Timelike Electrodynamics

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Abstract

The interval of propagation in Maxwell's theory is lightlike. If we assume that the interval is timelike, instead, by introducing a fundamental length, a rich new theory emerges.In this article it is shown that by varying the interval with respect to the fundamental length, the Lienard-Wiechert (LW) potential automatically emerges.Then, by varying the fundamental length in the expression for the moment of LW potential about the interval of propagation, the total field of a single point charge moving with arbitrary velocity and acceleration also emerges.They have exactly the same form as in Maxwell's theory but are finite everywhere, have finite self-effects and obey the Lorenz gauge condition.This is accomplished without using Maxwell's eqs at all.Only the interval of propagation is used.The Lorentz force, the field Lagrangian density, the energy stress tensor, all have the same form, are finite everywhere and have finite self effects.This shows that there exist at least one consistent theory different from Maxwell's theory which gives the same results while eliminating the infinities of Maxwell's theory in a Lorentz covariant way.

1 Introduction

Maxwell's theory is a very successful theory and it uses an interval of interaction which is lightlike.What has not been investigated is what happens if the interval of interaction is timelike but still obeys the laws of special relativity. Can such a theory be consistent and what does it mean? In this article, it is shown that if the interval is timelike , so that its square is the negative of the square of a "fundamental length", it is sufficient to vary the interval with respect to the square of the length to obtain the Lienard Wiechert potential. The latter automatically appears, has the same form as the Maxwell one, is finite everywhere and has finite self effects. Furthermore, the moment of potential about the interval can be put in the form of a rotation tensor. Varying this rotation tensor or the moment of potential, one obtains the total field of a single point charge moving with arbitrary velocity and acceleration. The total field becomes the curl of the LW potential as expected but here it is derived from the rotation tensor. The LW potential also obeys the Lorenz gauge condition. Again, the total field has the same form as in Maxwell's theory, is finite everywhere and has finite self-effects. Using the usual Maxwell formulas for the Lorentz force, the field Lagrangian density, and the energy stress tensor, we obtain an electrodynamics which is identical in form to Maxwell's, is finite everywhere and has finite self effects.Yet it does not satisfy Maxwell's eqs and does not seem to satisfy any differential eqs. All the physics is contained in the interval! The timelike nature of the interval defines a formal 4-vector velocity of the em influence and a formal 3- velocity of em influence. This 3 velocity is less than the speed of light. This does not mean that the influence moves ,of course ,it is only a formal statement. It does mean however that an inertial observer can move instantaneously with that velocity .

In section 2, we derive the LW potential and other formulas. In section 3, we derive the total field and analyze the total field, the acceleration field and the nonacceleration field and discuss how they are constructed.

The notation

4-vectors are in capital, 3-vector are not. Unit vectors are written with a caret to their right not on top. 4 vectors and 3 vectors have no arrows only their size differentiates them. The metric is [1,1,1,i] to avoid having to use covariant and contravariant indices, so whether an index appears as a subscript or as a superscript has no special meaning . Indices ijk run from 1 to 4. Indices abc if they occur run from 1 to 3. The summation convention for repeated indices is used unless specified otherwise. Dot product are specified by a dot. Cross product by the old fashioned x . Examples : 4 vector E . 3 vector e. Unit 4 vector A^ , E.B dot product. ExB cross product. $[ExB]_{ij} = E_iB_j - E_jB_i$ When the meaning is clear the indices will be omitted as in ExB . The speed of light c will not be put =1 unless doing so does not lead to confusion.

Preliminary formulas.

The interval of propagation (of influence, of interaction) is given by

E = R – R' (1a) ; R is the field point 4 vector. Its components are: R = [r ,ict] (1b)

r is the 3 position vector of the field point or the 3 position r(t) of a test charge q.

t is the instantaneous time of the field point or the test charge q.

R'(τ') is the retarded 4- position of the source charge q'.

R'(τ') = [r'(t'), ict'] (1c) r'(t')is the retarded 3 position of q'. t' is the retarded time .

U'(τ') = dR'(τ')/dτ' is the 4 velocity of q'. a' = $d^2R'(\tau')/d\tau'^2$ is the 4 acceleration of q'

The interval of influence is assumed to be timelike.

$E.E = -l^2$ (1d) ; l is a small invariant length. Not specified. Not a smallest length.

