Integrated Formulas of the Fine-structure Constant and Feigenbaum Constants (viXra:2021.0162v2)

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper is a subsequent paper to the previous paper “Formulas of Feigenbaum Constants and Their Physical Meanings” (viXra:2101.0187). In the previous paper, some formulas of Feigenbaum constants in fractional number format were given and the physical meanings of the factors in the formulas were exhibited, especially their relationships with nuclides, the fine-structure constant and $2\pi$. In the previous paper, some integrated formulas of the fine-structure constant, Feigenbaum constants and $2\pi$ were also given, briefly denoted as $\alpha_1\delta(2\pi)\approx1$, and their relationships with nuclides were illustrated. In this paper, some formulas for $\alpha_1\delta(2\pi)\approx1$ are supplemented, some formulas for $\alpha_2(\delta\alpha)^2\approx1$, $[\alpha_1(2\pi)]/(\alpha_2\alpha^2)\approx1$ and $(2\pi)/\alpha^2\approx1$ are given, some formulas of the fine-structure constant ($\alpha_1$ and $\alpha_2$) based on the key number 103 instead of 112, 173, 137, 83 and 29 are supplemented. In the end, by introducing correction factors $\gamma_1$, $\gamma_2$ and $\gamma$, accurate formulas $\alpha_1(\delta\gamma_1)^2(2\pi)=1$, $\alpha_2(\delta\alpha\gamma_2)^2=1$ and $2\pi/(\alpha\gamma)^2=1$ are gained.

**Keywords:** Formulas; the fine-structure constant; Feigenbaum constants; $2\pi$.

1. Introduction

In our previous papers\textsuperscript{1,2,3,4,5}, we gave or exhibited the following formulas.

\[
(2\pi)_{\text{Chen-}k} = e^2 \frac{e^2}{2!} \frac{e^2}{3!} \frac{e^2}{4!} \cdots \frac{e^2}{(k+1)!}; \quad (2\pi)_{\text{Wallis-}k} = 4 \cdot \left( \frac{2}{3} \frac{4}{5} \frac{6}{7} \cdots \frac{2k}{2k+1} \right),
\]

\[
(2\pi)_{\text{GL-}k} = 8 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + (-1)^{k+1} \frac{1}{2k+1}) \quad (GL \text{ means Gregory-Leibniz})
\]

\[
(2\pi)_{\text{NC-}k} = 6 + \sum_{n=1}^{k} \frac{(-1)^{n+1}}{n(n+1/2)(n+1)} \quad (NC \text{ means Nilakantha-Chen})
\]
\[
\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7 \cdot (2\pi) \mathrm{Chen}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435
\]
\[
\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13 \cdot (2\pi) \mathrm{Chen}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} = 1/137.035999111818
\]
\[
c_{au} = \frac{1}{\alpha_2} = \frac{1}{\alpha \alpha_2} = \sqrt{\frac{112 \times (168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{14 \cdot 112 \cdot (2 \cdot 173 + 1)})}{1}} = 137.035999074626
\]
\[
1/\alpha_1 = 56 + 81 + \frac{1}{28 - \frac{13 \cdot (2 \cdot 56 \cdot 11 - 1)}{3 \cdot 5 \cdot (2 \cdot 56 \cdot 43 + 1)}} = 137.03599907435
\]
\[
1/\alpha_2 = 56 + 81 + \frac{1}{28 - \frac{2 \cdot (16 \cdot 27 - 1)}{3 \cdot (16 \cdot 81 + 1)}} = 137.035999111818
\]
\[
c_{au} = \frac{1}{\alpha_2} = 56 + 81 + \frac{1}{28 - \frac{5 \cdot (4 \cdot 3 \cdot 7 \cdot 17 - 1)}{2 \cdot 5 \cdot (4 \cdot 5 \cdot 7 \cdot 23 + 1) + 1}} = 137.035999074626
\]
\[\text{Note: } c_{au} \text{ refers to the speed of light in vacuum in atomic units} \]

