Existence of a Prime Number Between the Double of Other Primes Conjecture.

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0- Abstract:

In this paper I show how is possible to do a new application to the Bertrand’s postulate doing a conjecture with 3 prime numbers and the double of 2 of them.

1- Introduction:

First we are going to define some useful tools: the set of the naturals, the subset of the composite numbers and the subset of the prime numbers,

(1) \( A = m \cdot n \quad \forall (m, n) \in (\mathbb{N} - \{1\}) \)

(2) \( P = (\mathbb{N} - A) - 1 \)

Bertrand’s postulate says that is always a prime between a natural number and the double of that number,

(3) \( n < p < 2n \quad \forall n \in \mathbb{N}; p \in P \)

Following the logic, if the prime numbers are a subset of the naturals,

(4) \( P \subseteq \mathbb{N} \)

We can affirm that the next inequality is true.

(5) \( p_1 < p_2 < 2p_1 \quad \forall (p_1, p_2) \in P \)

2- Conjecture:

There always exists a prime \( p_{n+1} \) between \( p_n \) and \( 2p_n \) and there always exists a prime \( p_m \) between \( 2p_n \) and \( 2p_{n+1} \).

(6) \( p_n < p_{n+1} < 2p_n < p_m < 2p_{n+1} \quad \forall (p_n, p_m) \in P \)

References: https://en.wikipedia.org/wiki/Bertrand%27s_postulate