Maxwell theory's interval is lightlike E.E = 0

The Lienard Wiechert potential (LW) is :

$-q'U'/(E.U') = [A, i\phi]$ A is the 3dim vector potential. Φ is the scalar potential.

The 4 velocity of the influence is $W/c = E/l$ (1e) ;

$W.W/c^2 = -1$ (1f) ; $W/c = [(r- r')/l, ic(t-t')/l]$ (1g)

$W/c = [(w/c)(1-w^2/c^2)^{-1/2}, i(1-w^2/c^2)^{-1/2}]$ (1h)

$(r-r')/l = (w/c)(1-w^2/c^2)^{-1/2}$ (1i) ; $c(t-t')/l = (1-w^2/c^2)^{-1/2}$ (1j) ;

2 Derivation of the Lienard-Wiechert (LW) potential by varying the interval of propagation.

A) Vary the interval keeping the field position constant.

$E = R - R'$ $E.E = - l^2$ $dR/dl^2 = 0$ $d(E.E)/dl^2 = -1$(1bis); $E.dE/dl^2 = -1/2$ (2) ;

$dR'/dl^2 = (dR'/d\tau')d\tau'/dl^2 = U'd\tau'/dl^2$ (3) ; $-(E.U')d\tau'/dl^2 = -1/2$ (4) ;

finally $d\tau'/dl^2 = 1/2(E.U')$ (5) ; $dR'/dl^2 = U'd\tau'/dl^2 = U'/2(E.U')$;

$-2q'dR'/dl^2 = -q'U'/E.U' = 2q'dE/dl^2$ (6)

Eq 6 has exactly the form of the Lienard Wiechert potential, it is finite everywhere and has finite self effects. It has been derived solely by varying the fundamental length l, without resorting to Maxwell's eqs at all.

Let $A_{LW} = [A, i\phi] = -q'U'/E.U' = -2q'dR'/dl^2 = 2q'dE/dl^2$ (7)

A is the the 3dim vector potential, ϕ is the scalar potential.

We also have $E.A = -q'$ (8) ;

B Vary the field point without varying the length.

We write E not as R-R' but as $(x^i - x'^i(t'))$ and $(x^k - x'^k)(x^k - x'^k) = -l^2$;

We also use ordinary derivatives to describe partial derivatives.

So that $dl^2/dx^i = 0$ since the length is not varied. $d(E.E)/dx^i = 0$

$E_k(dE_k/dx^i) = E_k[\delta_{ik} - dx'_k/dx^i] = 0$; $E_i = (E_k dx'_k/d\tau')d\tau'/dx_i = E.U'\, d\tau'/dx^i$;

So that $d\tau'/dx^i = E_i/E.U'$ (9) ; from eq 9 we can find important relations.

Namely : $dx'^i/dx^j = (dx'^l/d\tau')d\tau'/dx^j = U'_i E_j/E.U'$ (10) ;

Therefore $dx'_l/dx_i = E.U'/E.U' = 1$ (11) ;

The LW vector potential should be in the Lorenz gauge so we should have

$dA^i/dx^i = 0$ To verify use eq7 $dA^i/dx^i = -2q'd(dx'_I/dl^2)/dx^i$

$= -2q'd(dx'_i/dx_i)/dl^2 = 0$. So our LW potential satisfies $dA^i/dx^i = 0$ (12)

The Lorenz gauge condition. Moreover it satisfies this condition because

Of eq (11).

From eq10 we get another important result:

Namely $dx'_i/dx_j - dx'_j/dx_i = [U'_i E_j - U'_j E_i]/E.U' = -[ExU']_{ij}/E.U'$ (12) ;

So $q'[dx'_i/dx_j - dx'_j/dx_i] = -q'[ExU']_{ij}/E.U' = [Ex A_{LW}]_{ij}$ (13) ;

The rotation like tensor of eq13 is equal to the moment of LW potential about

The interval of propagation.

C When a test charge q is present at the field point $E= R(\tau) - R'(\tau')$

The source charge q' is always connected to test charge q via E so the trajectories

$R(\tau)$ and $R'(\tau')$ depend on each other through E. This means that $\tau'=\tau'(\tau)$

And $\tau = \tau(\tau')$. We do not vary l^2 in what follows.