Feigenbaum Constants: \( \delta = 4.66920160910299 \)
\[\alpha = 2.50290787509589 \]
\[
\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.21416937706236
\]
\[
= \frac{1}{4} - \frac{1}{27} + \frac{1}{4 \cdot 9 \cdot 23} - \frac{1}{2 \cdot 3 \cdot 7 \cdot 23 \cdot (2 \cdot 3 \cdot (4 \cdot 3 \cdot 11 - 1) + 1) + \frac{2 \cdot 23}{3 \cdot 19}}
\]
\[
\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135
\]
\[
= \frac{1}{2} - \frac{1}{9} + \frac{1}{3 \cdot 31} - \frac{1}{23 \cdot (8 \cdot 3 \cdot 17 + 1)} + \frac{1}{17 \cdot 23 \cdot (8 \cdot 3 \cdot 11^4 - 1)}
\]
\[\text{Note: } 136=8 \cdot 17, 138=6 \cdot 23 \]
\[\alpha \cdot \delta^2 (2\pi) \approx 1 \]

On Feb. 8, 2021, we also noticed that Hieb uploaded a paper\(^6\) in viXra in April of 2017, and gave an approximate formula of the fine-structure constant and Feigenbaum constant as follows, but without any explanations to its physical meanings.
\[
\delta' = (1 / (2\pi \alpha))^{1/2} = 4.670114 \approx \delta = 4.669201609
\]
\[\delta' - \delta = 0.000912 \]
\[\alpha : \text{ the fine-structure constant, } \alpha \approx 1/137.036 \]
2. Integrated Formulas of $\alpha_1$, $\delta$ and $2\pi$

A Concise Deduction

The Fine-structure Constant:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7 \cdot (2\pi)_{Chen-112}} \cdot \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

Feigenbaum Constants: $\delta = 4.66920160910299$

$$\alpha_1 = \frac{36}{7 \cdot (2\pi)_{Chen-112}} \cdot \frac{1}{112 + \frac{1}{75^2}} \approx \frac{36}{7 \cdot (2\pi)_{Chen-112}} \cdot \frac{1}{112} = \left(\frac{3}{14}\right)^2 \cdot \frac{1}{2\pi} \approx \frac{1}{\delta^2 (2\pi)}$$

$$\approx \frac{1}{4.66920160910299^2 \cdot (2 \cdot 3.14159265358979)} \approx 136.982$$

So it should be reasonable to assume the following approximate formulas:

$$\alpha_1, \delta^2 (2\pi) \approx 1 \text{ or } \frac{1}{\alpha_1 \delta^2 (2\pi)} \approx 1$$

Numerically: $\alpha_1 \delta^2 (2\pi) = \frac{4 \cdot 6692^2 \times 6.2832}{137.036} = 0.99961 \approx 1$

2021/ 2/1-3

The above approximate formula $\alpha_1 \delta^2 (2\pi) \approx 1$ is assumed to be the brief form of integrated formulas of $\alpha_1$, $\delta$ and $2\pi$. There should be some corresponding accurate forms of integrated formulas of $\alpha_1$, $\delta$ and $2\pi$ as follows.

$$\alpha_1 \delta^2 (2\pi)_{Chen-2517} = \frac{4.66920160910299^2 \cdot (e^2 \cdot \frac{e^2}{2} \cdot \frac{e^2}{3} \cdot \frac{e^2}{5} \cdot \frac{e^2}{7} \cdots)}{(2 \cdot 3 \cdot 71 \cdot 2 \cdot 5 \cdot 23 \cdot 4 \cdot 37 \cdot 2 \cdot 4 \cdot 36 \cdot 47 \cdot 48 \cdot 50 \cdot 46 \cdot 59 \cdot 64 \cdot 63 \cdot 64 \cdot 68 \cdot 71 \cdot 82 \cdot 8 \cdot 8 \cdot 9 \cdot 8 \cdot 7 \cdot 8 \cdot 16 \cdot 51 \cdot 128 \cdot 9 \cdot 25 \cdot 17 \cdot 14, 1 \cdot 7 \cdot 4.66920160910299^2 \cdot 6.28564399787948}}$$

$$= \frac{1}{137.035999037435}$$

$$= \frac{1}{128.9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} = 1.0000008354717 \approx 1$$

