, for MF transforms (MFT), we assume the source of particles is situated in the MF and the particles are detected in either frame, opposed to the RF transform (RFT) that addresses the scenario when the particle-source is located in the RF but observed from either frame. Further, complying with the second axiom i.e. to save the MF time from being illusory, the factors carrying odd-order dependence such as m can only appear in spatial transforms not in the temporal one, and complying with the third axiom i.e. even order factors such as e devoid of any directionality must affect all the spatial coordinates symmetrically, we begin with the mathematical form of the RDT,

$$x' = em(x - vt)\,,\; y' = em_{\perp}\, y\,,\; z' = em_{\perp}\, z \qquad (2)$$

$$t' = e_t(v^2/c^2)\, t \qquad (3)$$

where (x',y',z',t') are the primed frame coordinates of a particle that originated at the MF's origin at $t'=t=0$ and (x,y,z,t) are the same for the RF observer. Because of directionality, axial scaling factor m is differentiated from transverse scaling factor $m_{\perp}$. Similarly to start with there is no reason to use the same e factors for spatial and temporal coordinates. Arguments of e_t are to show that e_t is a function of v^2/c^2 or even orders v/c alone. Likewise, e is also a function of v^2/c^2 and so e can also be written as $e(v^2/c^2)$ but the arguments of e are omitted in (1) for brevity. Factors m can have linear order dependence in v/c or x beside the others.

2.1 Axial scaling factor *m*

Consider a rod of length L when stationary, which is set in the MF along x' with its one end lying at O' and the other at x'. MF observer sends a light signal from O' to A' at $t=t'=0$, and confirms its length to be $x'=L=ct'=e_t(ct)$, claiming that the light hit the other end at x' at t'. However, for the stationary observer, his estimate for the length of the moving rod, $(c-v)t'$, falls short by a value $vt' = e_t vt = (e_t vx)/c$ from the actual length of the rod, where x corresponds to the end of the rod at x'. To recover the proper length of the rod, $L=ct'$ for the MF observer, the RF observer has to scale his estimates by a factor $L/(L-e_t vx/c)$ This gives him the required m factor, as

$$m = \frac{1}{1-(v/c^2)(x/t)} \tag{4}$$

Thus, x coordinate transform becomes,

$$x' = e.\ \frac{1}{1-(v/c^2)(x/t)}\ (x - vt) \tag{5}$$

2.2 The temporal scaling factor e_t

For a photon, put $x/t=c$ or $x=ct$ in the RHS of (5) and divide it by (3) to yield $x'/t' = (e/e_t)c$. To conserve the speed of light in the two frames, both even order scaling factors have to be equal, $e_t(v^2/c^2) = e(v^2/c^2)$, and hence the temporal transform becomes

$$t' = e\,t \tag{6}$$

2.3 Transverse scaling factor

Consider an oblique ray of light in the $x'y'$ plane originating at the origin of the MF at $t=t'=0$, and reaching to point (x', y') at t'. For such a ray,

$$x'^2 + y'^2 = c^2 t'^2 \tag{7}$$

Putting x', y' and t' from eq (2), (5), and (6) in eq (7) and after following elementary algebra, we have

$$x^2 + m_\perp{}^2\ \frac{[1-(v/c^2)(x/t)]^2}{(1-v^2/c^2)}\ y^2 = c^2 t^2\ ,$$

where the coefficient of y^2 has to be 1 to preserve lightspeed and hence,

$$m_\perp = \frac{\sqrt{1-v^2/c^2}}{1-(v/c^2)(x/t)}\ .$$

Thus, transformations for the transverse coordinates are:

$$y' = e\ \frac{\sqrt{1-v^2/c^2}}{1-(v/c^2)(x/t)}\ y\ ,\ \ z' = e\ \frac{\sqrt{1-v^2/c^2}}{1-(v/c^2)(x/t)}\ z \tag{8}$$

2.4 Even order scaling factor e

Consider a light ray going on y'-axis in the MF from O' to hit a mirror M'. Mirror and ray's axial velocity is $x/t=v$ in the RF and the ray-path OM' is oblique, whose projection on y is OM such that $y=OM=O'M'=y'$. Substituting this along with $x/t=v$ in the first equation of (8), we get,

$$e = \sqrt{(1 - v^2/c^2)} \tag{9}$$

3. RDT summarised

Equations (5) through (9) summarize MFT reproduced here.

