Classical Doppler Shift Explains the Michelson-Morley Null Result

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Abstract

Here we review Michelson-Morley's original analysis of their interferometer experiment and discuss its use of optical distance. We derive a formula for transverse Doppler shift from geometric considerations, apply this to the Michelson-Morley interferometer, and present a phase analysis for the experiment. Furthermore, we present an equation for Doppler shift at a general angle and use this to derive the null phase shift result for round-trip interferometer paths at any arbitrary angle. We do not dispute the validity of the null result, nor the prediction of an arrival-time difference for the transverse and longitudinal arms of the interferometer; rather, we challenge the implicit assumption that an arrival time difference will necessarily result in an observable fringe shift.

1 Introduction

> If the Michelson–Morley experiment had not brought us into serious embarrassment, no one would have regarded the relativity theory as a (halfway) redemption.
>
> Albert Einstein

The Michelson-Morley experiment [1], conducted multiple times throughout 1887, was devised as an attempt to measure the absolute motion of the Earth through a hypothesized light-carrying-medium permeating space, known as the *aether*. It was believed that the speed of light is constant in all directions in the stationary frame of the aether (similar to how sound waves are constant in all directions in relation to a stationary observer on Earth's surface), but only in that frame. The failure of the Michelson-Morley interferometer to detect any effect attributable to the aether played a major role in the motivations for the development and acceptance of Einstein's theory of special relativity, proposed in 1905 [2].

In the Michelson-Morley interferometer, shown in figure (10), a collimated light source is directed toward a beam-splitter, which directs the beam toward two separate mirrors along two perpendicular paths each with length d. The light is reflected from

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each mirror, travels back, recombines, and is sent toward a detector for observation. The collimated light source contains at least two different frequencies of light, so that an interference pattern is formed consisting of multiple "fringes" appearing as rings of higher and lower intensity.

Michelson and Morley hypothesized that if their laboratory was moving at some velocity with respect to the aether's stationary frame, they would observe a visual interference pattern in the form of additional fringes—a separation between areas of intensity. If the aether caused a phase difference between light along the two paths, each full wavelength of phase shift would result in an additional fringe observed. A fringe shift was therefore considered to be the number of wavelengths along which the phase was shifted.

2 Historical Background

> The history of science, like the history of all human ideas, is a history of irresponsible dreams, of obstinacy, and of error. But science is one of the very few human activities–perhaps the only one–in which errors are systematically criticized and fairly often, in time, corrected.
>
> Karl Popper

2.1 Absolute Reference Frames

The concept of an absolute universal frame of reference dates back to ancient Greek philosophers such as Plato, Aristotle, and Ptolemy, who developed a layered model of *celestial spheres* based on the observation that the positions of celestial objects such as the sun, moon, and planets appeared to change position rapidly in relation to one another, while the stars remained relatively fixed. This led the Greeks to envision layers of celestial spheres, thought to be embedded in an aethereal elemental substance referred to as *quintessence*, with each celestial sphere rotating independently with respect to its neighbors.

Although the Earth was placed at the center of the universe (incorrectly, as we now understand), the ancient model had its successes. For example, Mercury, Venus, Mars, Jupiter, and Saturn were all placed in their correct ordinal positions. The observation that planets occasionally exhibited retrograde motion (appearing to move backward in relation to their usual direction of travel) led to Ptolemy adding *epicycles* to his geocentric model to more accurately track these planetary movements. The Ptolemaic system lasted well over 1000 years, but eventually complexity of the epicycles required to maintain the consistency of the model became burdensome, and the Ptolemaic system gave way to the Copernican model, published in 1543, which replaced the Earth with the sun as its center.

The ancient Greeks were somewhat divided on the nature of light and vision. One theory, known as the "tactile" theory, postulated that sight originated from the eyes themselves, which sent out very fine, invisible probes to "feel" objects too distant to physically reach. The competing hypothesis, known as "emission" theory, advanced that light was emitted from bright objects[1], traveling from there to enter the eyes, producing vision [3]. There was no clear consensus yet on whether the speed of light was finite or infinite.

2.2 The Speed of Light

In 1677, the Danish astronomer Ole Roemer used the timings of eclipses of Jupiter's moon Io, which occur roughly every 42.5 hours, to estimate the velocity of light [4]. Roemer's observations are important to understand, because the observation of the eclipses of Io is analogous to the beam path to mirror 1 in the Michelson-Morley experiment, shown in figure (10). As the Earth and Jupiter orbit the sun, they transition from receding away from the sun to moving toward it. At the beginning and end of this transition period, the relative distance between the planets is unchanged, as shown in figure (1).

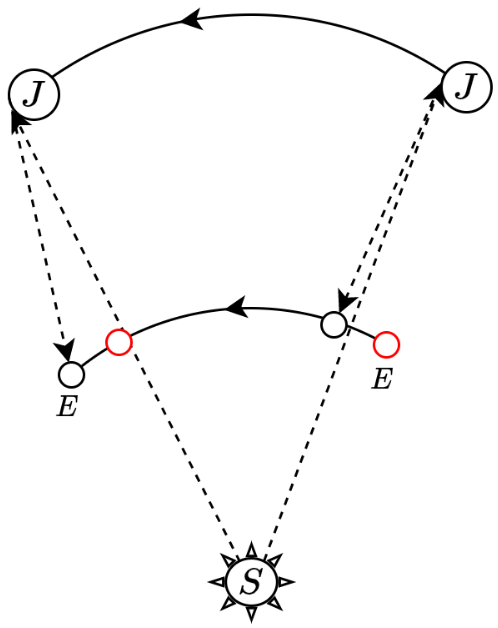

Figure 1: The Earth-Jupiter system during Roemer's observations

Despite the fact that the distance between the planets remains the same, the period between eclipses does not. The period is shorter while the Earth is receding from the sun, and longer while the Earth is moving toward the sun. Roemer understood this to mean that even though the relative distance between the two planets is equal in both cases, light leaving Io would reach the Earth sooner in the first case, since the Earth-Jupiter system was moving toward the light during its period of transit, and would reach the Earth later in the second case, while the Earth-Jupiter system was moving away from the light. Roemer was able to express the observed eclipse period

[1]We understand this today to be the correct theory, of course.

in the first case as

$$T_{\uparrow} = T_0\left(1 - \frac{v}{c}\right) \tag{1}$$

where $T_{\uparrow}$ is the observed period while the Earth is receding, T_0 is the absolute period, v is the Earth's orbital velocity, and c is the speed of light. In the second case, the period is expressed as

$$T_{\downarrow} = T_0\left(1 + \frac{v}{c}\right) \tag{2}$$

From these two equations, the speed of light can be expressed as[2]

$$c = \frac{v(T_{\downarrow} + T_{\uparrow})}{T_{\downarrow} - T_{\uparrow}} \tag{3}$$

Critically, Roemer was able to correctly interpret his observations by assuming that the speed of light is constant only with respect to an absolute reference frame (that of the sun, in this case), while in the Earth's frame the observed speed of light could be greater or less than its absolute speed. Interestingly, Michelson and Morley mentioned observations of the eclipses of Jupiter in their original paper as a potential means of determining the absolute aethereal motion of the Earth. It is unclear whether they were aware of Roemer's work two centuries prior.

In 1728, the English astronomer James Bradley applied Roemer's technique to starlight in order to distinguish the true position of stars from their apparent positions at different points in the Earth's orbit [4]. He was attempting to gauge the distance to a star using parallax, but instead he found that the position of the star varied depending on the relative *velocity* of the Earth to the star rather than its position. This was an unusual discovery; as it turned out, the parallax he was attempting to measure was too minute for him to detect, but in the process he discovered another effect varying the star's apparent position, which is now known as Bradley stellar aberration. The aberration caused an effect similar to parallax, except that the star's observed position was ahead of its expected position with parallax.

Again, this discovery required interpreting the observed speed of light from the vantage point of an observer on Earth as being alternately faster or slower (depending on the direction of the Earth's velocity) than the absolute speed of light as emitted from the stars. Thus we see that astronomers several centuries ago were able to successfully account for astronomical observations using purely geometrical arguments that assumed an absolute reference frame for the speed of light, outside of which the observed speed of light could be faster or slower.

2.3 The Aether

From the observation that light refracted and diffracted around surfaces as a wave, and that waves propagate through mediums, Christopher Huygens presented a wave theory of light in his 1690 book, *Treatise of Light* [4], which posited an aether, similar to air, as a medium for light to propagate. Light was understood to propagate at speeds much faster than sound (for example, from the observation that lightning is

[2]To calculate the speed of light with greater accuracy, Roemer would have had to account for the absolute velocity of the solar system, but it is possible to use his data to determine the correct value for the solar system's velocity as well.

seen before thunder), so the aether was hypothesized to be a very sparse yet rigid elastic medium permeating the universe. Using these assumptions, and positing that light's velocity was slowed in materials so that $v = c/n$ where v is the velocity of light through a material, and n is some refractive index for the material such that $n > 1$, Huygens was able to use his model to derive the known laws of reflection and refraction.

Huygens' model was successful in many ways, however, there were a number of observations his model did not account for, such as the fact that light did not appear to diffract into shadows[3], and the fact that light could be polarized by different materials, which was incompatible with existing observations of wave behavior. These inconsistencies led Newton to formulate a competing *corpuscular* model of light [5], which proposed that light consisted of particles of varying sizes and shapes to account for their varying reflectivity and polarization when interacting with materials.

In this paper we will not weigh the question of whether light is fundamentally a particle or a wave-like peturbation of an aether[4], whether light consists of particles traveling from one location to another or whether it is merely a transfer of energy, as in the case of sound. For the purposes of our analysis, we will simply assume that light can travel in a manner analagous to classical waves. We will also postulate the existence of an aether (similar to that envisioned by Le Sage, comprised of "ultramundane corpuscles") as well as an absolute reference frame in which the aether is stationary.

2.4 Action at a Distance

When light encounters a new medium, some portion of the light is reflected, while another portion is refracted toward the line normal to the surface. From the observation that polished surfaces reflect light coherently despite the fact that on a microscopic scale, these surfaces must have many imperfections, Newton posited that light must be interacting with surfaces at a distance before making contact, over an area larger than that of the imperfections, rather than interacting with the surfaces directly. Likewise, his laws of gravitation required instantaneous action at a distance to account for the orbital motions of planets. Despite the "corrections" to these orbits required by general relativity[5], it is well-understood that the speed of gravitational attraction must exceed the speed of light by many orders of magnitude to correctly compute planetary orbits[6], despite the recent detection by LIGO of "gravitational waves"[7].

[3] If the light was very close to the edge of the object, however, it would bend into shadows to some degree, as Newton noted in his observation of diffraction around the edge of a knife.

[4] Generally speaking however, we subscribe to a model of photons similar to that envisioned by Newton: as solid, massy, hard, impenetrable, moveable particles, that exist in a single position at any given time, and have a physical radius and a physical, spinning surface.

[5] These corrections essentially amount to a post-hoc justification for an extremely small "anomalous" precession in Mercury's orbit (amounting to 42 arcseconds per century), which was not accurately computed to begin with (Le Verrier's estimation of Mercury's mass was off by a factor of 2, for example [6]), and is already accounted for in large part by classical Newtonian effects.

[6] There are a variety of observations that easily support this, such as the Poynting-Robertson effect, which could not exist unless there was an aberration between radiation pressure and gravitational force, as well as the fact that introducing an eight-minute time delay (roughly the delay of light from the sun to the Earth) to numerical orbit calculations causes the Earth to roughly double its orbital distance around the sun in a mere 1200 years [7].