2021/1/31
\[
\alpha \delta^2(2\pi)_{\text{Chen-2517}} = \frac{1}{137.035999037435} \cdot 4.66920160910299^2 \cdot \left( e^2 \frac{e^2}{2^1} \frac{e^2}{3^2} \frac{e^2}{4^3} \cdots \frac{e^2}{(2\cdot3\cdot71)^{23\cdot37}} \right)
\]

\[
= \frac{1}{4.66920160910299^2 \cdot 6.28564399787948} = \frac{1}{0.999999164529037} = 1
\]

\[
\alpha \delta^2(2\pi)_{\text{Wallis-971}} = \frac{1}{37.035999037435} \cdot 4.66920160910299^2 \cdot 6.28564015562186
\]

\[
= \frac{1}{137.035999037435} \cdot 4.66920160910299^2 \cdot 6.28564015562186
\]

\[
= \frac{1}{0.999999775803991} \approx 1
\]
\[ \alpha_i \delta^2 (2\pi)_{GL-22.37} = \frac{4.66920160910299^2 \cdot 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 2 \cdot 11 \cdot 37 + 1}\right)}{137.035999037435} \]

\[ = \frac{4.66920160910299^2 \cdot 6.28563929398602}{137.035999037435} \]

\[ = 1 + \frac{1}{163 \cdot (6 \cdot 11^2 \cdot 97 + 1) + \frac{9}{10}} = 1.00000008711598 \approx 1 \]

2021/2/1

\[ \alpha_i \delta^2 (2\pi)_{GL-22.37} = \frac{4.66920160910299^2 \cdot 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 2 \cdot 11 \cdot 37 + 1}\right)}{137.035999037435} \]

\[ = \frac{4.66920160910299^2 \cdot 6.28563929398602}{137.035999037435} \]

\[ = 1 - \frac{1}{9 \cdot 11 \cdot 47 \cdot (2 \cdot 9 \cdot 137 + 1) - \frac{2}{25}} = 0.999999912884025 \approx 1 \]

2021/2/19

\[ \alpha_i \delta^2 (2\pi)_{NC-3} = \frac{4.66920160910299^2 \cdot (6 + \sum_{n=1}^{3} \frac{(-1)^{n+1}}{n(n+1/2)(n+1)})}{137.035999037435} \]

\[ = \frac{4.66920160910299^2 \cdot 6.29047619047619}{137.035999037435} \]

\[ = 1 + \frac{1}{3 \cdot (2 \cdot 7 \cdot 31 + 1) - \frac{1}{13 \cdot 89 \cdot (2 \cdot 37 \cdot 53 + 1) - \frac{2 \cdot 9}{25}}} = 1.000769602623252 \approx 1 \]

2021/2/1,3

\[ \alpha_i \delta^2 (2\pi)_{NC-3} = \frac{4.66920160910299^2 \cdot (6 + \sum_{n=1}^{3} \frac{(-1)^{n+1}}{n(n+1/2)(n+1)})}{137.035999037435} \]

\[ = \frac{4.66920160910299^2 \cdot 6.29047619047619}{137.035999037435} \]
\[ \alpha_0 \delta^2(2\pi) = \frac{4.66920160910299^2 \cdot (2 \cdot 3.14159265358979)}{137.035999037435} \]

\[ = 1 - \frac{1}{1.97} + \frac{1}{2 \cdot 4.7 \cdot (4.5 \cdot 19.31 - 1)} \approx 0.9996096754323 \approx 1 \]

\[ \alpha_2 = 1 + \frac{1}{512.5 \cdot 4 \cdot 9.7 \cdot 17 \cdot 19 \cdot 83} \approx 1.00039047698053 \approx 1 \]

\[ \alpha_2(\delta) = \frac{1}{1 \cdot 1 \cdot 1 \cdot 1} = 1/136.575 \]

\[ \alpha_2(\delta) \approx 1 \text{ or } \frac{1}{\alpha_2(\delta)} \approx 1 \]

\[ \text{Numerically: } \alpha_2(\delta) = \frac{(4.6692 \times 2.5029)^2}{137.036} = 0.99664 \approx 1 \]

2021/2/7

3. Integrated Formulas of \( \alpha_2, \delta \) and \( \alpha \)

The Fine-structure Constant:

\[ \alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13 \cdot (2\pi)_{\text{Chen}-278}}{100} \]

\[ = 1/137.035999111818 \]

Feigenbaum Constants: \( \delta = 4.66920160910299 \)

\[ \alpha = 2.502907878509589 \]

\[ \alpha_2 = \frac{13 \cdot (2\pi)_{\text{Chen}-278}}{100} \approx 13 \cdot (2\pi) \frac{1}{100} \approx \frac{(2\pi)}{100} \approx \frac{(\delta \alpha)^2}{(2\pi)} \]

\[ \approx \frac{1}{(\delta \alpha)^2} = \frac{1}{(4.66920160910299 \cdot 2.502907878509589)^2} \approx 1/136.575 \]

