Proof of Goldbach Conjecture

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Abstract

Conjecture:
Any even number greater than 2 can be written as the sum of two prime numbers.
Does the prime pair exist universally? If does, is the prime pair unique relatively? If not, how many prime pairs are there in an even?
Method:
Triangular lattice.
Result:
The number of prime pairs in an even can be expressed analytically and graphically.

Keywords

Goldbach, Euler, even, prime.

Catalogue

Preface

Let x>0, and π(x) represents the number of prime numbers not exceeding x.
π(x)~x/ln(x)

1. Structures

1.1. Concepts

Set of natural numbers is recorded as N, N={n}. n=0, 1, 2…
One variable belongs to N, mark it as n;
Two variables belong to N, mark them as n1 and n2.
Set of positive integral numbers is recorded as P, P={p}. p=1, 2, 3…
One variable belongs to P, mark it as p;
Two variables belong to P, mark them as p1 and p2.
Set of even numbers is recorded as A, A={a|a=2*n}. a=0, 2, 4…
One variable belongs to A, mark it as a.
Two variables belong to A, mark them as a1 and a2.
Set of odd numbers is recorded as B, B={b|b=2*n+1}. b=1, 3, 5…
One variable belongs to B, mark it as b.
Two variables belong to B, mark them as b1 and b2.
Set of odd composite numbers is recorded as C, C={c|c=(2*p1+1)*(2*p2+1)}. c=9, 15, 21…
One variable belongs to C, mark it as c.
Two variables belong to C, mark them as c1 and c2.
Set of one is recorded as Q, Q={1}.
Set of two is recorded as R, R={2}.
Set of prime numbers is recorded as D,
D={d|d belongs to B and R, d does not belong to C or Q.}.d=2, 3, 5…
One variable belongs to D, mark it as d.
Two variables belong to D, mark them as d1 and d2.

1.2. Discussions

Let a>0, and T(a) represents the number of prime pairs in the even a,
Let a>0, and N(a) represents the number of odd pairs in the even a;
In the same a, T(a) does not exceed N(a).

2. Functions

2.1. Function: N(a)

a=a/2+a/2, a>0.
If a/2 belongs to A, a=[(a/2-1)-2n)]+[(a/2+1)+2n].

(a/2+1)-2n is noted as bL, (a/2+1)+2n is noted as bR.

n<(a-2)/4, Card(n)=a/4.

If a/2 belongs to B, a=(a/2-2n)+(a/2+2n).

a/2-2n is noted as bL, a/2+2n is noted as bR.

n<a/4, Card(n)=(a+2)/4.

Three functions: bL, bR; N(a).

bL=(a/2+1)-2n, a/2 belongs to A; bL=a/2-2n, a/2 belongs to B.

bR=(a/2+1)+2n, a/2 belongs to A; bR=a/2+2n, a/2 belongs to B.

Card(n) is noted as N(a):

N(a)=a/4, a/2 belongs to A; N(a)=(a+2)/4, a/2 belongs to B.

N(a)~a/4, a>0.

Let P(a)=bR-bL:

The vertical axis represents positive even numbers, the horizontal axis represents P(a). Any intersection corresponds to (a, P(a)) and (bL, bR).

	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42
2	1,1																					
4		1,3																				
6	3,3		1,5																			
8		3,5		1,7																		
10	5,5		3,7		1,9																	
12		5,7		3,9		1,11																
14	7,7		5,9		3,11		1,13															
16		7,9		5,11		3,13		1,15														
18	9,9		7,11		5,13		3,15		1,17													
20		9,11		7,13		5,15		3,17		1,19												
22	11,11		9,13		7,15		5,17		3,19		1,21											
24		11,13		9,15		7,17		5,19		3,21		1,23										
26	13,13		11,15		9,17		7,19		5,21		3,23		1,25									
28		13,15		11,17		9,19		7,21		5,23		3,25		1,27								
30	15,15		13,17		11,19		9,21		7,23		5,25		3,27		1,29							
32		15,17		13,19		11,21		9,23		7,25		5,27		3,29		1,31						
34	17,17		15,19		13,21		11,23		9,25		7,27		5,29		3,31		1,33					
36		17,19		15,21		13,23		11,25		9,27		7,29		5,31		3,33		1,35				
38	19,19		17,21		15,23		13,25		11,27		9,29		7,31		5,33		3,35		1,37			
40		19,21		17,23		15,25		13,27		11,29		9,31		7,33		5,35		3,37		1,39		
42	21,21		19,23		17,25		15,27		13,29		11,31		9,33		7,35		5,37		3,39		1,41	
44		21,23		19,25		17,27		15,29		13,31		11,33		9,35		7,37		5,39		3,41		1,43