Therefore: $d(E.E)/d\tau = 0$; $E.dE/d\tau = 0$; $E.U - E.U'd\tau'/d\tau = 0$

$E.U/E.U' = d\tau'/d\tau$ (14)   ; $E.U\, d\tau = E.U'\, d\tau'$ (15) ;

What about $d(E.E)/dx_i$? We keep l^2 constant and $x_i = x_i(\tau)$

So $d(E.E)/dx_i = 0 = [x^k - x'^k][dx^k/dx^i - dx'^k/dx^i]$

$=[x^k - x'^k][\delta^{ik} - U'^k d\tau'/dx^i]$; $E_i = (E.U')d\tau'/dx_i$;

So $d\tau'/dx^i(\tau) = E^i/E.U'$ (16) ;   This is the same as eq 9 for the field point as it should. What about $d\tau/dx^i$? $d\tau'/dx^i = (d\tau'/d\tau)d\tau/dx^i = [E.U/E.U']d\tau/dx^i$

$= E^i/E.U'$; $d\tau/dx^i = E^i/E.U$ (17) ;

3 Derivation of the total field of a single point charge moving with arbitrary velocity and acceleration.

A Varying the moment of LW potential about the interval of propagation, by varying l^2 keeping the field point constant.

Write $d[ExA]_{ij}/dl^2 = 2q'd[ExdE/dl^2]/dl^2 = 2q'[Exd^2E/(dl^2)^2] = -2q'[Exd^2R'/(dl^2)^2]$

$= [ExdA/dl^2]_{ij}$ (18) ; $d[ExA]/dl^2 = [ExdA/dl^2] = -2q'[Exd^2R'/d(l^2)^2]$ (19) ;

It remains to calculate $d^2R'/(dl^2)^2$. Since $dR'/dl^2 = U'/2E.U'$

and $dU'/dl^2 = a'd\tau'/dl^2 = a'/2E.U'$ where a' is the 4 acceleration of q',

and with $d(E.U')/dl^2 = [E.a' - U'.U']d\tau'/dl^2$ we eventually get, making sure not to set c=1 but keeping it as c to avoid computational errors.

The result : $d^2R'/d(l^2) = \{[1+E.a'/c^2]U'/c + (a'/c)(E.U'/-c^2)\}/4(E.U'/-c)^3$ (20) ;

Eq20 gives:

$d(ExA)/dl^2 = -(q'/2)\{Ex[(1+E.a'/c^2)U'/c + (a'/c)(E.U'/-c^2)]\}(E.U'/-c)^{-3}$ (21)

$-2d[ExA]/dl^2 = q'\{Ex[(1+E.a'/c^2)U'/c + (a'/c)(E.U'/-c^2)]\}(E.U'/-c)^{-3} = F_{ij}^{\ total}$ (22);

This is the required result. Except for the factor of -2 eq22 is the variation of the moment of LW potential about the interval of propagation. Our notation is slightly different than the textbook ones (see the references.).

From eq13 we have: $-2q'd[dx'^i/dx^j - dx'^j/dx^i]/dl^2 = -2d[ExA]/dl^2$

$=[dA^i/dx^j - dA^j/dx^i] = F_{ij}^{\ total}$ (23) ; We have interchanged the variations d/dl^2 with the partial derivative d/dx^i in eqs13 and22.

We have just proved that varying the moment of potential gives the total field as the 4dim curl of the vector potential.

B The total field, the acceleration field, the non-acceleration field.

From eq 22, the total field can be written as:

$F_{ij}^{tot} = q'[ExU'_h */c]_{ij}(E.U'/-c)^{-3}$ (24)

With $U'_h */c = [(1+ E.a'/c^2)U'/c + (a'/c)(E.U'/-c^2]$ (24bis) ;

Eq24 is broken down into $F_{ij}^{total} = F_{ij}^{acc} + F_{ij}^{nonacc}$ (25) ;

Where $F_{ij}^{nonacc} = q'[ExU'/c](E.U'/-c)^{-3}$ (26) ;

$F_{ij}^{acc} = q'[Ex\{(E.a'/c^2)U'/c +(a'/c)(E.U'/-c^2)\}](E.U'/-c)^{-3}$ (26bis) ;

The Lorentz force will as usual be $F = qF_{ij}U_j/c$. The field Lagrangian density

And the energy-stress tensor will be taken as having the same form as

Maxwell's. Again, they will be finite everywhere and have finite self effects.