So it should be resonable to assume the following approximate formulas:

\[ \alpha_2(\delta \alpha)^2 \approx 1 \text{ or } \frac{1}{\alpha_2(\delta \alpha)^2} \approx 1 \]

2021/2/7
The above approximate formula $\alpha(\delta\alpha)^2 \approx 1$ is assumed to be the brief form of integrated formulas of $\alpha_2$, $\delta$ and $\alpha$. There should be some corresponding accurate forms of integrated formulas of $\alpha_2$, $\delta$ and $\alpha$ as follows.

$$\alpha_2(\delta\alpha)^2 = \frac{(4.66920160910299 \cdot 2.50290787509589)^2}{137.035999111818}$$

$$= 1 - \frac{1}{2.149} + \frac{1}{29 \cdot 31 \cdot (2 \cdot 3.49 \cdot 13 + 1) - 16} = 0.996644586263908 \approx 1$$

$\frac{1}{\alpha_2(\delta\alpha)^2} = \frac{1}{137.035999111818}$

$$= 1 + \frac{1}{27 \cdot 11} - \frac{1}{2 \cdot 3.25 \cdot 11 \cdot (4 \cdot 11.47 + 1) - \frac{2}{5}}$$

$$= 1 + \frac{1}{27 \cdot 11} - \frac{1}{2 \cdot 3.25 \cdot 11 \cdot (2.9 \cdot 5 \cdot 23 + 1) - \frac{2}{5}} = 1.00336671044256 \approx 1$$

$\frac{1}{\alpha_2(\delta\alpha)^2}$

$\frac{\alpha_1(\delta\alpha)^2}{\alpha_2(\delta\alpha)^2} = \frac{\alpha_1(2\pi)}{\alpha_2(2\pi)} \approx \frac{2\pi}{\alpha_2(\delta\alpha)^2} \approx 1.002975 \approx 1$

2021/2/11

4. Integrated Formulas of $a_1$, $a_2$, $a$ and $2\pi$

$$\alpha_1(2\pi) = 0.99961 \approx 1$$

$$\alpha_2(\delta\alpha)^2 = 0.99664 \approx 1$$

$$\alpha_1(\delta\alpha)^2 = \frac{\alpha_1(2\pi)}{\alpha_2(2\pi)} \approx \frac{2\pi}{\alpha_2(\delta\alpha)^2} \approx 1.002975 \approx 1$$

2021/2/11
5. Marvelous Coincidences

There are some marvelous coincidences of factors with nuclides in the above formulas. One typical example of these coincidences is listed as follows, which indicates the methodology and the formulas in this paper should be correct.
6. Formulas of the Fine-structure Constant based on 103

In our previous paper\(^1\), many formulas of the fine-structure constant based on the key numbers 112, 173, 137, 83 and 29 were given. As shown in the above two formulas in Section 5, it seems 103 is another key number comparable to the above stated key numbers, so some formulas of the fine-structure constant based on the key number 103 instead of them are constructed as follows.

\[
\alpha_1 \approx \frac{\alpha_\delta^2 (2\pi)_{Chen-2517} = 1 + \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} = 1.0000008354717}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} = 1.0000008354717 \approx 1}
\]

\[
\alpha_\delta^2 (2\pi)_{Chen-2517} = 1 - \frac{1}{2 \cdot 5 \cdot 7 \cdot (2 \cdot 83 \cdot 103 + 1) - \frac{9}{29}} = 0.999999164529037 \approx 1}
\]

\[
(2\pi)_{Chen-2517} = e^2 \left( \frac{e^2}{(\frac{1}{2})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots \frac{e^2}{(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17})^{23.37}} \right)
\]

\[
\frac{28,29,30}{29,31} Si, 14,15,16 \frac{35,37}{37} Cl, 18,20 \frac{50,51}{51} V, 27,28 \frac{63,65}{65} Cu, 34,36 \frac{79,81}{81} Br, 44,46 \frac{82,83,84,86}{86} Kr, 46,47,48,50
\]

\[
\frac{85,87}{87} Rb, 48,50 \frac{103}{103} Rh, 58 \frac{112}{112} Cd, 64 \frac{128}{128} Xe, 74 \frac{136,137,138}{138} Br, 44,46 \frac{140,142}{142} Ce, 82,84 \frac{173}{173} Yb, 103 \frac{209}{209} Bi
\]

\[
\frac{84,85}{85} Po, 125 \frac{9,29,26,29}{29} At
\]