Delete pairs in cells and put P(a) in, then there is the triangular lattice:

2	0																					
4		2																				
6	0		4																			
8		2		6																		
10	0		4		8																	
12		2		6		10																
14	0		4		8		12															
16		2		6		10		14														
18	0		4		8		12		16													
20		2		6		10		14		18												
22	0		4		8		12		16		20											
24		2		6		10		14		18		22										
26	0		4		8		12		16		20		24									
28		2		6		10		14		18		22		26								
30	0		4		8		12		16		20		24		28							
32		2		6		10		14		18		22		26		30						
34	0		4		8		12		16		20		24		28		32					
36		2		6		10		14		18		22		26		30		34				
38	0		4		8		12		16		20		24		28		32		36			
40		2		6		10		14		18		22		26		30		34		38		
42	0		4		8		12		16		20		24		28		32		36		40	
44		2		6		10		14		18		22		26		30		34		38		42

2.2. Function: P(a)

2	0																			
4		2																		
6	0		4																	
8		2		6																
10	0		4		8															
12		2		6		10														
14	0		4		8		12													
16		2		6		10		14												
18																				
20		2		6		10		14		18										
22	0		4		8		12		16		20									
24		2		6		10		14		18		22								
26	0		4		8		12		16		20		24							
28		2		6		10		14		18		22		26						
30	0		4		8		12		16		20		24		28					
32		2		6		10		14		18		22		26		30				
34	0		4		8		12		16		20		24		28		32			
36		2		6		10		14		18		22		26		30		34		
38	0		4		8		12		16		20		24		28		32		36	
40		2		6		10		14		18		22		26		30		34		38

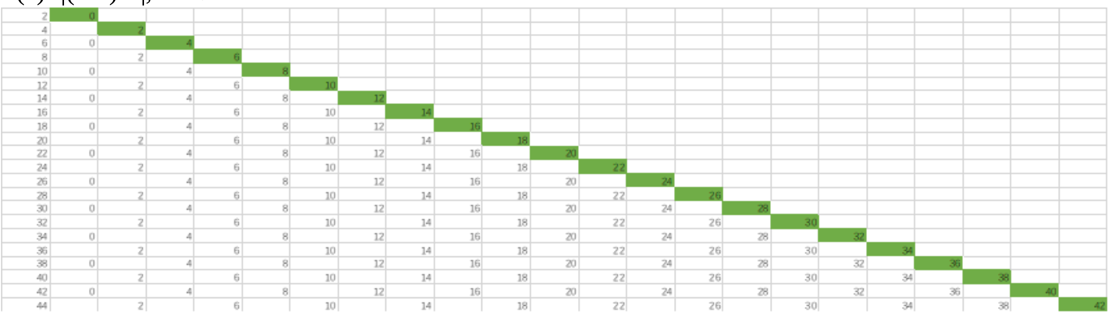
If f belongs to A, then {(a, P(a))|a=f} is noted as {L=f}.

If g belongs to B, then G={(bL, bR)|bL=g or bR=g} is noted as {R=g}.