$$x' = em(x - vt)\ ,\ y' = em_\perp\, y\ ,\ z' = e\, m_\perp z\ , \tag{10}$$

$$t' = et\ , \tag{11}$$

where,

$$e = \sqrt{1 - v^2/c^2}\ ,\ m = \frac{1}{1-(v/c^2)(x/t)}\ ,\ m_\perp = \frac{\sqrt{1-v^2/c^2}}{1-(v/c^2)(x/t)} \tag{12}$$

MFT along with their inverse transforms apply for the events of the moving frame observed from either frame, to get the view in one frame from the data of the view collected in the other frame. For back transforming, inverse the MFT,

$$x = \gamma m'(x' + vt')\ ,\ y = \gamma m'_\perp y'\ ,\ z = \gamma m'_\perp z' \tag{13}$$

$$t = \gamma t'\ . \tag{14}$$

We can derive on the lines of MFT, a separate set of RFT for the events of the RF viewed from either frame to predict their respective coordinates in the RF or vice-versa:

$$x = em'\,(x' + vt')\ ,\ y = em'_\perp\, y'\ ,\ z = e\, m_\perp z' \tag{15}$$

$$t = et'\ , \tag{16}$$

where,

$$m' = \frac{1}{1+(v/c^2)(x'/t)}\ ,\ m'_\perp = \frac{\sqrt{1-v^2/c^2}}{1+(v/c^2)(x'/t)} \tag{17}$$

Use inverse RFT to back transform,

$$x' = \gamma m(x - vt)\ ,\ y' = \gamma m_\perp\, y\ ,\ z' = \gamma m_\perp z \tag{18}$$

$$t' = \gamma g t\ . \tag{19}$$

For RDT, x and x' in general are interpreted as effective lengths explored by the probe or particle having speed $v_p = x/t$ used in the m factor. If the velocity of the particle is v_p for time t, then eq (10-12) become,

$$x' = em(x - vt)\ ,\ y' = em_{\perp}\, y\ ,\ z' = em_{\perp} z \qquad (20)$$

$$t' = e\, t\ , \qquad (21)$$

$$e = \sqrt{1 - v^2/c^2}\ ,\ m = \frac{1}{1-(vv_p/c^2)}\ ,\ m_{\perp} = \frac{\sqrt{1-v^2/c^2}}{1-(vv_p/c^2)} \qquad (22)$$

If the particle is stationed in the MF i.e. $v_p=v$, (20-22) provides the transform between the frames for the special case when the particle's frame is used as the MF,

$$X' = (x - vt)\ ,\ y' = y\ ,\ z' = z\ ,\ t' = et \qquad (23)$$

Both NT and LT reduce to this common eq (23) also called the transform of agreed spatial overlap of two frames and is useful to the particles or probes stationed in either frame. But for a particle having non-zero velocity in either of the frames, (23) does not apply. Also, instead of origin, if a particle starts its journey from $x=x_0$ in the RF at time $t=0$, and its final positions in the RF is $x_f=x_0+x$, then in the MF both x_0 and x have to be separately transformed and then added to give its MF position at t

$$x_f' = (1/e)\{x_0 + m(x - vt)\} \qquad (24)$$

4. The Impact of RDT

RDT provides an alternative view of relativistic phenomena by mapping the events in the real domain. Both the views, the Minkowski domain view by LT and the real domain view by RDT, can benefit and enrich each other. Let us list both the common and different aspects of both views.

4.1 RDT are shown in [6] to reproduce the following relativistic phenomena in the real domain: the time-dilation, length-contraction, velocity-addition, preservation of lightspeed, and sphericity of growing lightsphere, correct phase and interval relationships, and the Doppler shift.

4.2 However, the synchronization term of the Minkowski domain disappears in the real domain suggesting the spacetime mixing of LT might just be a mathematical tool used by LT, which does not translate to the real domain. Such a possibility at least requires consideration by the physicists. In the real domain, instead of x and t it is the orientation of the transverse spatial dimensions for a moving observer that couples them with the axial dimension.