\[
\frac{112}{112} Lr^* , 173 \frac{344,143,129}{129} Fy^ie, 208,209,210 \frac{434,152,29}{29} Ch^ie, 173 \frac{9,29,262}{262}
\]

\[
1 \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} = 1.0000008354717 \approx 1}
\]

\[
\frac{1}{2 \cdot 5 \cdot 7 \cdot (2 \cdot 83 \cdot 103 + 1) - \frac{9}{29}} = 0.999999164529037 \approx 1}
\]

\[
(2\pi)_{Chen-2517} = e^2 \left( \frac{e^2}{(\frac{1}{2})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots \frac{e^2}{(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17})^{23.37}} \right)
\]

\[
\frac{28,29,30}{29,31} Si, 14,15,16 \frac{35,37}{37} Cl, 18,20 \frac{50,51}{51} V, 27,28 \frac{63,65}{65} Cu, 34,36 \frac{79,81}{81} Br, 44,46 \frac{82,83,84,86}{86} Kr, 46,47,48,50
\]

\[
\frac{85,87}{87} Rb, 48,50 \frac{103}{103} Rh, 58 \frac{112}{112} Cd, 64 \frac{128}{128} Xe, 74 \frac{136,137,138}{138} Br, 44,46 \frac{140,142}{142} Ce, 82,84 \frac{173}{173} Yb, 103 \frac{209}{209} Bi
\]

\[
\frac{84,85}{85} Po, 125 \frac{9,29,26,29}{29} At
\]

\[
\frac{112}{112} Lr^* , 173 \frac{344,143,129}{129} Fy^ie, 208,209,210 \frac{434,152,29}{29} Ch^ie, 173 \frac{9,29,262}{262}
\]
\[ \alpha_1 = \frac{137}{29 \cdot 4 \cdot (2 \cdot 4 \cdot 6 \cdot 3 \cdot 5 \cdot 5 \cdot 1548 \cdot 2 \cdot 2 \cdot 9 \cdot 43 + 1)} + \frac{1}{103 + 2 \cdot 3 \cdot 5 \cdot (2 \cdot 3 \cdot 7 \cdot 13 - 1) - \frac{3}{17}} \]

\[ = \frac{1}{173.035999037435} \]

\[ \alpha_1 = \frac{137}{29 \cdot 8 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 17 \cdot 29 + 1})} + \frac{1}{103 + \frac{1}{7 \cdot (4 \cdot 7 \cdot 199 + 1) + \frac{4}{7}}} \]

\[ = \frac{1}{173.035999037435} \]

\[ \alpha_1 = \frac{28 \cdot 29 \cdot 30}{14 \cdot 15 \cdot 16} + \frac{35 \cdot 37 \cdot 39 \cdot 40 \cdot 41}{17 \cdot 18 \cdot 20 \cdot 21 \cdot 22} + \frac{63 \cdot 65 \cdot 67 \cdot 69 \cdot 71}{29 \cdot 31 \cdot 33 \cdot 35 \cdot 37} + \frac{8 \cdot 11}{4 \cdot 7} \]

\[ = \frac{1}{173.035999037435} \]

\[ \alpha_1 = \frac{23 \cdot 35}{12 \cdot 11} + \frac{39 \cdot 40 \cdot 41 \cdot 42}{19 \cdot 20 \cdot 21 \cdot 22} + \frac{55 \cdot 56}{25 \cdot 26} + \frac{83 \cdot 84 \cdot 85 \cdot 86}{63 \cdot 64 \cdot 65 \cdot 66} + \frac{119 \cdot 120 \cdot 121 \cdot 122}{99 \cdot 100 \cdot 101 \cdot 102} \]

\[ = \frac{1}{173.035999037435} \]

\[ \alpha_2 = \frac{25 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2 \cdots}{e^2} \frac{\left( \frac{2}{1} \right)^3 \left( \frac{3}{2} \right)^3 \left( \frac{4}{3} \right)^3 \cdots}{\left( \frac{2500}{3 \cdot 49.17} \right)} = \frac{1}{11.19} \]

\[ \alpha_2 = \frac{1}{11.19} \left( 103 - \frac{32 \cdot (512 \cdot 25 - 1) + \frac{3}{10}}{} \right) \]

\[ = \frac{1}{173.0359991111818} \]

\[ \alpha_2 = \frac{25 \cdot 4 \cdot (2 \cdot 4 \cdot 6 \cdot 3 \cdot 5 \cdot 5 \cdots)}{7496 \cdot 2 \cdot 23 \cdot 163} = \frac{1}{11.19} \left( 103 - \frac{7 \cdot (16 \cdot 3 \cdot 19 \cdot 257 - 1)}{} \right) \]

\[ = \frac{1}{173.0359991111818} \]
7. Integrated Formulas of $\alpha_1$, $\delta$, $2\pi$ and $\gamma_1$