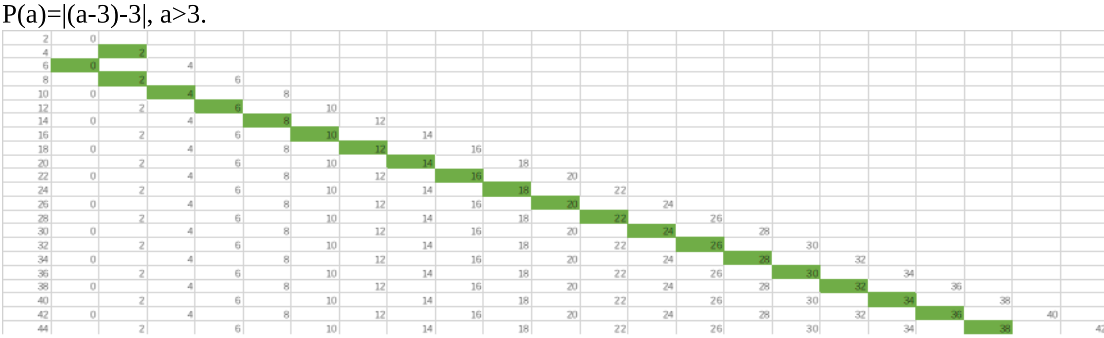
Any odd composite number belongs to (0, a) corresponds to one cell in {L=a};

P(a)=|(a-g)-g|, a>g.

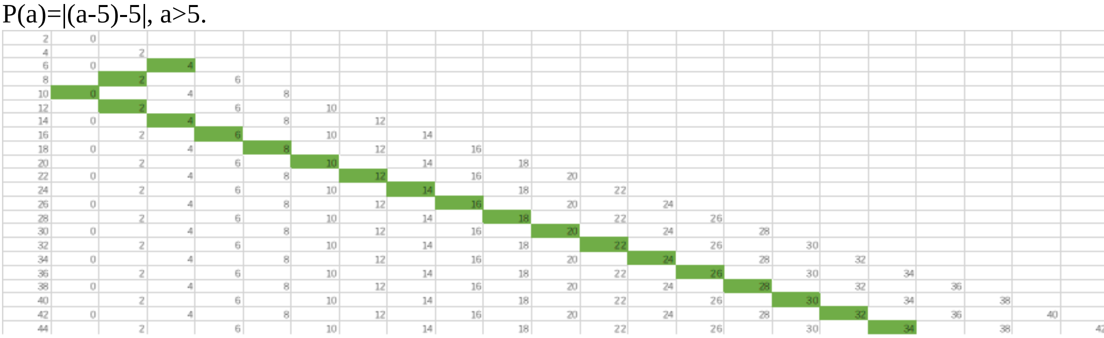
P(a)=|(a-1)-1|, a>1.



P(a)=|(a-3)-3|, a>3.



P(a)=|(a-5)-5|, a>5.

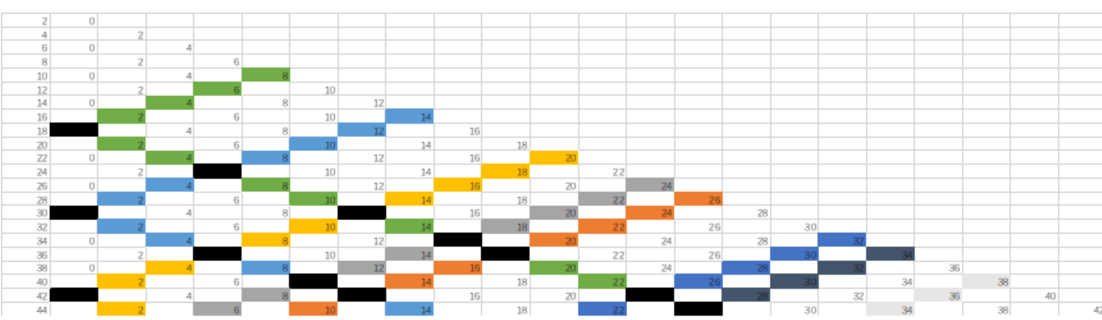


…

3. Analysis

3.1. U(a)-T(a)=S(a)-N(a)

Number in blank named cell, white the cells.



