On the Polignac's Conjecture

Jean-Max CORANSON-BEAUDU, January 2021

Le Lamentin, Martinique, F.W.I,

jmcoranson@me.com

Abstract: The approach of this proof, is to show that whatever of the even number, it is always be decomposed into the difference of two odd numbers. Then, with the fundamental theorem of arithmetic, it can be show the necessary existence of a prime number *Pi* less than *2n* when *2n* is more than 3. We deduce that *2n* is the difference of a prime number and an odd number. An arithmetic sequence of parameter *n* and first term *Pi* will be constructed to deduce that there is at least one prime number in the terms of this sequence by Dirichlet-Lejeune's Theorem. We will show that whatever of *2n>3*, there are two prime numbers P_i and $-2n + P_i$ whose difference is equal to *2n*.

For n=1, i.e 2=$P_{i+1} - P_i$ the twin prime's conjecture is demonstrated.

Key words: Twin Prime number, Polignac conjecture

Introduction

The prime numbers have fascinated men for millennia. From China through Ancient Egypt and passing by Europe. The immediate utility of prime numbers is cryptography and more recently, links between quantum physics and prime number have been discussed (Montgomery and Dyson 1970). Some are still debating the usage of prime number within the string theory (Kurokawa 1986). The notion of prime number is related to the study of the multiplier structure of the ring of relative integers that define one of the foundations of mathematics.

The Polignac's Conjecture is expressed as following:

$$\forall\ 2n > 3 \text{ there is an infinit couple } (p, q) \text{ of prime } number\ such\ that\ 2n = p - q$$

At the origin, in 1849, Alphonse de Polignac [1] made the more general conjecture that for every natural number n, there are infinitely many primes p such that p + 2n is also prime. The case n = 1 of de Polignac's conjecture is the twin prime conjecture.

This apparently simple problem turns out to be of great difficulty to demonstrate. In 2014 an important breakthrough was made by Zhang Yitang [2] who proved that there were an infinite number of main gaps of size n for a value of n <70,000,000. Later that year, James Maynard announced that there are an infinite number of main gaps of a certain size up to 600. From 2015, one year after Zhang's announcement, according to the Polymath project wiki, the main gap has been reduced to 246; assuming the Elliott - Halberstam conjecture and its generalized form, the gap can be reduced to 12 and 6, respectively.

Moreover, computers have allowed us to confirm this conjecture to gigantic numbers. Obviously, this is not proof, but all the same, a good tendency. Many mathematical articles can give hope that Polignac's conjecture is true. This is the case of Zhang Yitang [2] (2014) and Alphonse de Polignac [1] (1849) mentioned previously, but also of J. Maynard (2015) [3]. Recently, the work of Mbakiso F.

Mothebe , Dintle N. Kagiso and Ben T. Modise (2020) [4] showed that hat for all integers n sufficiently large there is a arithmetical sequence witch contain primes number who verifying Polignac conjecture.

Evidence

Notation

Either $P_1, P_2, P_3, P_4, P_5, \ldots.$ the continuation of the prime numbers 2,3,5,7,11,.....

Either $\mathbb{P}$ the set of the prime numbers.

Whatever of the common even number $2n$, $2n > 3$ it is possible to break it down into two odd numbers.

$$\exists\, a, b \in \mathbb{N} \text{ odd } \; such\ that \;\; 2n = a - b$$

The prime number's theorem demonstrated by Jacques Hadamard and Charles-Jean de La Vallée Poussin [5] shows that whatever of the number $2n > 3$, there are primes numbers. $P_i < 2n$

It is then possible to break down *2n* by the difference of a prime number and an odd number:

$2n = P_i - b$ with $P_i \neq 2$ and b is odd number.

Step1:$\forall\, n \in \mathbb{N}, 2n > 3, \exists\, P_i \in \mathbb{P}$,
and b an odd number such that

$$b = P_i - 2n$$

We deduce the existence of a numerical sequence $S_n(P_i)$ of numbers $\{3 - 2n, 5 - 2n, 7 - 2n, \ldots P_i - 2n, \ldots P_{Max} - 2n\}$ for any *2n>3.*

Step2: The objective is to show the following proposal, which is an equivalent formulation of Polignac's conjecture:

$\mathcal{P}_G$"There is at least a prime number in the suite $P_i - 2n$ whatever of the even number *2n>3*, knowing that P_i is prime. »

This proposal can be also:

$$\mathcal{P}_G: \forall\, n \in \mathbb{N} \text{ with } 2n > 3, \exists\; P_i \in \mathbb{P}$$

$$such\ than\ b = P_i - 2n \in \mathbb{P}$$

To demonstrate that this proposal is true, suppose it is wrong and deduce the contradictory consequences.