By introducing a correction factor $\gamma_1$, some integrated formulas of $\alpha_1$, $\delta$, $2\pi$ and $\gamma_1$ in the format of $\alpha_1(\delta\gamma_1)^2(2\pi)=1$ could be obtained as follows.

\[ \alpha_1 = \sqrt{2\pi\alpha_1\delta} = \sqrt{\frac{137.035999037435}{1}} = \frac{1}{2.3\cdot13.197} \]

\[ = (1 + \frac{1}{2.3\cdot13.197})^2 = 1.00019521943495 \approx 1 \]

\[ \frac{1}{\gamma_1} = \sqrt{2\pi\alpha_1\delta} = \sqrt{\frac{6.28564399787948}{137.035999037435}} \]

\[ = (1 - \frac{1}{47.109} + \frac{1}{27.7\cdot(3-8\cdot(3-8\cdot(4-137-1)-1))})^2 = 0.999804818668238 \]

2021/2/28
\[ \alpha_1(\delta^2 \gamma_{\text{Chen}_{-2517}})^2(2\pi)_{\text{Chen}_{-2517}} = 1 \]

\[ \gamma'_{\text{Chen}_{-2517}} = \frac{1}{\sqrt{(2\pi)_{\text{Chen}_{-2517}}} \alpha_1 \delta} \]

\[ = \frac{1}{\sqrt{137.035999037435}} \cdot \frac{2}{\sqrt{2} \cdot \frac{\left( \frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdots \cdot \frac{23}{22} \right)}{25 \cdot 17}} \cdot 4.66920160910299 \]

\[ = \frac{1}{\sqrt{137.035999037435}} \cdot \frac{2}{\sqrt{2} \cdot \frac{\left( \frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdots \cdot \frac{23}{22} \right)}{25 \cdot 17}} \cdot 4.66920160910299 \]

\[ = \frac{1}{2} \cdot 79 \cdot 109 \cdot 139 + \frac{15}{17} \text{or} \frac{22}{25} = 0.99999582264432 \]

\[ \gamma'_{\text{Chen}_{-2517}} = \frac{1}{\sqrt{(2\pi)_{\text{Chen}_{-2517}}} \alpha_1 \delta} \]

\[ = 1 + \frac{1}{2} \cdot 79 \cdot 109 \cdot 139 - \frac{2}{17} \cdot \frac{3}{25} = 1.0000004177354 \]

\[ \gamma'_{\text{Chen}_{-2517}} = \frac{1}{\sqrt{(2\pi)_{\text{Chen}_{-2517}}} \alpha_1 \delta} \]

\[ = 1 - \frac{1}{2} \cdot 79 \cdot 109 \cdot 139 + \frac{15}{17} \text{or} \frac{22}{25} = 0.999999887901990 \]

\[ \alpha_1(\delta^2 \gamma_{\text{Wallis}_{-971}})(2\pi)_{\text{Wallis}_{-971}} = 1 \]

\[ \gamma'_{\text{Wallis}_{-971}} = \frac{1}{\sqrt{(2\pi)_{\text{Wallis}_{-971}}} \alpha_1 \delta} \]

\[ = \frac{1}{\sqrt{137.035999037435}} \cdot \frac{4 \cdot (2 \cdot \frac{4}{3} \cdot \frac{5}{4} \cdots \cdot \frac{1279}{1278} \cdot \frac{1280}{1279} \cdot 2 \cdot \frac{9}{71} + 1)}{25 \cdot 17} \cdot 4.66920160910299 \]

\[ = \frac{1}{\sqrt{137.035999037435}} \cdot \frac{4 \cdot (2 \cdot \frac{4}{3} \cdot \frac{5}{4} \cdots \cdot \frac{1279}{1278} \cdot \frac{1280}{1279} \cdot 2 \cdot \frac{9}{71} + 1)}{25 \cdot 17} \cdot 4.66920160910299 \]

\[ = 1 - \frac{1}{5 \cdot 7 \cdot (32 \cdot 27 \cdot 5 - 1)} = 0.999999887901990 \]