[7] If we are to be charitable, we may accept that these experiments are detecting authentic fluctuations in the aethereal wind, but these fluctuations cannot be representative of the ordinary gravita-

From his observation that light reflected off surfaces at equal angles, Newton hypothesized that surfaces must exert a force against light at a normal angle to reflect them away. During refraction, on the other hand, light bent into surfaces *toward* the normal, as if an opposite normal force was pulling light *into* the surface. By considering momentum to be conserved along the direction parallel to the surface, Newton established the formula $p \sin\alpha = p' \sin\beta$, where p and α are light's momentum and angle (relative to the normal) entering a new medium, and p' and β are light's momentum and angle within the new medium, so that

$$\frac{p'}{p} = \frac{\sin\alpha}{\sin\beta} > 1 \tag{4}$$

Newton understandably concluded that light must have a greater velocity within the material [4] (as one would expect with a force pulling light into the material). However, the velocity of light is measurably slower in materials, indicative of an inverse relationship between momentum and speed. We can find this relation from our modern formulation of light's momentum given by de Broglie's formula

$$p = \frac{h}{\lambda} = \frac{hf}{c} = \frac{E}{c} \propto \frac{1}{c} \tag{5}$$

Using this inverse relationship, we can express the ratio between momentums as

$$\frac{p'}{p} = \frac{c}{v} = n > 1 \tag{6}$$

where c is the velocity of light in a vacuum, v is the velocity of light in the material, and n is considered the refractive index of the material. This in turn correctly yields Snell's law for light moving from a vacuum into a new medium: $\sin\alpha = n \sin\beta$.

We observe light to refract in an analogous manner around celestial bodies. As light is drawn toward a gravitational source, its wavelength is blue-shifted, indicating an increase in momentum, while its trajectory is refracted toward the gravitational source, indicating a decrease in velocity. We deduce that in both cases (the refraction of light into a medium and the refraction of light around celestial bodies) that light is traveling from a region in which the aether is more dense to a region in which the aether is less dense[8]. This attraction to regions of lower density is consistent with the principle that Nature takes the path of least resistance. The bending of light rays also remains consistent with Fermat's principle of least time, that "Nature always acts by the shortest course".

The decrease in the velocity of light associated with a corresponding increase in momentum seems paradoxical compared to classical physics. However, it is likely consistent with the behavior of an incompressible fluid moving against a drag force, creating an effect similar to that observed in water flowing through a pipe with varying diameter sections. An increase in diameter causes a decrease in velocity, while a decrease in diameter causes an increase in velocity. This effect is counter-intuitive to many, yet it is entirely classical.

In any case, action at a distance appears to reliably describe the motions of celestial objects under gravitational attraction, the deflections of light around gravitational sources, and the refraction of light through various mediums. In the absence of a

tional forces that maintain the orbits of planets and stars.

[8]This is consistent with the mechanism of Le Sage gravity, for example.

relativistic theory of light, one might profitably deduce that—similar to other observed particles—particles of light carry mass. Furthermore, these particles must interact at highly superluminal velocities.

2.5 Emission Theories

Following the relativistic experiments of Michelson-Morley, Fizeau, Sagnac, Hafale-Keating, and others, we are left with three possible options for emission theories of light. Either the velocity of light is: dependent on both the source and the observer[9], dependent on the observer but not the source[10], or is dependent on neither the source nor the observer[11].

The ballistic theory is Newtonian and easily explains the results of Michelson and Morley. However, it is clearly violated by the Sagnac experiment[12] (since it would predict no observable interference), as well as variety of other observations including Bradley's stellar aberration as discussed earlier. Thus, it must be discounted.

Relativity certainly explains the results of Michelson and Morley, although it is less unambiguously clear whether the results of Sagnac and Hafele-Keating really support the theory as relativists claim[13]. Furthermore, relativity encounters serious issues in dealing with scenarios such as the twin paradox in its attempts to remove absolute frames of reference from physics.

The general theory of relativity, which is invoked to resolve many of these paradoxes, suffers from its own deficiencies, including the creation of "singularities" within black holes, violations of the equivalence principle for charged particles (which are predicted to radiate in one frame but not another), and requiring an infinite amount of energy to assemble an electron, which in turn predicts an infinite electron mass, due to its assumption of mass-energy equivalence.

The aether theory, as we will see, not only fits early observations of light such as the timings of eclipses and Bradley stellar aberration, but can also simply account for "relativistic" observations such as the Michelson-Morley experiment.

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[9] We may refer to this as ballistic theory, Newtonian theory, or alternatively Ritzian theory, after the physicist Walther Ritz who developed a version of ballistic theory compatible with Maxwell's equations.

[10] This is the aether theory, although it does not necessarily require the aether to be a medium distinct from light itself, as we have previously noted.

[11] This is, of course, the theory of relativity.

[12] A rapidly rotating variation of the Michelson-Morley experiment.

[13] Relativistic experiments such as the Hafele-Keating experiment can be interpreted as a type of Sagnac interferometer using planes and clocks. The clocks in this scenario perform a function similar to light clocks as envisioned by Max Born—since they rely on an electromagnetic mechanism, the difference in synchronization is an optomechanical effect rather than a dilation of time itself. The Hafele-Keating experiment is, if anything, evidence against the theory of relativity since—all reference frames being equal—each plane travels the same distance, therefore one should be able to argue that one plane's clock should run ahead as easily as the other. Or, one might expect both clocks to remain synchronized since each travels the same distance at the same speed, and should experience the same amount of time dilation. The fact that this is not observed is evidence that the planes are traveling in opposing directions within a frame (the Earth's) that is rotating with respect to a separate frame of reference. Nevertheless, these experiments are commonly interpreted as evidence *for* relativity rather than against.

3 Doppler Shift and Frequency Invariance

> Is the ocean composed of water or of waves or of both? Some of my fellow passengers on the Atlantic were emphatically of the opinion that it is composed of waves; but I think the ordinary unprejudiced answer would be that it is composed of water.
>
> Sir Arthur Eddington, *New Pathways in Science* (1935)

Here we examine several scenarios involving a wave-emitting source and a receiver, and gradually derive the equations for Doppler shift at a general angle, which will be helpful for analyzing the Michelson-Morley experiment. We show that in general, for a source moving at velocity v_S and a receiver moving at velocity v_R, if $v_S = v_R$ then the frequency observed by the receiver will be the same as the case when the source and receiver are both stationary; that is, $v_S = v_R$ implies $f' = f$. This frequency invariance holds for motion at any arbitrary angle.

3.1 Stationary Source and Stationary Receiver

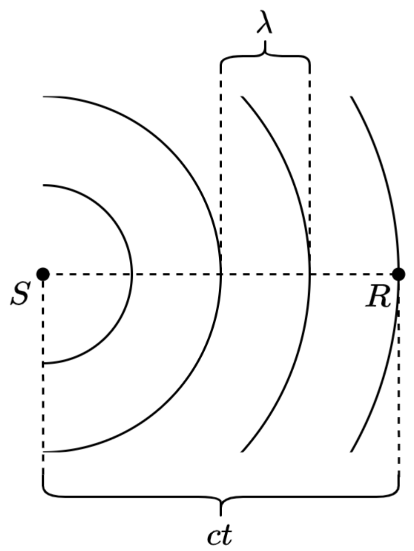

Figure 2: A stationary source S and stationary receiver R

In figure (2) we have a stationary source at S and a stationary receiver at R. The source emits waves, which move at speed c regardless of the velocity of the source v_S.

We consider a wavefront to be the locus of all points where the wave's phase is equal to some initial value (which we can consider to be zero for convenience). The distance d between S and R is then given by $d = ct$.

The source emits wavefronts moving at speed c every T seconds with frequency $f = \dfrac{1}{T}$. At time zero, the source emits a wavefront. Let t be the time at which the wavefront emitted at time zero reaches the receiver, and let λ be the distance between any two wavefronts.

In figure (2), we have

$$n = \frac{d}{\lambda} = \frac{ct}{\lambda} \tag{7}$$

where n is the number of wavefronts between the source and the receiver. If we consider this to be the situation at time zero, then we will have n wavefronts pass the receiver within time t, so the frequency of wavefronts observed by the receiver is given by

$$f' = \frac{n}{t'} \tag{8}$$

where we are using an apostrophe to distinguish between emitted frequency and observed frequency. The source and the receiver will always agree on the time taken for the initial wavefront to reach the receiver, so we will have $t' = t$. Thus,

$$f' = \frac{n}{t'} = \frac{n}{t} = f \tag{9}$$

and we see that the observed frequency is the same as the emitted frequency, and thus it is invariant. Similarly, we find that since the source and receiver also agree on distance and time, speed c is invariant:

$$c' = \frac{d}{t'} = \frac{d}{t} = \frac{ct}{t} = c \tag{10}$$

We can rearrange equation (7) and substitute (8) to obtain the velocity of the wavefronts to obtain the familiar equation:

$$c = \frac{n\lambda}{t} = f\lambda \tag{11}$$

From our previous discussion, we also have:

$$f' = f \tag{12}$$

$$c' = f'\lambda' = c \tag{13}$$

$$\lambda' = \frac{c'}{f'} = \frac{c}{f} = \lambda \tag{14}$$

The invariance of speed, frequency, and wavelength in the case of a stationary source and receiver should not be surprising, but this analysis is helpful in laying the groundwork for further discussion.

3.2 Longitudinally Moving Source and Stationary Receiver

Let us consider a source at S moving directly (longitudinally) toward a stationary receiver at R with a velocity $v_S < c$. The source will move a distance $v_S t$ in time t, which will cause the wavefronts ahead of the source to appear compressed, as shown

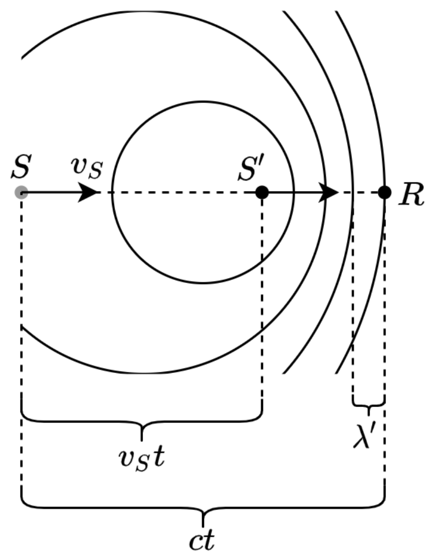

Figure 3: A longitudinally moving source and a stationary receiver

in figure (3). At time t, the initial wavefront from S reaches R and the number of wavefronts between the source (now at S') and the receiver is then given by

$$\begin{aligned} n &= \frac{ct - v_S t}{\lambda'} \\ &= \frac{ct\left(1 - \frac{v_S}{c}\right)}{\lambda'} \end{aligned} \tag{15}$$

Rearranging for λ' and substituting equation (7), we have

$$\begin{aligned} \lambda' &= \frac{ct\left(1 - \frac{v_S}{c}\right)}{n} \\ &= \lambda\left(1 - \frac{v_S}{c}\right) \end{aligned} \tag{16}$$

The time required for the initial wavefront released at S at time zero to reach R is still t regardless of v_S, so $t' = t$ and

$$\begin{aligned} c' &= \frac{d}{t'} \\ &= \frac{d}{t} \\ &= c \end{aligned} \tag{17}$$

so the observed speed of the wavefront is invariant. Thus,

$$\begin{aligned} f' &= \frac{c'}{\lambda'} \\ &= \frac{c}{\lambda'} \\ &= \frac{c}{\lambda\left(1 - \dfrac{v_S}{c}\right)} \\ &= \frac{f}{1 - \dfrac{v_S}{c}} \end{aligned} \tag{18}$$

