On the conformal energy tensor defined as a combination of Weyl tensor and the Hodge dual of Weyl tensor

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Abstract

In four dimensions with the Minkowski metric $\eta_{\mu\nu} = diag(+1, -1, -1, -1)$ the electromagnetic field tensor is defined as a differential 2-form F that constructs the electromagnetic stress–energy tensor as a combination of F and the Hodge dual of F, in Einstein field equations this role is played by the Weyl tensor C, the conformal tensor curvature is the only part of the curvature that exists in free space and governs the propagation of gravitational waves, so the conformal energy tensor can be defined as a combination of C and the Hodge dual of C

The electromagnetic field tensor defined as a differential 2-form

In four dimensions with the Minkowski metric $\eta_{\mu\nu} = diag(+1, -1, -1, -1)$ and c = 1

Electromagnetic field tensor is defined as a differential 2-form **F** [1]

$$F_{\mu\nu} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{bmatrix}$$

$\star$**F** is the Hodge dual

$$*F_{\mu\nu} = \begin{bmatrix} 0 & -B_1 & -B_2 & -B_3 \\ B_1 & 0 & -E_3 & E_2 \\ B_2 & E_3 & 0 & -E_1 \\ B_3 & -E_2 & E_1 & 0 \end{bmatrix}$$

The electromagnetic stress–energy tensor **T** as a combination of **F** and the Hodge dual of **F** [2]

$$T_{\mu\nu} = 1/8\pi(F_{\mu\gamma}F_{\nu}^{\gamma} + *F_{\mu\gamma}*F_{\nu}^{\gamma})$$

Conformal energy tensor defined as a combination of Weyl tensor C and the Hodge dual of C

In Einstein field equations the Weyl tensor **C**, the conformal tensor is the only part of the curvature that exists in free space and governs the propagation of gravitational waves [3]

$$C_{ik\ell m} = R_{ik\ell m} + \frac{1}{n-2}\left(R_{im}g_{k\ell} - R_{i\ell}g_{km} + R_{k\ell}g_{im} - R_{km}g_{i\ell}\right) + \frac{1}{(n-1)(n-2)}R\left(g_{i\ell}g_{km} - g_{im}g_{k\ell}\right).$$

where R_{abcd} is the Riemann tensor, R_{ab} is the Ricci tensor, R is the Ricci scalar (the scalar curvature)

The conformal energy tensor **T** can be defined as a combination of **C** and the Hodge dual of **C**

$$T_{abcd} = 1/8\pi(C_{alcd}C^{l}_{bcd} + {*C_{alcd}}{*C^{l}_{bcd}} + C_{abcn}C^{n}_{abd} + {*C_{abcn}}{*C^{n}_{abd}})$$

References

[1] See Chapter 19, 19.2 in: Roger Penrose, “The Road to Reality”, Jonathan Cape, London 2004

[2] See Chapter 19, 19.5 in: Roger Penrose, “The Road to Reality”, Jonathan Cape, London 2004

[3] See Chapter 19, 19.7 in: Roger Penrose, “The Road to Reality”, Jonathan Cape, London 2004