\[ \gamma'_{\text{Wallis}_{-971}} = \frac{1}{\sqrt{(2\pi)_{\text{Wallis}_{-971}}} \alpha_1 \delta} \]

\[ = 1 - \frac{1}{5 \cdot 7 \cdot (32 \cdot 27 \cdot 5 - 1)} = 0.999999887901990 \]
\[
\gamma_{\text{Wallis} - 9.71} = \frac{1}{\sqrt{(2\pi)_{\text{Wallis} - 9.71} \alpha_1 \delta}}
\]
\[
= 1 + \frac{1}{4.9 \cdot (8.3 \cdot 25 \cdot 7 \cdot 59 - 1)} = 1.00000011209802
\]

\[
\begin{array}{c|c}
27 & Al \\
13 & 14 \\
23 & 27, 28 \\
29 & Co \\
32 & 31, 38, 40 \\
31 & Ga \\
36 & Kr \\
46, 47, 48, 50 & 46 \\
50, 51 & 69, 71 \\
82, 83, 84, 86 & 112 \\
112, 114, 118, 120, 122 & 51, 70, 72 \\
80 & Sn \\
62, 64, 68, 70, 72 & 50 \\
141 & Pr \\
177 & Yb \\
175, 176 & 71 \\
104, 105 & 75 \\
71 & Lu \\
82 & 70 \\
103 & 175, 176 \\
112 & 114, 118, 120, 122 \\
123 & 121, 123 \\
128 & 128 \\
137 & 137 \\
138 & 138 \\
208 & 208 \\
209 & 209 \\
126 & 126 \\
210 & 210
\end{array}
\]

\[
209 \text{ Po}^* \quad 210 \text{ Al}^* \quad 285 \text{ Cn}^* \quad 344, 2173, 348 \quad F_{\gamma}^* \quad 208, 209, 210
\]

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\[
\alpha_1 (\delta \gamma_{1 - \text{GL} - 22.37})^2 (2\pi)_{\text{GL} - 22.37} = 1
\]

\[
\gamma_{1 - \text{GL} - 22.37} = \frac{1}{\sqrt{(2\pi)_{\text{GL} - 22.37} \alpha_1 \delta}}
\]
\[
= \frac{1}{\sqrt{137.035999037435}}
\]
\[
= \frac{1}{\sqrt{6.2856392998602 \cdot 4.66920160910299}}
\]
\[
= 1 - \frac{1}{2 \cdot 9 \cdot 11 \cdot 47 \cdot (2 \cdot 9 \cdot 137 + 1) - \frac{5}{11}} = 0.999999956442012
\]

\[
\begin{array}{c|c}
23 & Na \\
11 & 12 \\
25 & 25 \text{ Ti} \\
55 & 30 \text{ Mn} \\
83, 84 & 36 \text { As} \\
57, 48 & 37 \text{ Kr} \\
85, 87 & 37 \text{ Rb} \\
99 & 48, 50 \text{ Ru} \\
44 & 107, 109 \text{ Ag} \\
56 & 47 \text{ Ba} \\
80 & 137 \text{ Ba} \\
162 & 169 \text{ Dy} \\
94, 96 & 74 \text{ W} \\
108, 110 & 83 \text{ Bi} \\
209 & 126 \text{ Po}^* \\
84 & 209 \text{ Bi}^* \\
285 & 112 \text{ Cn}^* \\
213 & 2173 \text{ Fy}^* \\
209 & 309
\end{array}
\]

2021/3/2

\[
\gamma_{1 - \text{GL} - 22.37} = \frac{1}{\sqrt{(2\pi)_{\text{GL} - 22.37} \alpha_1 \delta}}
\]
\[
= 1 + \frac{1}{4.25 \cdot 7 \cdot (2 \cdot 23^2 \cdot 31 - 1) + \frac{6}{11}} = 1.0000004355799
\]

\[
\begin{array}{c|c}
23 & Na \\
11 & 12 \\
25 & 25 \text{ Ti} \\
55 & 30 \text{ Mn} \\
83, 84 & 36 \text{ As} \\
57, 48 & 37 \text{ Kr} \\
85, 87 & 37 \text{ Rb} \\
99 & 48, 50 \text{ Ru} \\
44 & 107, 109 \text{ Ag} \\
56 & 47 \text{ Ba} \\
80 & 137 \text{ Ba} \\
162 & 169 \text{ Dy} \\
94, 96 & 74 \text{ W} \\
108, 110 & 83 \text{ Bi} \\
209 & 126 \text{ Po}^* \\
84 & 209 \text{ Bi}^* \\
285 & 112 \text{ Cn}^* \\
213 & 2173 \text{ Fy}^* \\
209 & 309
\end{array}
\]