3.3 Stationary Source and Longitudinally Moving Receiver

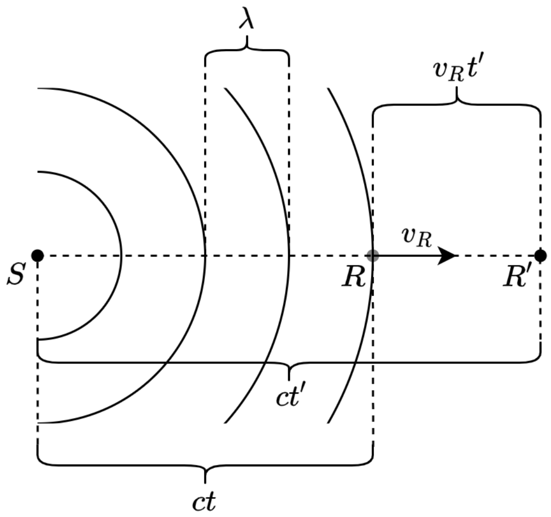

Figure 4: A stationary source and a longitudinally moving receiver

Let us consider a receiver at R moving longitudinally away from a stationary source at S with velocity $v_R < c$. Here, the initial wavefront emitted at time zero catches up to the receiver at R' after time t'. During this time, the receiver travels a distance $v_R t'$ so that the total distance the initial wavefront travels is given by $ct' = ct + v_R t'$. Rearranging for t', we have

$$\begin{aligned} t' &= \frac{ct}{c - v_R} \\ &= \frac{t}{1 - \dfrac{v_R}{c}} \end{aligned} \tag{19}$$

so that the observed time t' for the initial wavefront to reach the observer is longer than it would be in the stationary case.

Since the receiver is moving at less than c, it does not pass any wavefronts ahead of it, but it is passed by the n wavefronts between S and R within time t'. The frequency observed by the receiver then is given by:

$$\begin{aligned} f' &= \frac{n}{t'} \\ &= \frac{n\left(1 - \frac{v_R}{c}\right)}{t} \\ &= f\left(1 - \frac{v_R}{c}\right) \end{aligned} \tag{20}$$

Since the source is not moving, the receiver still measures the distance between successive wavefronts to be the same as in the stationary case, so that

$$\lambda' = \lambda \tag{21}$$

The observed speed c' of the wavefront by the receiver is then given by

$$\begin{aligned} c' &= f'\lambda' \\ &= f'\lambda \\ &= f\left(1 - \frac{v_R}{c}\right)\lambda \\ &= c\left(1 - \frac{v_R}{c}\right) \end{aligned} \tag{22}$$

3.4 Longitudinally Moving Source and Receiver

We can now combine our work from the previous two sections to obtain the more general longitudinal formulas for a moving source and receiver. The wavelength observed is dependent only on the motion of the source, so

$$\lambda' = \lambda\left(1 - \frac{v_S}{c}\right) \tag{23}$$

On the other hand, the observed frequency is dependent on both the motion of the source and the receiver as previously shown. Combining equations (20) and (18), we have

$$f' = f\left(\frac{1 - \frac{v_R}{c}}{1 - \frac{v_S}{c}}\right) \tag{24}$$

Thus, the observed speed is given by

$$\begin{aligned} c' &= f'\lambda' \\ &= c\left(1 - \frac{v_R}{c}\right) \end{aligned} \tag{25}$$

To summarize, for a longitudinally moving source and receiver, the observed wavelength is only dependent on the velocity of the source, the observed frequency is dependent on both the velocity of the source and receiver, and the observed speed is dependent only on the velocity of the receiver. Furthermore, from equation (24) we see that when $v_R = v_S$, we have $f' = f$.

3.5 Source and Receiver Moving at Same Speed and Angle

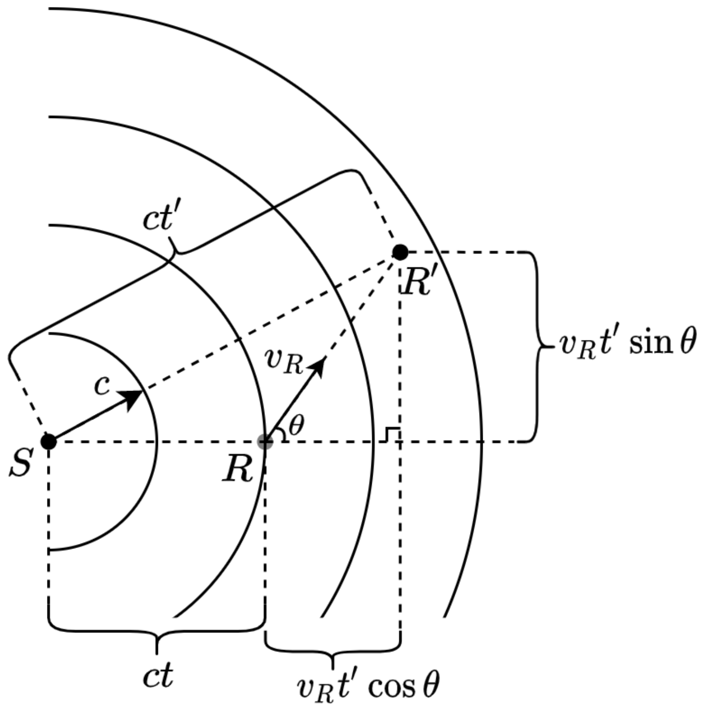

Figure 5: A stationary source and a receiver moving at an arbitrary angle

We do not consider the case of a receiver moving at an arbitrary angle away from a stationary source (see figure (5) or a source moving at an arbitrary angle toward a stationary receiver here (as shown in figure (6)), because the observed values of frequency, wavelength, and speed change continuously with time due to the variance of the angle $\angle SS'R$ or $\angle SR'R$ as the source or receiver moves (respectively), thus they do not have a fixed value and must be parameterized by time.

Instead, let us consider the case of a source and receiver at S and R respectively, both moving with velocity v in the same direction, as shown in figure (7). As usual, we consider t' to be the time taken for a wavefront emitted at S to reach the receiver; this event occurs at R'. Here we define t to be the time for the initial wavefront from S to reach R' in the case when the source is stationary[14], so we have $t = t'$, and both source and receiver move a distance $vt' = vt$ during this time.

Now, let us examine how many wavefronts the receiver crosses as it moves from R to R'. From the figure, we see that the outermost wavefront around S' at time t coincides with the wavefront from S that was released at time zero (which reaches R' at time t by our definition of t). We can also see by geometric translation that the number of wavefronts between S' and R' at time t (shown in black) is the same as the number of wavefronts between S and R at time zero (shown in red). Thus, the number of wavefronts crossed by the receiver is exactly equal to the number of wavefronts (shown in red) between R and S at time zero.

In other words, at time zero, S emits a new wavefront; at time t this initial wavefront reaches R' and in the time interval during which the receiver moves from R to

[14]Note that this is not the same as the time taken when both S and R are stationary.

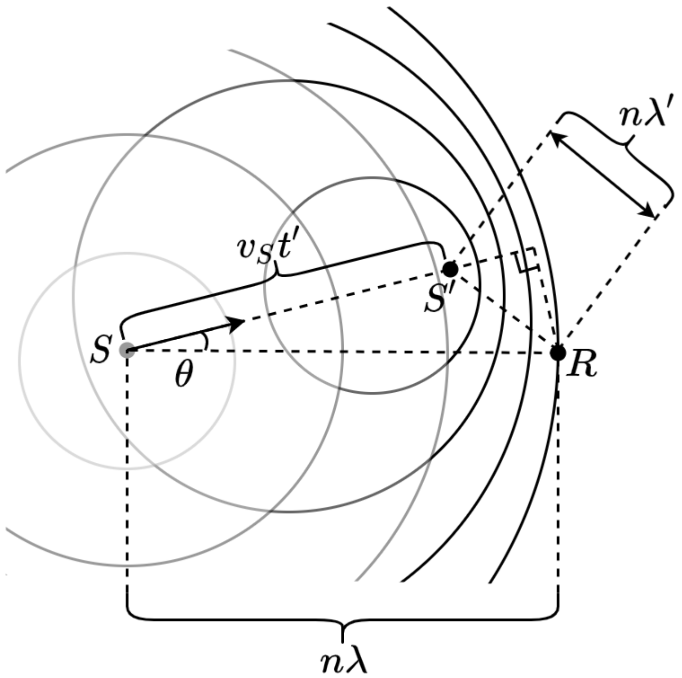

Figure 6: A source moving at an arbitrary angle toward a stationary receiver

R', all of the red wavefronts between S and R (no more and no less) must pass the receiver.

So, the number of wavefronts n crossed by the receiver in time t when the source and receiver are both moving with the same velocity v is the same as the number crossed in the case when the source and receiver are both stationary[15]. Thus, in the case of a source and receiver both moving at the same speed and angle, *we have frequency invariance*; i.e., the frequency observed in the case when both the source and receiver are moving in concert is the same as when they are both stationary. This principle holds in the case of a Michelson-Morley interferometer experiment, for example, because the interferometer is a rigid object in which the sources and receivers (e.g., the beam splitter and mirrors) all move together.

Thus, while we can assume frequency invariance, as we can see from figure (7) we clearly will not have wavelength invariance, so we will also not have speed invariance. The observed speed will be given by $c' = f\lambda'$.

We can use apply the Pythagorean theorem to the smaller right triangle in figure (7) to determine the wavelength shift:

$$(n\lambda)^2 = (n\lambda' \cos\theta - vt)^2 + (n\lambda' \sin\theta)^2 \tag{26}$$

Since $n\lambda = ct$, we can substitute $t = \dfrac{n\lambda}{c}$ into equation (26) and divide both sides

[15]Note that the source and receiver can be placed anywhere in the stationary case without changing any observed values, for example, they could be placed at S and R or at S and R' and the frequency observed by the receiver remains unchanged.