If bL and bR belong to C, then black the cell.

If bL or bR belongs to C, then color (non-black and non-white) the cell.

The number of white cells in {L=a} is noted as T(a),

The number of black cells in {L=a} is noted as U(a);

The number of colored (non-black and non-white) cells in {L=a} is noted as V(a).

The number of prime numbers in (0, a] is noted as I(a),

The number of odd composite numbers in (0, a] is noted as S(a).

V(a)+T(a)+U(a)=N(a), V(a)=S(a)-2*U(a).

{Any even number greater than 2 can be written as the sum of two prime numbers} can be noted as {Any T(a)>1, a>4.}.

3.2. H(a)=Z(a)/Y(a)

W={(bL, bR)|bL belongs to(0, a/2], bR belongs to [a/2, a).}

Card(bL, bR) is noted as W(a), W(a)=N(a)^2.

W(22)=36

2	0										
4		2									
6	0		4								
8		2		6							
10	0		4		8						
12		2		6		10					
14	0		4		8		12				
16		2		6		10		14			
18	0		4		8		12		16		
20		2		6		10		14		18	
22	0		4		8		12		16		20
24		2		6		10		14		18	
26	0		4		8		12		16		20
28		2		6		10		14		18	
30	0		4		8		12		16		20
32		2		6		10		14		18	

W(24)=36

2	0											
4		2										
6	0		4									
8		2		6								
10	0		4		8							
12		2		6		10						
14	0		4		8		12					
16		2		6		10		14				
18	0		4		8		12		16			
20		2		6		10		14		18		
22	0		4		8		12		16		20	
24		2		6		10		14		18		22
26	0		4		8		12		16		20	
28		2		6		10		14		18		22
30	0		4		8		12		16		20	
32		2		6		10		14		18		22
34	0		4		8		12		16		20	

If bL and bR belong to C, then the cell is noted as (cL, cR).

X={(cL, cR)|cL and cR belong to (0, a/2-1].};

Card(cL, cR) is noted as X(a), X(a)=S(a/2-1)*(S(a/2-1)+1)/2.

Y={(cL, cR)|cL belongs to (0, a/2] and cR belongs to [a/2, a-1].};

Card(cL, cR) is noted as Y(a), Y(a)=S(a/2)*(S(a-1)-S(a/2-1)).

If bL and bR belong to Y, then the cell is noted as (yL, yR).

Z={(yL, yR)|yL+yR belongs to (0, a].};

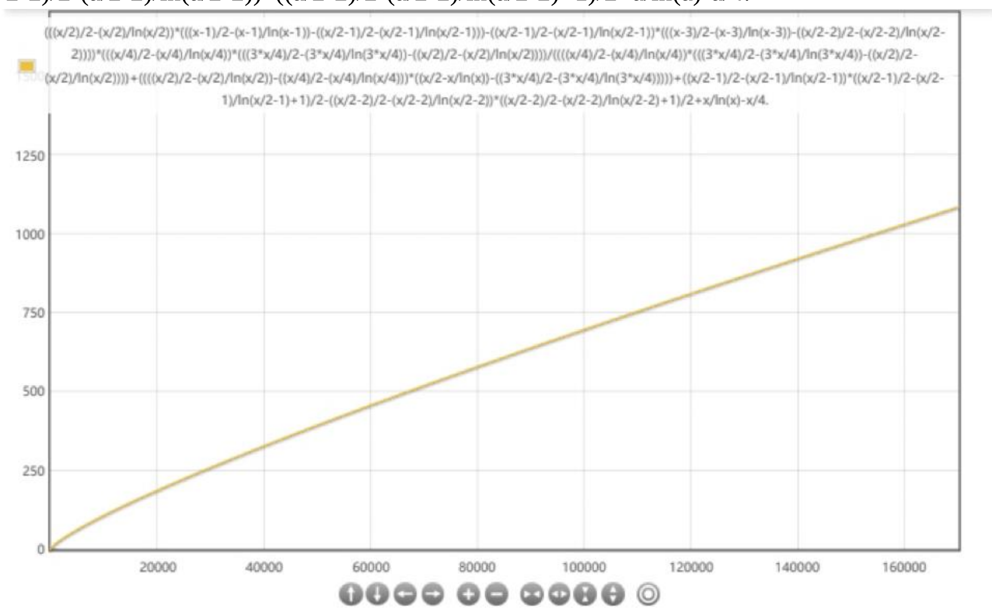
Card(yL, yR) is noted as Z(a), Z(a)=H(a)*Y(a).