2021/3/2

\[
\alpha_1 (\delta \gamma_{1 - \text{NC} - 3})^2 (2\pi)_{\text{NC} - 3} = 1
\]

\[
\gamma_{1 - \text{NC} - 3} = \frac{1}{\sqrt{(2\pi)_{\text{NC} - 3} \alpha_1 \delta}}
\]
\[
= \frac{\sqrt{137.035999037435}}{\sqrt{6 + \sum_{n=1}^{n} (-1)^{n+1}} n(n+1/2)(n+1)} \cdot 4.66920160910299
\]
Integrated Formulas of $\alpha_2$, $\alpha$, $\delta$ and $\gamma_2$

By introducing a correction factor $\gamma_2$, some integrated formulas of $\alpha_2$, $\alpha$, $\delta$ and $\gamma_2$ in the format of $\alpha_2(\delta\alpha\gamma_2)^2 = 1$ could be obtained as follows.

$$\alpha_2(\delta\alpha\gamma_2)^2 = 1$$

$$\gamma_2 = \frac{1}{\sqrt{\alpha_2\delta\alpha}} = \sqrt{137.035999111818}$$

$$= 1 + \frac{1}{5.17 \cdot (16 \cdot 3 \cdot 157 + 1) + \frac{16}{17}} = 1.00168194075892$$

$$\gamma_2 = \frac{1}{\alpha_2 \delta \alpha} = \sqrt{137.035999111818}$$

$$= 1 + \frac{1}{4.3 \cdot 17 \cdot 23 - 137 - \frac{11}{59}} = 0.998320883415699$$
9. Integrated Formulas of $\alpha_1$, $\alpha_2$, $\alpha$, $2\pi$, $\gamma_1$ and $\gamma_2$

\[
\frac{\alpha_1(2\pi)}{\alpha_2(\gamma_1 \gamma_2)} = 1
\]

\[
\gamma_2 = \sqrt{\frac{137.035999037435 \cdot 2.50290787509589^2}{137.035999111818 \cdot (2 \cdot 3.14159265358979)}}
\]

\[
\gamma_1 = \frac{1}{32 \cdot 3 \cdot 7} - \frac{1}{32 \cdot 89 \cdot (4 \cdot 53 - 1)} = \frac{25}{2 \cdot 23} = 1.00148643114372
\]

\[
\gamma_2 = \frac{1}{32 \cdot 3 \cdot 7 + 1} + \frac{1}{4 \cdot 53 \cdot (2 \cdot 49 \cdot 29 + 1)} = \frac{2 \cdot 3}{23} = 0.99851577505446
\]

10. Integrated Formulas $\alpha$, $2\pi$ and $\gamma$

By introducing a correction factor $\gamma$, some integrated formulas of $2\pi$, $\alpha$ and $\gamma$ in the format of $2\pi/(\gamma \gamma) = 1$ could be obtained as follows.

\[
\frac{2\pi}{(\gamma \gamma)} = 1
\]

\[
\gamma = \sqrt{\frac{(2 \cdot 3 \cdot 1.4159265358979)}{2.50290787509589^2}}
\]

\[
= 1 + \frac{1}{32 \cdot 3 \cdot 7} - \frac{1}{23 \cdot 151 \cdot 173 + \frac{9}{4.7}} = 1.00148643087192
\]

\[
\gamma = 1 - \frac{1}{32 \cdot 3 \cdot 7 + 1} + \frac{1}{2 \cdot 41 \cdot (2 \cdot 3 \cdot 25 \cdot 49 - 1)} = \frac{1}{6} = 0.99851577532546
\]
References:


Acknowledgements

Yichang Huifu Silicon Material Co., Ltd., Guangzhou Huifu Research Institute Co., Ltd. and Yichang Huifu Nanometer Material Co., Ltd. have been giving Dr. Gang Chen a part-time employment since Dec. 2018. Thank these companies for their financial support. Specially thank Dr. Yuelin Wang and other colleagues of these companies for their appreciation, support and help.

Thank Prof. Wenhao Hu, the dean of School of Pharmaceutical Sciences, Sun Yet-Sen University, for providing us an apartment in Shanghai since January of 2021 and hence facilitating the process of writing this paper.
## Appendix I: Research History

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Note: Time was recorded according to Beijing Time.