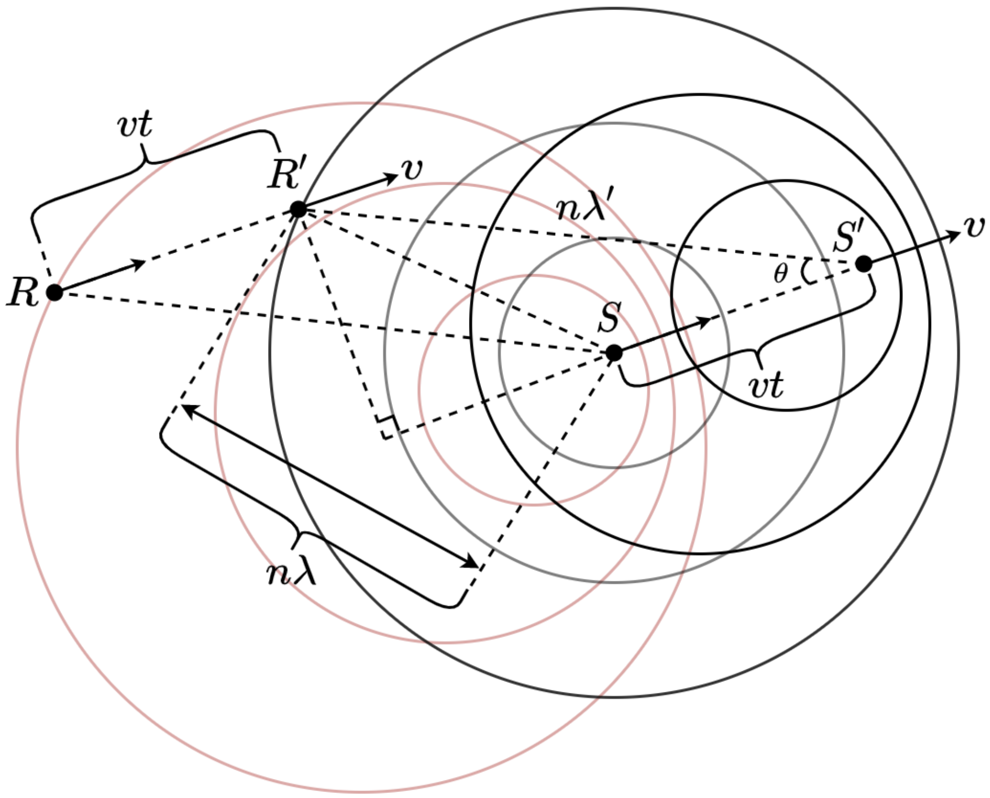

Figure 7: A source and receiver moving at the same speed and angle

by n^2 to obtain:

$$\begin{aligned}\lambda^2 &= \left(\lambda' \cos\theta - \frac{v\lambda}{c}\right)^2 + (\lambda' \sin\theta)^2 \\ &= (\lambda' \cos\theta)^2 - 2\lambda\lambda' \left(\frac{v\cos\theta}{c}\right) + \left(\frac{v\lambda}{c}\right)^2 + (\lambda' \sin\theta)^2 \\ &= \lambda'^2 - 2\lambda\lambda' \left(\frac{v\cos\theta}{c}\right) + \left(\frac{v\lambda}{c}\right)^2 \\ &= \left(\lambda' - \left(\frac{v\lambda\cos\theta}{c}\right)\right)^2 + \left(\frac{v\lambda}{c}\right)^2 - \left(\frac{v\lambda\cos\theta}{c}\right)^2 \\ &= \left(\lambda' - \left(\frac{v\lambda\cos\theta}{c}\right)\right)^2 + \left(\frac{v\lambda}{c}\right)^2 (1 - \cos^2\theta) \\ &= \left(\lambda' - \left(\frac{v\lambda\cos\theta}{c}\right)\right)^2 + \left(\frac{v\lambda\sin\theta}{c}\right)^2\end{aligned} \tag{27}$$

Thus,

$$\begin{aligned}\left(\lambda' - \left(\frac{v\lambda\cos\theta}{c}\right)\right)^2 &= \lambda^2 - \left(\frac{v\lambda\sin\theta}{c}\right)^2 \\ &= \lambda^2 \left(1 - \left(\frac{v\sin\theta}{c}\right)^2\right)\end{aligned} \tag{28}$$

Taking the square root of both sides,

$$\lambda' - \left(\frac{v\lambda\cos\theta}{c}\right) = \lambda\sqrt{1 - \left(\frac{v\sin\theta}{c}\right)^2} \tag{29}$$

and factoring out λ we have the result[16]:

$$\lambda' = \lambda\left(\frac{v\cos\theta}{c} + \sqrt{1 - \left(\frac{v\sin\theta}{c}\right)^2}\right) \tag{30}$$

Note that for $\theta = 0$ and $\theta = \pi$, equation (30) reduces to the familiar form for longitudinal Doppler shift,

$$\lambda' = \lambda\left(1 \pm \frac{v}{c}\right) \tag{31}$$

and for $\theta = \frac{\pi}{2}$, this becomes

$$\lambda' = \lambda\sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{\lambda}{\gamma} \tag{32}$$

where $\gamma = \dfrac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$ is the familiar Lorentz factor. Equations (31) and (32) play an important role in the analysis of the Michelson-Morley experiment.

Since $f' = f$ and $c' = f'\lambda' = f\lambda'$, from equation (30) we also have

$$c' = c\left(\frac{v\cos\theta}{c} + \sqrt{1 - \left(\frac{v\sin\theta}{c}\right)^2}\right) \tag{33}$$

We can further simplify this to

$$c' = v\cos\theta + \frac{c}{\gamma_\theta} \tag{34}$$

with γ_θ defined as

$$\gamma_\theta = \frac{1}{\sqrt{1 - \left(\frac{v\sin\theta}{c}\right)^2}} \tag{35}$$

and we will see that this is the same as equation (67) as derived from the Michelson-Morley interferometer tilted at an arbitrary angle with respect to the aether wind.

3.6 General Classical Doppler Shift

In figure (8) we have the commonly accepted depiction of classical Doppler shift for a source moving at an angle. Aside from the fact, as we have noted with figure (6), that a moving source and a stationary receiver will not produce a constant Doppler

Figure 8: Doppler shift for a moving source

shift, this figure is also shown without wavefronts, which makes it easier to misjudge the resulting wavelengths.

The commonly accepted formulation of Doppler shift in this scenario simply modifies the equation for longitudinal Doppler shift by taking the component of the velocity along the vector from the observer to the source, so the equation for longitudinal Doppler shift,

$$\lambda' = \lambda \left(1 + \frac{v}{c}\right) \tag{36}$$

becomes:

$$\lambda' = \lambda \left(1 + \frac{v \cos\theta}{c}\right) \tag{37}$$

Equation (37), however, is *not* the correct formula for Doppler shift at a general angle. We know that it must not be correct, since for $\theta = \frac{\pi}{2}$, equation (37) predicts *zero* Doppler shift, but as we have previously established by equation (30), there is a measurable wavelength contraction due to Doppler shift at 90 degrees.

We can see this contraction clearly in figure (9), which shows two sets of n wavefronts, one moving (shown in black) and one stationary (shown in red). Here, C_1 and C_2 are the respective centers of the outermost wavefront in each set, while S is the location of the moving source, given that it emitted its initial wavefront at C_2. From the geometry of the figure we can see that $n\lambda' < n\lambda$, so we must have $\lambda' < \lambda$ for an observer at a 90 degree angle to the velocity of a moving source.

For $v \ll c$, there is very little practical difference between equation (37) and the correct general equation, given by (30). However, when v becomes a non-negligible portion of c (as may be the case for particles traveling in particle accelerators or stars orbiting around galaxies, for example), the difference between the two equations becomes significant. We include a more in-depth discussion of the derivation of equation (37) in appendix A.

[16]Klinaku [8] also derives this formula using a similar method.

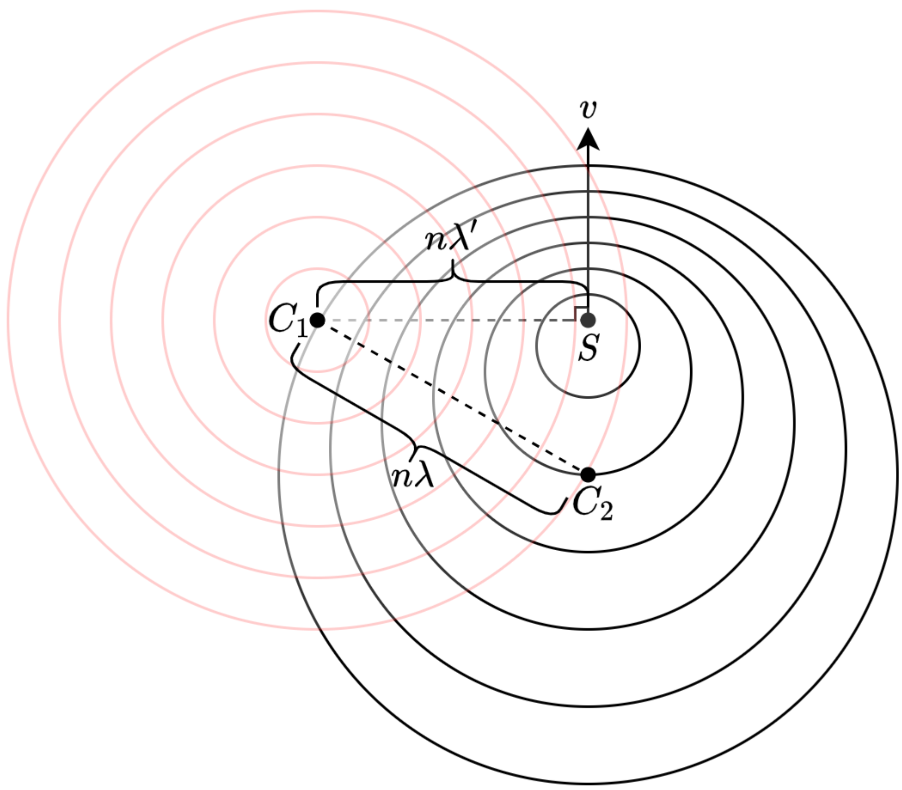

Figure 9: Transverse Doppler shift for a moving source

4 The Michelson-Morley Experiment

> The devil can cite Scripture for his purpose.
>
> Shakespeare, *Merchant of Venice*

4.1 Michelson-Morley's Fringe Shift Analysis

Here we will review the original derivation [1] of Michelson-Morley's fringe shift calculation. Michelson-Morley's experimental apparatus could be rotated in different orientations with respect to the hypothesized aether, however, to simplify our analysis we will consider the case in which the laboratory is moving in parallel along the path to mirror 1 with respect to the aether, as shown in figure (10).

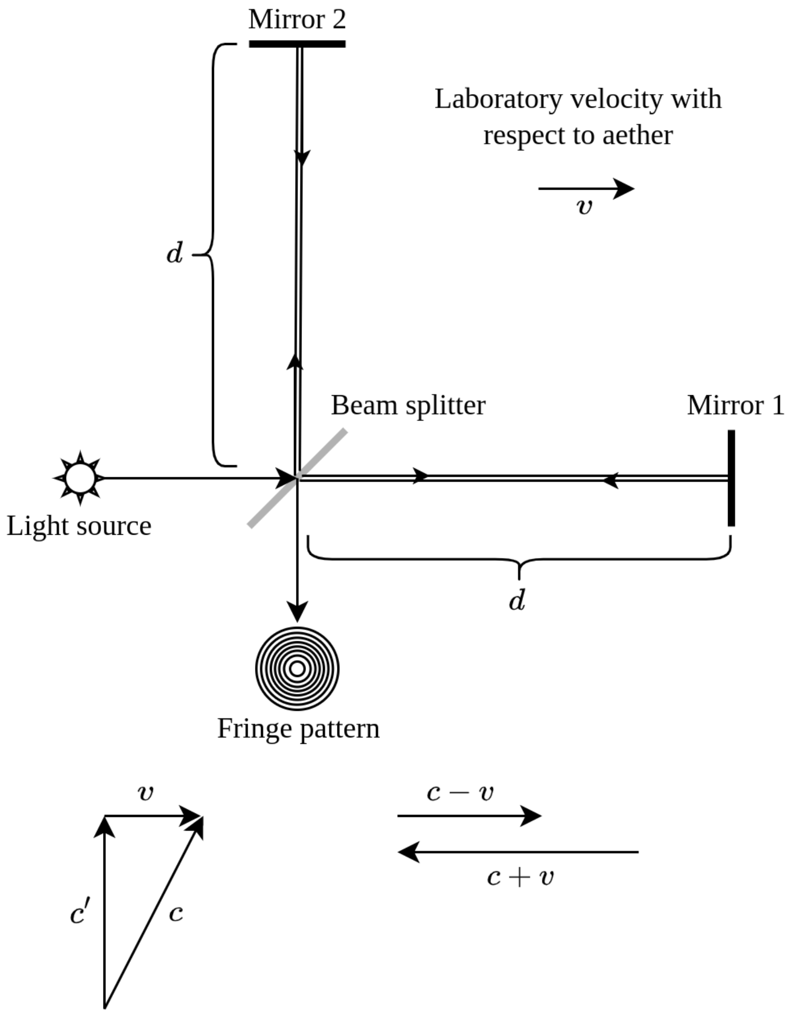

Figure 10: Michelson-Morley experimental setup

The time for light to traverse the round trip path to mirror 1 is given by

$$
\begin{aligned}
t_1' &= \frac{d}{c-v} + \frac{d}{c+v} \\
&= \frac{2dc}{c^2 - v^2} \\
&= \frac{2d}{c} \cdot \frac{1}{1 - \dfrac{v^2}{c^2}} \\
&= \frac{2d}{c} \cdot \gamma^2 \\
&= t\gamma^2
\end{aligned}
\tag{38}
$$

where γ is the Lorentz factor given by $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$, and t is the expected round trip time in the stationary aether frame (i.e., in an apparatus with no aethereal "wind"). We are using variables with apostrophes to indicate variables observed in the laboratory frame, so for example, t' is the observed time in the laboratory frame.

