How to visualize divide by a bigger number

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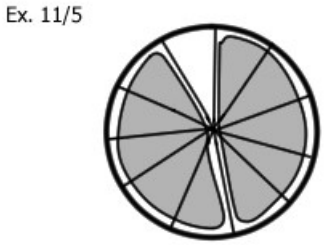
Abstract

How to visualize divide by a bigger number using modular arithmetic and the Pronic numbers.

1 Visualize divide by a smaller number

To visualize divide by a smaller number you take a circle and divide it into sections by the number of the numerator and than you can colour in the number of sections according to the number in the denominator.



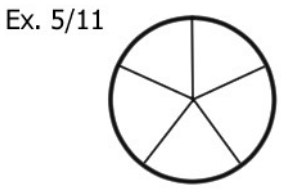
$\frac{11}{5} = 2$ with 1 remainder

$$= 2\frac{1}{5}$$
$$= 2.2$$

But this does not work when the denominator is bigger, because the denominator number is bigger than the amount you can section from the numerator.

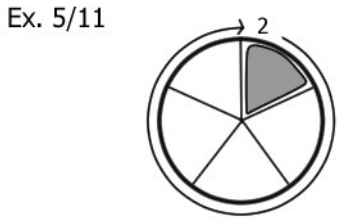
2 Visualize divide by a bigger number

To visualize divide by a bigger number take a circle and divide it into sections by the number in the numerator like before



Now go around the circle like you do in modular arithmetic which allows you to go around the circle multiple times to get to the number in the denominator.

Once around the circle is 5, the second time around the circle is another 5 and the third time around you have a remainder of 1. Colour in the remainder and show the number of turns around the circle.



To evaluate what this means think about going around the circle the first time as equaling 1. Say $\frac{5}{5}$ you would go around the circle once. After that you are taking away from 1. The second time around the circle $\frac{5}{10}$ you get $\frac{1}{2}$ this is equal to $1-\frac{1}{2}=\frac{1}{2}$. If you go around the circle 3 times $\frac{5}{15}$ you get $\frac{1}{3}$ this equals to $1-\frac{1}{2}-\frac{1}{6}=\frac{1}{3}$. 4 times around the circle $\frac{5}{20}$ equals $1-\frac{1}{2}-\frac{1}{6}-\frac{1}{12}=\frac{1}{4}$

A pattern emerges that is.

$$1-\frac{1}{2}-\frac{1}{6}-\frac{1}{12}-\frac{1}{20}-\frac{1}{30}-\frac{1}{42}-\frac{1}{56}\cdots$$

This is the reciprocals of the Pronic numbers.

$$\frac{1}{1(2)}-\frac{1}{2(3)}-\frac{1}{3(4)}-\frac{1}{4(5)}-\frac{1}{5(6)}-\frac{1}{6(7)}-\frac{1}{7(8)}\cdots$$

The remainder term is the remainder divided by the number of times around the circle multiplied by the denominator. The remainder term is subtracted after the series.

$$\frac{5}{11} = 1 - \frac{1}{2} - \frac{1}{2(11)}$$

$$= 1 - \frac{1}{2} - \frac{1}{22}$$

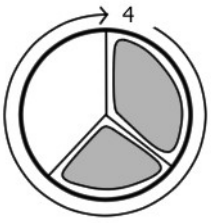
$$= 0.4545...$$

The formula can be reduced to

$$\frac{N}{D} = \frac{1}{q} - \frac{R}{q(D)}$$

Where q is the number of full turns around the circle , R is the remainder, D is the denominator and N is the numerator.

Ex. 3/14



$$\frac{3}{14} = \frac{1}{4} - \frac{2}{4(14)}$$

$$= \frac{1}{4} - \frac{2}{56}$$

$$=0.21428...$$