4.3 Also, the real domain solves the interval and phase invariance paradox of LT discussed in [7], which are not clock compatible. For a source located in the MF,

$$x'^2+y'^2+z'^2 - c^2t'^2 = e^2(x^2+y^2+z^2 - c^2t^2) \qquad (25)$$

$$X'^2 + Y'^2 + Z'^2 - c^2T'^2 = x^2 + y^2 + z^2 - c^2t^2 \qquad (26)$$

4.4 Next is the list of new phenomena that are implicitly present in the Minkowski domain but become explicit in the real domain. One of them is the availability of particles at different positions in different frames (DPDF) at a given instant. Suppose a photon emitted at $t=t'=0$ when origins of both the frames coincide, is found at $OP=x$ in RF. This point P coincides with P' from (23) as $OP' =\gamma(x\text{-}vt)$, while the point at which photon is available in the MF at that instant is Q', $OQ'=x'=ex$ from (10), giving rise to the relativity of spatial concurrence (RSC) shift,

$$P'Q' = evx/c\ , \qquad (27)$$

4.5 From above emerges the appeal to reconsider the current interpretation of LT based on the assumption of availability of a particle at an overlapped position in different frames (OPDF) or assuming the moving particle to be relativistically localized. One of the outcomes of OPDF is the relativity of simultaneity (RoS) based interpretation of LT, however, neither the Minkowski nor the real domain supports OPDF [6,7]. Yes, even LT does not support OPDF because they do map the position to another frame at an overlapped point given by (23) but assign a different time to occupy it. Once OPDF is rejected then new phenomena like DPDF, RSC, ASW, and RNL pave the way forward, which are favored using both the Minkowski and real domain views [8-11].

4.6 The second axiom as mentioned in section 2 prefers linear or odd order warping of space over time. The support for this comes from experiments [12,13] that show a moving clock's time is dilated in second or even orders of *v/c* and no trace of linear or anisotropic warping of time is seen. In [6], the Sagnac effect is explained as a proof of ASW.

4.7 Mechanism behind lightspeed constancy: RNL not only brings relativity and quantum physics closer, but it can also be the mechanism behind the preservation of lightspeed [6]. If so, the possibility to get around this mechanism can also not be ruled out [12].

4.8 Equivalence of RDT and LT: The main purpose of RDT is to facilitate interpretation of LT so that so far unexplored relativistic phenomena are brought to light, and the efforts to test RoS, OPDF, and spacetime mixing are initiated. Both RDT and LT are equivalent as the Minkowski domain *(X',Y',Z',T')* and the real domain *(x',y',z',t')* coordinates are reducible to each other using the following relationship in (28), which can be found dividing respective transforms in (1) and (10-11). Similarly, backward LT and RDT can also be related.

$$x' = e^2mX',\ t' = e^2mT',\ y' = e^2mY',\ z' = e^2mZ' \qquad (28)$$

5. How events are mapped by LT and RDT?

In the Minkowski domain, a set of simultaneous events in one frame are mapped to a set of non-simultaneous events in the other, but in the real domain, a set of simultaneous events are mapped to a set of simultaneous events. However, the mapping in the Minkowski domain does not contradict the mapping of the real domain because the output sets of events in both cases are different. Consider five particles spread on the *x*-axis out of infinite atoms mentioned in the first section of this paper. In fig 2 are listed chronologically the events *(S,T)* of a set of five particles lying on the x-axis symmetric about the origin, doing to and fro motion in *y*, as observed from both the frames independently. LT in split domain maps a horizontal set of simultaneous events to a diagonal set of non-simultaneous events, while RDT in the real domain maps them to a horizontal set of simultaneous events.

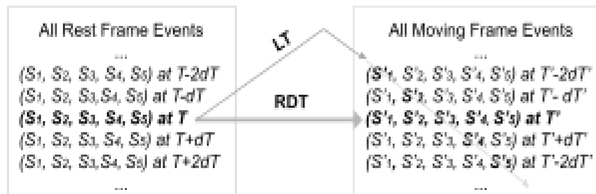

Fig 2. The bolded set of events in the RF is mapped to a horizontal set by RDT but a diagonal set by LT in the MF.

6. Conclusion

RDT is derived from scratch and their equivalence with LT is established. The main purpose of RDT is to facilitate the right interpretation of LT by providing a real domain view of relativistic physics. The impact of new transforms is summarised point by point in section 4 of this paper such as replacing RoS-based interpretation of LT with that based on RSC. In summary, new perceptions and phenomena that were implicitly present hiding under the spacetime mixing of the Minkowski domain, usher explicitly in the real domain. One of them is RNL that perhaps can provide better integration of quantum and relativity and it can also be a mechanism behind lightspeed preservation [14]. New phenomena are explored further [8-11].

Acknowledgment: I am thankful to Mukt Mind Lab for always standing with me during this extensive research.

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