Since $c^2 = (c')^2 + v^2$ by the Pythagorean theorem, the observed speed of light from the laboratory frame in the mirror 2 path is slower than the speed of light in the stationary aether frame and is given by $c' = \sqrt{c^2 - v^2}$. This holds for both directions and is due to the fact that the actual path the speed of light is taking is longer than the observed path in the laboratory frame.

The time for light to traverse the round trip path to mirror 2 is given by

$$\begin{aligned} t'_2 &= \frac{2d}{\sqrt{c^2 - v^2}} \\ &= \frac{2d}{c} \cdot \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} \\ &= \frac{2d}{c} \cdot \gamma \\ &= t\gamma \end{aligned} \tag{39}$$

It is important to note that in neither case is the speed of light c actually changing. Light is emitted at speed c with respect to the stationary aether frame regardless of the laboratory's velocity v. However, because the laboratory is moving, the light appears to travel faster or slower from the laboratory perspective depending on its direction. The situation for light in this scenario is analogous to the way sound waves travel in Earth's atmosphere. Regardless of our velocity within the Earth's atmosphere, sound waves always travel at the same speed with respect to the frame of a stationary observer on the ground. However, in a fast-moving vehicle, a jet for example, the sound waves generated by the jet traveling in the same direction of the jet appear to be moving more slowly, from the jet's perspective, while sound waves moving away from the jet in the generation of the exhaust appear to be moving more quickly.

Because the speed of light is constant with respect to the stationary aether frame in both scenarios, according to Michelson and Morley the optical path difference for the light is given by $ct'_2 - ct'_1$, and the fringe shift is given by[17]

$$\delta_n = \frac{ct'_1 - ct'_2}{\lambda_d} = \frac{2d}{\lambda_d}(\gamma^2 - \gamma) > 0 \tag{40}$$

where λ_d is the distance between fringes. Michelson and Morley did not observe any fringe shift during the course of their experiments[18], and this null result was taken as evidence against the aether hypothesis.

[17] In the original Michelson-Morley analysis, this fringe shift was multiplied by 2 since by rotating the interferometer 90 degrees after measurement, Michelson and Morley were able to double their fringe displacement. However, for the purposes of our analysis we do not need to introduce this extra factor.

[18] In fact, this statement is not entirely correct. They did not observe any fringe shift that they were able to attribute to motion of the aether, although they *did* consistently observe fringe shifts. We will discuss this more in depth later on.

4.2 Optical Distance

Michelson-Morley's fringe shift calculation is unusual. Generally, the method for comparing phase differences for a single frequency of light traveling through materials with different indices is to compute the optical distance for each material as $\delta_D = \eta x$, where δ_D is the optical distance, η is the material's index of refraction, and x is the distance traveled. Then, the phase difference is given by

$$\Delta\phi_n = \frac{\delta_{D_1} - \delta_{D_2}}{\lambda} = \frac{\eta_1 x_1 - \eta_2 x_2}{\lambda} \tag{41}$$

where $\Delta\phi_n$ is the change in phase (measured in cycles, not radians) and λ is the wavelength of light in a vacuum. Note that we are assuming that in all three cases–vacuum, the first material, and the second material–that we are examining a ray of light emitted at a single frequency, which is the same for all three.

In the Michelson-Morley experiment, light only travels through one material, air–although we will not consider the effect of air's index of refraction in this analysis, so we will assume a vacuum instead. In any case, it was the motion of the laboratory, rather than motion through different materials, which was expected to cause a phase difference. Because of this, Michelson and Morley could not simply apply equation (41) in its present state to their experiment. Instead, we suggest that they converted the optical distance into a usable form for their analysis as follows:

$$\delta_D = \eta x = \left(\frac{c}{c'}\right) x = c\left(\frac{x}{c'}\right) = ct' \tag{42}$$

Then, they divided by λ to obtain their fringe shift formula as expressed in equation (40). Superficially, their conversion seems to be logical and reasonable. However, it overlooks the reason *why* equation (41) actually works. Consider a single frequency of light traveling through two different materials with different indices of refraction, as shown in figure (11):

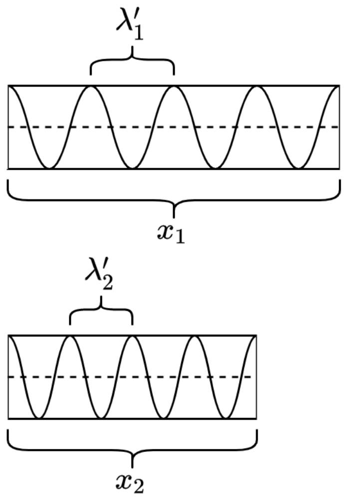

Figure 11: A standing wave in materials with different indices of refraction

The optical distances of these two waves are equal, despite the fact that the geometric distances traveled (x_1 and x_2) are not. Thus from figure (11) we have the relation

$$\frac{x_1}{\lambda_1'} = \frac{x_2}{\lambda_2'} \tag{43}$$

for two equal optical distances. Note that the travel time for light in both cases is also equal.

With this in mind, we can derive equation (41) by comparing the phases of moving waves in two different materials. The phase of a moving wave (measured in cycles) with frequency f after traveling a distance x is given by:

$$\phi = ft' - \frac{x}{\lambda'} \tag{44}$$

where t' is the measured time and λ' is the measured wavelength.

We can multiply both sides by λ so that

$$\begin{aligned}\lambda\phi &= \lambda ft' - \frac{\lambda x}{\lambda'} \\ &= \lambda ft' - \frac{f\lambda x}{f\lambda'} \\ &= \lambda ft' - \frac{cx}{c'} \\ &= \lambda ft' - \eta x\end{aligned} \tag{45}$$

And dividing back by λ again, we have:

$$\phi = ft' - \frac{\eta x}{\lambda} \tag{46}$$

When we compare materials of different refractions, we are making the comparison simultaneously at time t' for each material (otherwise, we could let a wave travel through one material for any arbitrarily long time period, and obtain any phase difference we please, rendering the comparison meaningless). Thus, when we apply this result to compute the phase shift, the time variables are eliminated:

$$\begin{aligned}\Delta\phi_n &= \phi_2 - \phi_1 \\ &= \left(ft' - \frac{\eta_2 x_2}{\lambda}\right) - \left(ft' - \frac{\eta_1 x_1}{\lambda}\right) \\ &= \frac{\eta_1 x_1 - \eta_2 x_2}{\lambda}\end{aligned} \tag{47}$$

This is the reason equation (41) works without considering time. However, notice what happens if we start with equation (44) and attempt to put it into the form of equation (41) for the Michelson-Morley experiment:

$$\begin{aligned}\Delta\phi_n &= \phi_2 - \phi_1 \\ &= \left(ft_2' - \frac{x}{\lambda_2'}\right) - \left(ft_1' - \frac{x}{\lambda_1'}\right) \\ &= f(t_2' - t_1') - \left(\frac{x}{\lambda_2'} - \frac{x}{\lambda_1'}\right)\end{aligned} \tag{48}$$

We are immediately faced with a problem. The observed times for each path are different, thus the time variables do not cancel. Likewise, the observed wavelengths along each path are different, and these will not cancel either. We will examine the rest of the phase shift calculation shortly, but here we've demonstrated the crux of the problem: Michelson-Morley's application of optical distance simply does not apply to an interferometer in which the path differences are not caused by refraction.

We suggest that perhaps a distinction should be made between optical path *distance*, which is defined as $\delta_D = \eta x$, and optical path *length*, which could be defined as $\delta_L = ct'$, so that

$$\phi = \frac{\delta_L - \delta_D}{\lambda} \tag{49}$$

Michelson and Morley expected that a difference in the travel times of the two beams of light would create an interference pattern. However, an interferometer does not measure differences in propagation times; rather, it measures differences in *phase*. The interference pattern observed depends only on the light at the location of the beam splitter at the moment of observation; whether different components of this light took different paths to the beam splitter or left the source at different times is irrelevant to the pattern observed. Once the light source is turned on, the light is flowing continuously and beams emitted at different times may be returning to the beam splitter at the same instant. We only need to concern ourselves with the question, *what is the phase of light upon its return to the beam-splitter?* This must be calculated for each separate path in the interferometer.

A difference in propagation time does not necessarily correspond to a difference in phase if the wavelength of light cannot be assumed to remain constant throughout its journey. In fact, the wavelength of light cannot be assumed constant throughout the experiment, because in order to have a difference in propagation time, there must be a corresponding change in the observed speed of light down each path. A change in wavelength then necessarily follows from our previously discussed principle of frequency invariance, which requires $c' = f\lambda'$.

By taking the speed of light to be the same along both optical paths, Michelson and Morley mistakenly assumed their conclusion (technically speaking, Einstein's conclusion), which is that the speed of light is constant. This is reasoning from a false premise. Simply put, equation (40) is meaningless; it does not compute the phase shift.

Thus, we cannot simply calculate fringe shift using the absolute speed of light—we must calculate the observed speed of light along each direction of travel, which affects the apparent wavelength of light along each direction of travel. Both travel time and wavelength shift must be accounted for in the phase shift calculation.

It can be difficult to imagine that two beams of light may arrive at the same position[19] at different times (in the laboratory frame), yet still not produce any interference. However, it is well-known that in the case of a beam of light traveling through glass, and a parallel beam traveling along the same path through air, the two beams will arrive out-of-phase at a detector[20] and produce interference. This is observed despite the fact that their geometric paths were the same, because their optical paths

[19]We can consider this position to be the location of the beam-splitter along the return trip.

[20]We assume that the source of light is at the same location for both beams, and that the detector is placed at the location where the first beam exits the glass.

were not.

Similarly, we can imagine that two beams may arrive in-phase despite having taken different geometrical paths. From the view of the stationary aether frame, the two beams in the interferometer were traveling at the same speed but arrived at different times (and different places) due to a difference in their geometric paths. Nevertheless, as we will see, due to the effect of Doppler shift they arrived in-phase.

4.3 Longitudinal and Transverse Doppler Shift

The observed round-trip time down either path can be expressed as

$$t' = \frac{d}{c'_{\uparrow}} + \frac{d}{c'_{\downarrow}} \tag{50}$$

where $c_{\uparrow}$ represents the forward trip and $c_{\downarrow}$ represents the return trip for either beam.