It will be shown in a future work that they have, each, a geometrical interpretation.

C Some properties of the three fields.

Let $S = [(E.a'/c^2)U'/c + (a'/c)(E.U'/-c^2)]$ (27) ;

$F_{ij}^{acc} =q'[ExS](E.U'/-c)^{-3}$ (28) ; We have $S.E =0$ (29) ;

S is a vector lying in the U',a' plane, perpendicular to E .

In components $S_i = (U'/c \times a'/c^2)_{ij} E_j$ (30) ;

Now project E perpendicularly onto the U', a' plane. Call the projection T.

$T = (E.a'\hat{})a'\hat{} + (E.U'/-c)U'/c$ (31) ; where $a'\hat{}$ is a unit vector in the a' direction. We easily find $S.T = 0$ (32) ;

Now take a vector D perpendicular to the U', a' plane.

$D = E - T$ (33) ; $D = [E - (E.a'\wedge)a'\wedge + (E.U'/c)U'/c]$ (34)

In components $D_i = [\delta_{ij} - a'_i \wedge a'_j\wedge + U'_l U'_j /c^2]E_j$ (35) ;

The vectors S, T, D should be mutually perpendicular to each other.

This means T.D = T.S = S.D = 0 (36) ; To check this use eq33

If D.T = E.T – T.T = 0 ; then E.T = T.T ;

$E.T = (E.a'\wedge)^2 - (E.U'/c)^2$; $T.T = (E.a'\wedge)^2 - (E.U'/-c)^2$; This proves that D.T =0 ;

D.S = E.S - T.S = 0 ; using eqs28 and32, so eq 36 is proved.

We also have $F_{ij}^{tot} = q'[Ex (S + U'/c)]_{ij} (E.U'/-c)^{-3}$ (37) ;

And $U'_h{*}/c = [S + U'/c]$ (38) ; $[U'_h{*}/c - U'/c] = S$ (39) ;

A dual can also be defined:

$U'_h{*}^{dual}/c = (1 + E.a'/c^2)a'\wedge + (a'/c)(E.U'/-c^2)U'/c$ (40) ;

$U'_h{*}^{dual}/c - a'\wedge = (E.a'/c^2)a'\wedge + (a'/c)(E.U'/-c^2)U'/c$

$= (a'/c^2)[(E.a'\wedge)a'\wedge + (E.U'/-c)U'/c] = (a'/c^2)T$ (41) ;

Finally: $U'_h{*}^{dual}/c - a'\wedge = (a'/c^2)T$ (42) ;

The purpose of all these formulas, in addition to their intrinsic geometric interest, is to eventually obtain a velocity unit 4 vector U'_h /c which will permit the elimination of the explicit 4 acceleration in the total field. This derivation is quite intricate and requires referring to geometrical constructions involving a hyperboloid of two sheet foliation of Minkowski spacetime as well as a hyperboloid of one sheet foliation of Minkowski spacetime. It will therefore be done in a separate publication.

4 Conclusion

We have shown in this article that Maxwell’s electrodynamics in the Lorenz gauge is not unique to Maxwell’s theory. There exist a consistent theory based solely on a timelike interval of propagation which gives the same results in the sense that they have the same form, are finite everywhere and have finite self effects. We are not dealing with a cutoff, the theory is Lorentz invariant so no artificial cutoff is needed, it is built into the theory. No wave eq is derived so we are not dealing with a massive photon or similar entities. We are outside of differential equations. Everything is contained in the interval.The “fundamental length “ is unspecified but must be small enough not to contradict all known experiments where classical electrodynamics is valid.

The Lienard Wiechert potential is derived by varying the interval. Once the potential is derived it is natural to take its moment about the interval. Doing this gives the total field by varying the moment of LW potential and also allows us to prove that it is the curl of the potential. The Lorenz gauge condition is also derived from the interval. The timelike interval implies the existence of a formal 3 velocity of influence which is less than that of light.

In future articles all the entities of electrodynamics : the potential, the fields, the Lorentz force, the field Lagrangian density and the energy stress tensor will be rewritten entirely in terms of 4 velocities and projective relative 4 velocity vectors and tensors,their geometric meanings explicated, and they will be connected to hyperboloid of one sheet and of two sheet foliations of Minkowski spacetime which involve virtual geodesic hyperbolic motions.

5 References

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