3.3.1. H(a)~J(a)/(J(a)+K(a)), a>0.

J={(cL, cR)|cL belongs to (0, a/4] and cR belongs to (a/2, 3*a/4]};
Card(cL, cR) is noted as J(a), J(a)=S(a/4)*(S(3*a/4)-S(a/2)).
K={(cL, cR)|cL belongs to (a/4, a/2] and cR belongs to (3*a/4, a]};
Card(cL, cR) is noted as K(a), K(a)=(S(a/2)-S(a/4))*(S(a)-S(3*a/4)).
J(a)/(J(a)+K(a))~(((a/4)/2-(a/4)/ln(a/4))*(((3*a/4)/2-(3*a/4)/ln(3*a/4))-((a/2)/2-(a/2)/ln(a/2))))/((((a/4)/2-(a/4)/ln(a/4))*(((3*a/4)/2-(3*a/4)/ln(3*a/4))-((a/2)/2-(a/2)/ln(a/2))))+((((a/2)/2-(a/2)/ln(a/2))-((a/4)/2-(a/4)/ln(a/4)))*((a/2-a/ln(a))-((3*a/4)/2-(3*a/4)/ln(3*a/4))))).

3.3.2. T(a)~(Y(a)-Y(a-2))*J(a)/(J(a)+K(a))+X(a)-X(a-2)-S(a)+N(a), a>0.

T(a)~(Y(a)-Y(a-2))*J(a)/(J(a)+K(a))+X(a)-X(a-2)-S(a)+N(a);
T(a)~(((a/2)/2-(a/2)/ln(a/2))*(((a-1)/2-(a-1)/ln(a-1))-((a/2-1)/2-(a/2-1)/ln(a/2-1)))-((a/2-1)/2-(a/2-1)/ln(a/2-1))*(((a-3)/2-(a-3)/ln(a-3))-((a/2-2)/2-(a/2-2)/ln(a/2-2))))*(((a/4)/2-(a/4)/ln(a/4))*(((3*a/4)/2-(3*a/4)/ln(3*a/4))-((a/2)/2-(a/2)/ln(a/2))))/((((a/4)/2-(a/4)/ln(a/4))*(((3*a/4)/2-(3*a/4)/ln(3*a/4))-((a/2)/2-(a/2)/ln(a/2))))+((((a/2)/2-(a/2)/ln(a/2))-((a/4)/2-(a/4)/ln(a/4)))*((a/2-a/ln(a))-((3*a/4)/2-(3*a/4)/ln(3*a/4)))))+((a/2-1)/2-(a/2-1)/ln(a/2-1))*((a/2-1)/2-(a/2-1)/ln(a/2-1)+1)/2-((a/2-2)/2-(a/2-2)/ln(a/2-2))*((a/2-2)/2-(a/2-2)/ln(a/2-2)+1)/2+a/ln(a)-a/4.



Question: How many prime pairs are there in an even?
Answer: T(a), what has been explained above is the analytical approximation to T(a) and its image.

Postscript

Prime number theorem shows that the number of prime numbers has an approaching, and the above explanation shows that if the number of prime numbers has an approaching then the number of prime pairs in an even has an approaching. T(a) is monotonically increasing in the interval {a>17}, errors do not affect the monotonicity of the function. That is, any even number greater than 2 can be written as the sum of two prime numbers

Conflict of interest statement:

On behalf of all authors, the corresponding author states that there is no conflict of interest.

Data availability statement:

My manuscript has no associated data.

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