Since $c' = \dfrac{2d}{t'}$, this implies

$$\frac{1}{c'} = \frac{1}{2}\left(\frac{1}{c'_{\uparrow}} + \frac{1}{c'_{\downarrow}}\right) \tag{51}$$

where c' is a harmonic mean of the forward and return speeds. Since $c' = f\lambda'$ due the principle of frequency invariance discussed earlier, this implies

$$\frac{1}{\lambda'} = \frac{1}{2}\left(\frac{1}{\lambda'_{\uparrow}} + \frac{1}{\lambda'_{\downarrow}}\right) \tag{52}$$

To understand the effect of longitudinal Doppler shift in the interferometer, let us analyze the mirror 1 path. Using equation (31) to substitute for $\lambda_{\uparrow}$ and $\lambda_{\downarrow}$, we calculate the observed wavelength adjusted for longitudinal Doppler shift:

$$\begin{aligned}
\frac{1}{\lambda'_1} &= \frac{1}{2\lambda}\left(\frac{1}{1-\dfrac{v}{c}} + \frac{1}{1+\dfrac{v}{c}}\right) \\
&= \frac{1}{2\lambda}\left(\frac{1+\dfrac{v}{c}}{1-\left(\dfrac{v}{c}\right)^2} + \frac{1-\dfrac{v}{c}}{1-\left(\dfrac{v}{c}\right)^2}\right) \\
&= \frac{1}{\lambda}\left(\frac{1}{1-\left(\dfrac{v}{c}\right)^2}\right) \\
&= \frac{\gamma^2}{\lambda}
\end{aligned} \tag{53}$$

Thus,

$$\lambda'_1 = \frac{\lambda}{\gamma^2} \tag{54}$$

The observed distance traveled along the mirror 1 path is then given by:

$$\begin{aligned} c_1' t_1' &= (f\lambda_1')t_1' \\ &= f\left(\frac{\lambda}{\gamma^2}\right)(t\gamma^2) \\ &= (f\lambda)t \\ &= ct \end{aligned} \tag{55}$$

For $t = \dfrac{2d}{c}$, we have $c_1' t_1' = 2d$. This is an important, if somewhat obvious result: The observed distance traveled along the mirror 1 path is the same as the distance traveled in the stationary case without any aethereal motion.

Next, we can calculate the effect of transverse Doppler shift along the mirror 2 path. From the previous section, the observed time for a beam of light traveling along the mirror 2 path is given by:

$$t_2' = t\gamma \tag{56}$$

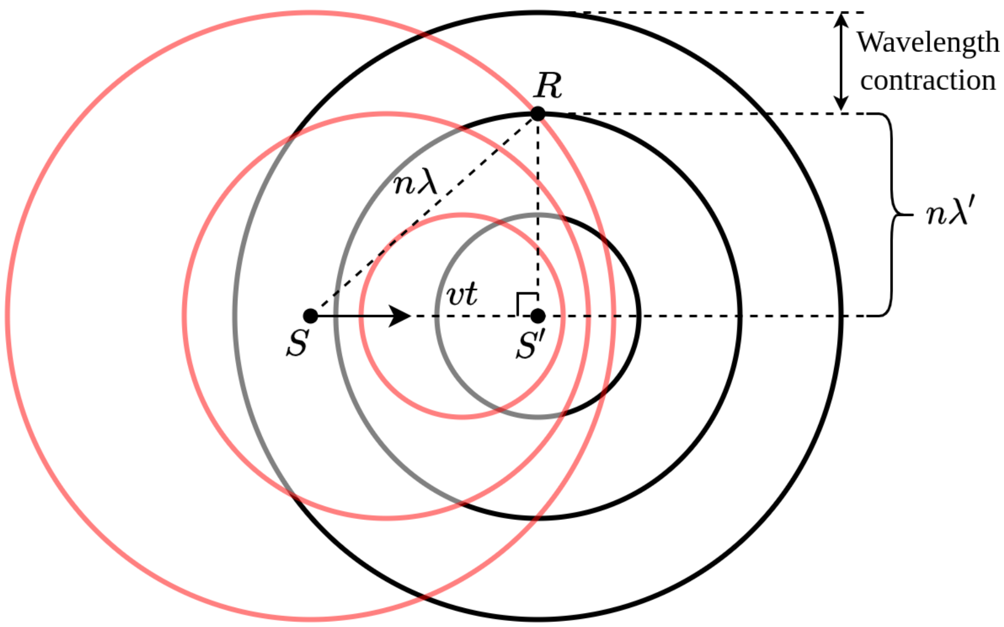

Figure 12: Transverse Doppler shift for a moving source

To calculate the observed wavelength, consider figure (12). At time zero, a source at S traveling at speed v emits a wavefront (shown in red) traveling with speed c, and at time t this wavefront reaches a stationary receiver at R. During this time, the source emits n wavefronts and moves from S to S'. Since S resides at the center of the outermost red wavefront, the distance from S to R is equal to n stationary wavelengths, $n\lambda$, and since these wavefronts travel at speed c, the travel time can be expressed as $t = \dfrac{n\lambda}{c}$. The distance from S' to R is equal to n observed wavelengths,

$n\lambda'$. Using the Pythagorean theorem, we can express the relation betwen distances as

$$\begin{aligned}(n\lambda)^2 &= (n\lambda')^2 + (vt)^2 \\ &= (n\lambda')^2 + \left(v\left(\frac{n\lambda}{c}\right)\right)^2\end{aligned} \tag{57}$$

Canceling the n's and rearranging, we have

$$\begin{aligned}\lambda'^2 &= \lambda^2 - \left(\frac{v\lambda}{c}\right)^2 \\ &= \lambda^2\left(1 - \left(\frac{v}{c}\right)^2\right)\end{aligned} \tag{58}$$

and taking the square root of both sides, we obtain:

$$\lambda' = \lambda\sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{\lambda}{\gamma} \tag{59}$$

This wavelength contraction, given by equation (59), is mathematically equivalent to the Lorentz contraction in special relativity; however, the nature of this contraction is classical, not relativistic: The beam's wavelength is contracting in the transverse direction along mirror 2 (rather than the arm of the apparatus contracting longitudinally along mirror 1, as the theory of relativity holds), and this wavelength contraction is derived purely from geometric considerations.

For the mirror 2 path,

$$\begin{aligned}\frac{1}{\lambda'} &= \frac{1}{2}\left(\frac{1}{\lambda'_\uparrow} + \frac{1}{\lambda'_\downarrow}\right) \\ &= \frac{1}{2\lambda}\left(\frac{2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}\right) \\ &= \frac{\gamma}{\lambda}\end{aligned} \tag{60}$$

in which we have made use of our equation for transverse Doppler shift to express $\lambda'_\uparrow = \lambda'_\downarrow = \lambda\sqrt{1 - \left(\frac{v}{c}\right)^2}$. Thus,

$$\lambda'_2 = \frac{\lambda}{\gamma} \tag{61}$$

The observed distance traveled along the mirror 2 path is then given by:

$$\begin{aligned}c'_2 t'_2 &= (f\lambda'_2)t'_2 \\ &= f\left(\frac{\lambda}{\gamma}\right)(t\gamma) \\ &= (f\lambda)t \\ &= ct\end{aligned} \tag{62}$$

While it is obvious from the experimental setup that both observed distances are equal, it is nevertheless worthwhile to see how the calculation is performed.

4.4 Corrected Phase Analysis

Here we calculate the phase shift along each path. Phase can be expressed as

$$\phi(x,t) = \omega t - kx \tag{63}$$

for a wave traveling a distance x in time t, where $k = \frac{2\pi}{\lambda}$ is the wave number and $\omega = 2\pi f$ is the angular frequency.

We can compute the phase difference for a round trip (starting from $\phi(0,0) = 0$) by substituting $x = 2d$, $t = t'$, $\omega = 2\pi f$, and $k = \frac{2\pi}{\lambda'}$ into $\phi(x,t)$. For path 1:

$$\begin{aligned}
\Delta\phi_1 &= 2\pi \left(f t_1' - \frac{x}{\lambda_1'} \right) \\
&= 2\pi \left(f(t\gamma^2) - 2d\left(\frac{\gamma^2}{\lambda}\right)\right) \\
&= \frac{2\pi}{\lambda}\left((f\lambda)(t\gamma^2) - 2d\gamma^2\right) \\
&= \frac{2\pi}{\lambda}(ct\gamma^2 - 2d\gamma^2) \\
&= \frac{2\pi}{\lambda}(2d\gamma^2 - 2d\gamma^2) \\
&= 0
\end{aligned} \tag{64}$$

keeping in mind $c = f\lambda$ and $ct = 2d$. The calculation for path 2 is similar:

$$\begin{aligned}
\Delta\phi_2 &= 2\pi \left(f t_2' - \frac{x}{\lambda_2'} \right) \\
&= 2\pi \left(f(t\gamma) - 2d\left(\frac{\gamma}{\lambda}\right)\right) \\
&= \frac{2\pi}{\lambda}\left((f\lambda)(t\gamma) - 2d\gamma\right) \\
&= \frac{2\pi}{\lambda}(ct\gamma - 2d\gamma) \\
&= \frac{2\pi}{\lambda}(2d\gamma - 2d\gamma) \\
&= 0
\end{aligned} \tag{65}$$

At this point it should be clear that whenever the wavelength and travel time are scaled by factors that are multiplicative inverses of each other, phase will remain invariant. Thus, the expected phase shift for a roundtrip route is zero for each path in the interferometer.

4.5 Phase Analysis For General Roundtrip Paths

Phase analysis using Doppler shift can also be applied to variations of the Michelson-Morley experiment using different length paths at arbitrary (not necessarily perpendicular) angles, such as the Kennedy-Thorndike experiment, by adjusting one path to rest at an angle of θ from horizontal and analyzing the resulting geometry.