Note $\neg\mathcal{P}_G$ the denial of the hypothesis $\mathcal{P}_G$

(If $\mathcal{P}_G$is true then $\neg\mathcal{P}_G$ is false and if is $\mathcal{P}_G$ is false then $\neg\mathcal{P}_G$is true.)

Suppose $\neg\mathcal{P}_G$ it true

$$\neg\mathcal{P}_G \;:\; \exists\, n \in \mathbb{N} \text{ with } 2n > 3, \forall\; P_i \in \mathbb{P}$$

$$such\ than\ b = P_i - 2n \notin \mathbb{P}$$

Step3

Let us express as an arithmetic progression, the numerical sequence $S_n(P_i) = P_i - 2n$

$n \leq \frac{P_i - 3}{2}$, which represents the same set of integers as in **Step1**

	P_i	3	5	7	11	13	17	19	23	…
n	2n	3-2n	5-2n	7-2n	11-2n	13-2n	17-2n	19-2n	23-2n	…-2n
2	4			3	7	9	13	15	19	…
3	6				5	7	11	13	17	…
4	8				3	5	9	11	15	…
5	10					3	7	9	13	…
6	12						5	7	11	…
7	14						3	5	9	…
8	16							3	7	…
9	18								5	…
10	20								3	…
11	22									…
12	24									…
13	26									…
…	…	…	…	…	…	…	…	…	…	…

Table of Sequences - In horizontal reading the sequence $S_n(P_i)$ and in vertical reading the arithmetic sequence of reason -2 and first term P_i .

The theorem of Lejeune Dirichlet (1949) [6] proves that in this arithmetic sequence of reason -2 and first term P_i , it is existing at least a prime number.

That is,

$$\exists\, n \in \mathbb{N} \text{ with } 2n > 3, \forall\ P_i \in \mathbb{P}\ such\ than\ S_n(P_i) = P_i - 2n \in \mathbb{P}$$

Which contradicts the hypothesis, $\neg\mathcal{P}_G$ so $\neg\mathcal{P}_G$ is wrong what leads to $\mathcal{P}_G$ is true.

In conclusion

$$\forall\, n \in \mathbb{N} \text{ with } 2n > 3, \exists\ P_i\ and\ P_k \in \mathbb{P}$$

$$such\ than\ P_k = P_i - 2n \in \mathbb{P}$$

This demonstrates the following proposition:

$$\boxed{\begin{array}{c}\forall\, n \in \mathbb{N}, 2n > 3, \exists\, P_k\, , P_i\ \ such\ that \\ 2n = P_k - P_i\ \ with\ P_k\ and\ P_i\ prime\end{array}}$$

It's Polignac's theorem.

CONFLICT OF INTERESTS

The author has not declared any conflict of interests.

References

[1] Alphonse de Polignac, Recherches nouvelles sur les nombres premiers, Comptes Rendus des Sciences de l'Academie des Sciences, 1849.

[2] Zhang Yitang, Bounded gaps between primes, Annals of Mathematics, 179(3) 1121-1174 (2014).

[3] J. Maynard, Small gaps between primes, Annals of Mathematics, 181(2) , 383-413, (2015).

[4] Mbakiso F. Mothebe , Dintle N. Kagiso and Ben T. Modise , A proof of cases of de Polignac's conjecture, http://arxiv.org/abs/1912.09290v4, 2020

[5] Charles-Jean de La Vallée Poussin. On the function ζ(s) de Riemann et le nombre des nombres premiers inférieurs à une limite donnée, mémoires couronnées et autres mémoires in-8-publiées par l'Académie Royale de Belgique, 49 (1899), 74 pp.

[6] Atle Selberg (1949), "An Elementary Proof of Dirichlet's Theorem About Primes in an Arithmetic Progression". Annals of Mathematics Second Series, Vol. 50, No. 2 (Apr., 1949), pp. 297-304.