The exterior connections

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February 15, 2021

Abstract

We define the notion of a exterior connection which is a connection for the exterior forms.

1 The Koszul connections

Let $M$ be a manifold and $E$, a vector fiber bundle over $M$. The Koszul connections are operators acting on sections such that:

$$\nabla_X (f.s) = X f.s + f. \nabla_X (s)$$

$f$ is a smooth function and $s$ is a section of the fiber bundle $E$.

2 The exterior connections

2.1 Definition

An exterior fiber bundle is a vector fiber bundle which is a modulus for the exterior algebra. The exterior connections $\nabla$ are such that:

$$(\alpha \wedge \beta).s = \alpha.(\beta.s)$$

$$\nabla(\alpha.s) = d\alpha.s + (-1)^{\deg(\alpha)} \alpha. \nabla(s)$$

with $\alpha, \beta \in \Lambda^*(TM)$ and $s$ a section of $E$.

2.2 Curvature

The curvature of the connection is $R = \nabla \circ \nabla$ and is linear, such that:

$$R(\alpha.s) = \alpha.R(s)$$

2.3 Characteristic classes

The characteristic classes are defined in the cohomology:

$$c_k = tr(R^k)$$
References