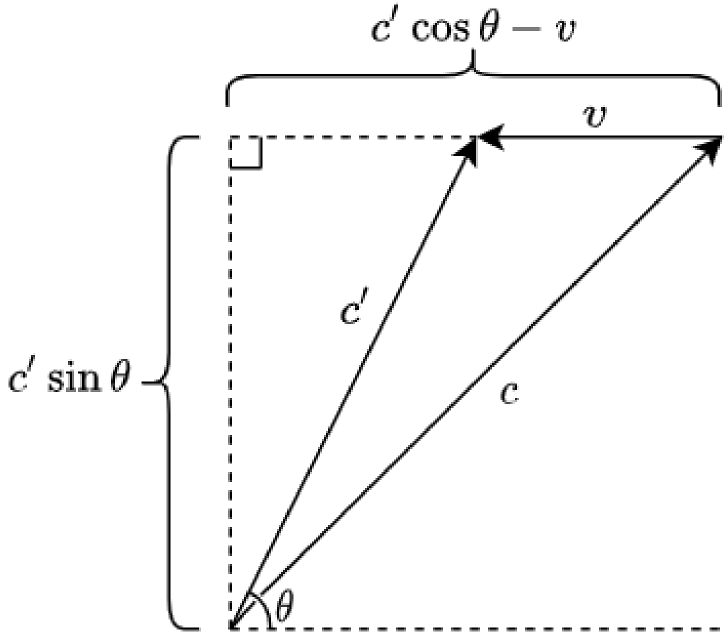

Figure 13: Real and apparent velocities at arbitrary interferometer angle

In figure (13), a beam of light is directed along a path that is angle θ from horizontal, and travels with an observed velocity of c' in the laboratory frame while traveling through the aether with velocity v. Meanwhile, in the stationary aether frame the beam travels at velocity c. From the geometry of the figure, we can use the Pythagorean theorem to express the relation between these variables:

$$\begin{aligned} c^2 &= (c'\cos\theta - v)^2 + (c'\sin\theta)^2 \\ &= (c'\cos\theta)^2 - 2c'v\cos\theta + v^2 + (c'\sin\theta)^2 \\ &= c'^2 - 2c'v\cos\theta + v^2 \\ &= (c' - v\cos\theta)^2 - (v\cos\theta)^2 + v^2 \\ &= (c' - v\cos\theta)^2 + v^2(1-\cos^2\theta) \\ &= (c' - v\cos\theta)^2 + (v\sin\theta)^2 \end{aligned} \tag{66}$$

Rearranging to solve for c', we have

$$\begin{aligned} c' &= v\cos\theta + c\sqrt{1 - \left(\frac{v\sin\theta}{c}\right)^2} \\ &= v\cos\theta + \frac{c}{\gamma_\theta} \end{aligned} \tag{67}$$

in which we have defined γ_θ as:

$$\gamma_\theta = \frac{1}{\sqrt{1 - \left(\frac{v\sin\theta}{c}\right)^2}} \tag{68}$$

Using the identity $\cos(\theta+\pi) = -\cos\theta$, we can calculate the observed time t' for a round trip:

$$
\begin{aligned}
t' &= \frac{d}{\dfrac{c}{\gamma_\theta} + v\cos\theta} + \frac{d}{\dfrac{c}{\gamma_\theta} - v\cos\theta} \\
&= d\left\{ \frac{\dfrac{c}{\gamma_\theta} - v\cos\theta}{\left(\dfrac{c}{\gamma_\theta}\right)^2 - (v\cos\theta)^2} + \frac{\dfrac{c}{\gamma_\theta} + v\cos\theta}{\left(\dfrac{c}{\gamma_\theta}\right)^2 - (v\cos\theta)^2} \right\} \\
&= \frac{2dc}{\gamma_\theta}\left\{ \frac{1}{\left(\dfrac{c}{\gamma_\theta}\right)^2 - (v\cos\theta)^2} \right\} \\
&= \frac{2d}{c\gamma_\theta}\left\{ \frac{1}{1 - \left(\dfrac{v\sin\theta}{c}\right)^2 - \left(\dfrac{v\cos\theta}{c}\right)^2} \right\} \\
&= \frac{2d}{c\gamma_\theta}\left\{ \frac{1}{1 - \left(\dfrac{v}{c}\right)^2} \right\} \\
&= \frac{2d}{c}\left(\frac{\gamma^2}{\gamma_\theta}\right)
\end{aligned}
\tag{69}
$$

Thus, for observed time we have the result:

$$
t' = t\left(\frac{\gamma^2}{\gamma_\theta}\right) \tag{70}
$$

From our previous discussion of the equation for general Doppler shift, we can express the observed wavelength for the foward trip as:

$$
\begin{aligned}
\lambda'_\uparrow &= \lambda\left(\frac{v}{c}\cos\theta + \sqrt{1 - \left(\frac{v\sin\theta}{c}\right)^2}\right) \\
&= \lambda\left(\frac{v}{c}\cos\theta + \frac{1}{\gamma_\theta}\right)
\end{aligned}
\tag{71}
$$

Likewise, the observed wavelength for the return trip is given by:

$$
\lambda'_\downarrow = \lambda\left(-\frac{v}{c}\cos\theta + \frac{1}{\gamma_\theta}\right) \tag{72}
$$

From our previous discussion of observed wavelength, starting with equation (52) we have:

$$
\begin{aligned}
\frac{1}{\lambda'} &= \frac{1}{2}\left(\frac{1}{\lambda'_{\uparrow}} + \frac{1}{\lambda'_{\downarrow}}\right) \\
&= \frac{1}{2\lambda}\left\{\frac{1}{\frac{1}{\gamma_\theta} + \frac{v}{c}\cos\theta} + \frac{1}{\frac{1}{\gamma_\theta} - \frac{v}{c}\cos\theta}\right\} \\
&= \frac{1}{2\lambda}\left\{\frac{\frac{1}{\gamma_\theta} - \frac{v}{c}\cos\theta}{\frac{1}{\gamma_\theta^2} - \left(\frac{v}{c}\cos\theta\right)^2} + \frac{\frac{1}{\gamma_\theta} + \frac{v}{c}\cos\theta}{\frac{1}{\gamma_\theta^2} - \left(\frac{v}{c}\cos\theta\right)^2}\right\} \\
&= \frac{1}{\lambda\gamma_\theta}\left\{\frac{1}{\frac{1}{\gamma_\theta^2} - \left(\frac{v}{c}\cos\theta\right)^2}\right\} \\
&= \frac{1}{\lambda\gamma_\theta}\left\{\frac{1}{1 - \left(\frac{v}{c}\sin\theta\right)^2 - \left(\frac{v}{c}\cos\theta\right)^2}\right\} \\
&= \frac{1}{\lambda\gamma_\theta}\left\{\frac{1}{1 - \left(\frac{v}{c}\right)^2}\right\} \\
&= \frac{1}{\lambda}\left(\frac{\gamma^2}{\gamma_\theta}\right)
\end{aligned}
\tag{73}
$$

Thus, for observed wavelength we have the result:

$$
\lambda' = \lambda\left(\frac{\gamma_\theta}{\gamma^2}\right) \tag{74}
$$

Using equations (74) and (70), we can show that the observed distance traveled in the laboratory frame is equal to the observed distance without any aethereal motion at all:

$$
\begin{aligned}
c't' &= (f\lambda')t' \\
&= f\left(\lambda\left(\frac{\gamma_\theta}{\gamma^2}\right)\right)\left(t\left(\frac{\gamma^2}{\gamma_\theta}\right)\right) \\
&= (f\lambda)\,t \\
&= ct
\end{aligned}
\tag{75}
$$

And since $t = \dfrac{2d}{c}$, we also see that the observed distance is $c't' = 2d$ as expected. Next, we apply these results to phase shift. Phase shift can be expressed as:

$$
\phi(x,t) = \omega t - kx \tag{76}
$$

for a wave traveling a distance x in time t, where $k = \dfrac{2\pi}{\lambda}$ is the wave number and $\omega = 2\pi f$ is the frequency.

We can compute the phase difference for a round trip (starting from $\phi(0,0) = 0$) by substituting $t = t'$ and $k = \dfrac{2\pi}{\lambda'}$ into $\phi(x,t)$. For $x = 2d$,

$$\begin{aligned}\Delta\phi &= 2\pi\left(ft' - \frac{x}{\lambda'}\right) \\ &= 2\pi\left(f\left(\frac{2d}{c}\right)\frac{\gamma^2}{\gamma_\theta} - \left(\frac{2d}{\lambda}\right)\frac{\gamma^2}{\gamma_\theta}\right) \\ &= 2\pi\left(\left(\frac{2d}{\lambda}\right)\frac{\gamma^2}{\gamma_\theta} - \left(\frac{2d}{\lambda}\right)\frac{\gamma^2}{\gamma_\theta}\right) \\ &= 0\end{aligned} \tag{77}$$

keeping in mind $c = f\lambda$. Thus, the expected phase shift for a round-trip route along any arbitrary angle is always zero.

5 Discussion

> *Rule I.* We are to admit no more causes of natural things, than such as are both true and sufficient to explain their appearances.
>
> Newton, *Principia Mathematica*

5.1 Comparison of Theories

A variety of theories have been proposed since 1887 to explain the Michelson-Morley null result. We will order these approximately from most well-known to least:

1. **Relativity**. This is of course, the most-well known and by far the most accepted theory explaining the null result. Einstein's theory of relativity can be succinctly expressed as two postulates: First, the laws of physics are identical in all inertial (non-accelerating) reference frames. Second, the speed of light in a vacuum is constant for all observers (and not dependent on the velocity of the source or the velocity of the observer).

   Relativity denies the existence of any absolute stationary reference frame or aether medium for light, and abandons classical mechanics in the process. Massless photons, mass-energy equivalence, spacetime, and Lorentz invariant field theories all follow.

2. **Lorentz aether theory**. Mathematically, this theory is equivalent to the relativistic explanation of the null result. Lorentzian aether theory postulates the existence of an absolute reference frame and proposes that the Michelson-Morley interferometer contracts longitudinally (with respect to its direction of motion) by the Lorentz factor to ensure equal transit times for each branch.

Lorentzian aether theory (LET) is usually presented as the main alternative to the relativistic explanation. The main difficulty of LET lies in providing a plausible mechanism for this contraction; "aether pressure" lacks a precise description. Rrelativity on the other hand, is seen as providing this mechanism.

3. **Frame-dragging**. This theory holds that the Earth drags the aether along with it, and is usually one of the immediate alternatives raised by individuals encountering the Michelson-Morley result for the first time. Unfortunately, this theory encounters several difficulties with respect to various experimental observations: In particular, it is inconsistent with Bradley stellar aberration and the Michelson-Gale-Pearson experiment (which used the Sagnac effect to experimentally detect the rotation of the Earth from an interferometer).

4. **Ballistic emission theory**. This theory posits that speed of light is dependent on the speed of its source, and that these velocities add in Galilean fashion (for example, a ball thrown forward at 40 mph from a car traveling 50 mph will have a total forward velocity of 90 mph). This can explain the Michelson-Morley null result, but is inconsistent with many other optical experiments, including the Sagnac experiment, Bradley stellar aberration, the timings of the eclipses of Io, Quirino Majorana's rotating mirror apparatus, and various other astronomical and experimental observations. It is very well-established that the speed of light is independent of the motion of its source.

5. **Geocentric theory**. This theory posits that the Earth resides in a preferred stationary frame, and the rest of the universe moves around it. The geocentric hypothesis is questionable from a philosophic standpoint, and is inconsistent with a number of observations, for example, the Coriolis effect (which can be demonstrated with Foucault's pendulum) and the equatorial bulge of the Earth; however, there is a documentary with interviews from prominent physicists that advances this view, so it warrants mention.

Notably, each of the previously mentioned theories offers an explanation for how the two branches of the interferometer do not actually cause different arrival times for different rays of light. The theory advanced by this paper is unique from the others because it does not dispute the prediction of different arrival times; instead, it explains how these differences can lead to the observed null result without abandoning classical mechanics.

6. **Classical Doppler shift**. Based on geometrical arguments, we show that if we have a source and observer moving in parallel with the same velocity, the frequency observed will be the same as in the stationary case. Furthermore, we advance (again based on geometrical arguments) that the wavelength of a moving source will be observed to contract along the transverse axis by a factor equivalent to the Lorentz factor in relativity and LET. We then apply a phase shift calculation to determine the phase of light waves on their return to the beam splitter, and conclude that zero phase shift (and thus, zero interference) should be expected along roundtrip paths at any arbitrary angle.

   The Doppler shift explanation is consistent with the Michelson-Morley experiment and all other optical "relativistic" experiments (e.g., the Sagnac experiment, the Fizeau experiment, Ives-Stillwell, Thorndike-Kennedy, Hafele-Keating, etc). It assumes an absolute reference frame for a stationary aether, and it does not require abandoning classical mechanics. It demonstrates that a careful and rigorous application of classical mechanics resolves the mystery of the Michelson-Morley experiment, and removes the need for the theory of relativity as well as other explanations that unnecessarily propose the abandonment of classical mechanics.

The possibility that the Michelson-Morley null result might be explained by Doppler shift was originally proposed by the German physicist Woldemar Voigt in 1887, although Voigt was not able to provide the correct analysis of the experiment at the time and later withdrew his objections after discussion with Lorentz. In 1983, J.P. Wesley published a paper hypothesizing that the Michelson-Morley result could be explained by a Voigt-Doppler effect, which involved plane waves and differed slightly from the classical Doppler effect [9].

In 2001, Norbert Feist conducted an experiment [13] to test the isotropy of sound waves, using an equation for wave velocity similar to equation (74) in this paper[21]. In 2006, after studying Feist's experiment, Wesley amended his argument and concluded that the Michelson-Morley result is satisfactorily explained by the classical Doppler effect [4]; however, Feist's result is inconsistent with this paper (as discussed in section 5.3) and Wesley's argument differs from the argument presented in this paper–he does not use the general equation for Doppler shift presented here, and he claims that the phase velocity of separate beams differs from the velocity of their energy propagation.

In 2016, Klinaku published the formula presented in this paper (equation (30)) for Doppler shift at an arbitrary angle [8], although similar forms of the equation have appeared in earlier work, as mentioned.

5.2 Predictions

> There are two possible outcomes: if the result confirms the hypothesis, then you've made a measurement. If the result is contrary to the hypothesis, then you've made a discovery.
>
> Enrico Fermi

To clarify how our theory diverges from existing theory, we make the following predictions:

1. Light waves travel within an aethereal medium, and there is an absolute reference frame for which the aether is stationary.

As corollaries, we predict:

2. The speed of light is independent of the motion of its source, but not its observer.

[21]Equation (74) in this paper can be transformed to equation (2) in Feist's 2001 paper by multiplying both sides of the equation by a constant frequency f and performing substitution for γ_θ and γ.

3. For measurements taken within the rotating frame of the Earth, there is an anisotropy in the one-way speed of light, which should vary in accordance with a sidereal day. In other words, it should be possible to conduct experiments that measure the absolute motion of the Earth with respect to the aether.

The anisotropy of light should be detectable as a second-order effect in Michelson-Morley interferometer experiments, if the experiment is not conducted in a vacuum. In fact, it is a common misconception that Michelson and Morley failed to observe any fringe shifts associated with the Earth's sidereal motion; the original Michelson-Morley paper contains an entire table full of fringe shifts containing evidence of absolute aethereal motion [1]. However, because the fringe shifts only corresponded to a nominal speed of roughly 8 km/s, which is significantly less than the Earth's known orbital velocity of 30 km/s, these shifts were nevertheless considered a null result.

In light of the foregoing analysis however, it is interesting that any fringe shift at all could be detected, less experimental error. As it turns out, this fringe shift can be attributed to the fact that the experiment was not conducted in a vacuum, and that the refractive index of air is slightly greater than that of a vacuum. If the analysis of the experiment is expanded to include Fresnel drag (as experimentally determined by D.C. Miller in 1933), then the observed fringe shifts actually correspond to an absolute aethereal motion of greater than 300 km/s [10].

A handful of other experiments have been conducted since D.C. Miller's replication of the Michelson-Morley experiment, which reportedly detected absolute motion. In the 1970's and 1980's, the Belgian physicist Stefan Marinov conducted a variety of coupled-mirror and toothed-wheel experiments (building on experimental work originally conducted by Fizeau) to measure the one-way anisotropy of light, and measured an absolute aethereal motion of roughly 300 km/s [11].

In 1991, Roland De Witte (a telecom engineer) measured variations in the one-way travel time of RF signals through a pair of buried 1.5 km coaxial cable over 178 days, and confirmed that the variations (as measured by two clusters of atomic clocks in separate buildings) clearly tracked the Earth's sidereal time [12]. The fact that two cables were utilized in this experiment is significant, because signals were sent in opposite directions down each cable, so that any external factor causing an anisotropy in travel time should have affected both cables equally. Furthermore, having the cables buried meant that temperature variations (which could have potentially changed the refractive index of gas within the cables) were minimized (and in any case, wouldn't have varied with sidereal time even if they weren't). A similar experiment was conducted by Torr and Kolen in 1984 [10].

In 1996, Monstein and Wesley measured the aethereal motion from the anisotropy of muon flux [4] (an effect often cited as evidence of relativistic time dilation). In 2013, Randy Wayne at Cornell conducted a reproduction of the Fizeau experiment in which he demonstrated that the interference pattern was more accurately predicted by classical Doppler theory than by Newtonian (Galilean) theory or special relativity [14].

4. Classical spherical waves emitted from a moving source will contract in wavelength along the transverse axis, to an extent equivalent to the Lorentz contraction.

5. For a source moving at velocity v_S and a receiver moving at velocity v_R, if $v_S = v_R$ (in magnitude and direction) then the frequency observed by the receiver will be

the same as the case when the source and receiver are both stationary; that is, $v_S = v_R$ implies $f' = f$.

These effects should be measurable in classical waves such as sound waves, or waves in a fluid.

5.3 Feist's Acoustic Experiment

In 2001, Norbert Feist conducted an acoustic experiment [13] similar to the Michelson-Morley experiment, using an ultrasonic range finder set across from a reflecting target surface, with the apparatus mounted on the roof of a BMW. He made a variety of test runs at different speeds with the apparatus mounted at different angles with respect to the vehicle's motion, and measured his speed along with the flight time of the ultrasonic pulses, and used these times to infer the observed speed of the pulses.

Feist tested two hypotheses for the roundtrip speed measurement. The first was that roundtrip speed would be equal to the harmonic mean speed for a longitudinal roundtrip, which can be derived from equation (51):

$$\begin{aligned}\frac{1}{c'} &= \frac{1}{2}\left(\frac{1}{c-v} + \frac{1}{c+v}\right) \\ &= \frac{c}{c^2 - v^2}\end{aligned} \tag{78}$$

Thus, his first hypothesis was that the observed speed for ultrasonic pulses would be:

$$c' = \frac{c^2 - v^2}{c} \tag{79}$$

Note that equation (79) is independent of the pulse's emission angle with respect to the car's motion. Feist's second hypothesis can be derived from equation (74) in our paper as follows:

$$\begin{aligned}\lambda' &= \lambda\left(\frac{\gamma_\theta}{\gamma^2}\right) \\ &= \lambda\left(\frac{1-\left(\frac{v}{c}\right)^2}{\sqrt{1-\left(\frac{v\sin\theta}{c}\right)^2}}\right) \\ &= \frac{\lambda}{c}\left(\frac{c^2-v^2}{\sqrt{c^2-(v\sin\theta)^2}}\right)\end{aligned} \tag{80}$$

Multiplying both sides by f, we obtain his second hypothesis:

$$c' = \frac{c^2 - v^2}{\sqrt{c^2 - (v\sin\theta)^2}} \tag{81}$$

Since the actual variable Feist measured was flight time, we can reformulate his hypotheses in terms of flight time. The first hypothesis is then equivalent to $t' = t\gamma^2$, while his second hypothesis is equivalent to equation (70):

$$t' = t\left(\frac{\gamma^2}{\gamma_\theta}\right) \tag{82}$$

Feist made a series of runs at speeds varying from 0 to 120 km/hr at a variety of angles from 0 and 90 degrees, and generated a series of curves indicating that the first hypothesis, not the second, fit the data. This is rather astonishing, because it suggests that the roundtrip flight-time for the ultrasonic pulses is independent of the angle of the apparatus with respect to its motion, rather than dependent on the angle as the second hypothesis would predict.

Wesley's explanation for Michelson-Morley result (that phase velocity differs from the velocity of energy propagation) cannot explain Feist's result, since Feist was only measuring energy propagation (thus, unlike the Michelson-Morley experiment, Feist's apparatus was not an interferometer). While Feist's result appears to mimic the Michelson-Morley result for acoustic waves, in fact it is as inconsistent with classical wave mechanics as relativity.

Feist suggests (based on the earlier work of a physicist named H. Varcollier) that acoustic waves may be propagated ellipsoidally. This theory seems unlikely for multiple reasons, however. First, it requires a plausible mechanism, which is difficult to envision because the acoustic medium of propagation (the atmosphere) is essentially isotropic. Second, the generated ellipsoidal waves would require an eccentricity that precisely mimics an additional Lorentz factor, out of all of the possible eccentricities they might take. This also seems unlikely.

While Feist's result superficially appears to cast doubt on the relativistic interpretation of the Michelson-Morley experiment, it is also inconsistent with the ordinary classical understanding of wave propagation. We have carefully read Feist's paper and have not found any error in his experimental methodology as described in his paper; however, we nevertheless reject his result and suspect an issue in his methodology.

We propose a replication of Feist's acoustic experiment to be conducted, which can serve to test our prediction of classical transverse wavelength contraction as well as our prediction of frequency invariance. We hypothesize that a careful replication of Feist's experiment should validate his second hypothesis rather than his first.

6 Conclusion

> See now the power of truth; the same experiment which at first glance seemed to show one thing, when more carefully examined, assures us of the contrary.
>
> Galileo Galilei

We conclude (as our title states) that classical Doppler shift, when applied to the Michelson-Morley interferometer experiment, satisfactorily accounts for the null result. Thus, the classical aether emission model for light (in which the velocity of light is source-independent, observer-dependent) does not need to be abandoned to explain this experimental result. Other well-known optical experiments (for example, Ives-Stillwell, Kennedy-Thorndike, and Sagnac) can also be interpreted using the classical emission model for light. In light of our analysis, we suggest that the classical emission model for light was abandoned prematurely, and the Michelson-Morley result did not require classical physics to be superseded by the theory of relativity.

Relativity is the theoretical foundation of much of the past century's efforts in theoretical physics. An overlooked issue in the foundation of modern physics would have a cascading effect on the rest of physics, casting doubt on many other theoretical analyses dependent on relativity. While this would be a personal tragedy for many practitioners in the field who have based much of their work on relativistic assumptions, resolving any underlying issues in the foundations of physics would be a great step forward for the field as a whole, paving the way for new progress to be made along many previously dismissed lines of investigation.

Furthermore, the absence of a cosmic speed limit would be exciting news. Many areas of the universe previously thought to be unreachable by communication or travel might be less disconnected than previously imagined. Many new technologies, previously thought to be impossible (as heavier-than-air human flight[22] and alternating current generators[23] were once perceived, for example), might become significantly more plausible. It would certainly make for an exciting new era in physics.

7 Acknowledgments

Thank you Gabriele Wesley-Modest for your assistance, and thank you Gerald Leb for your encouragement.

Appendices

A Classical Doppler Approximation

A.1 Derivation

Here we reproduce the figure and derivation for classical Doppler shift at an arbitrary angle, as found in the MIT educational textbook *Vibrations and Waves* by A.P. French, with some minor additions to improve clarity and differences in notation to maintain consistency with this paper. French's textbook does not cite the original source of this derivation, but we assume that it is not French's original work. We have also added additional wavefronts to the figure to clarify its meaning.

[22]In 1896, Lord Kelvin wrote in reply to an invitation to join the Aeronautical Society: "I have not the smallest molecule of faith in aerial navigation other than ballooning or of expectation of good results from any of the trials we hear of." The Wright Brothers' first sustained, powered flight followed in 1903.

[23]When Nikola Tesla was a young physics student in 1877 interested in the prospect of creating an AC generator, his physics professor remarked, "Mr. Tesla may achieve many great things, but he certainly will never do this. It would be equivalent to converting a steady pulling force, like that of gravity, into a rotary effort. It is a perpetual motion scheme, an impossible idea."

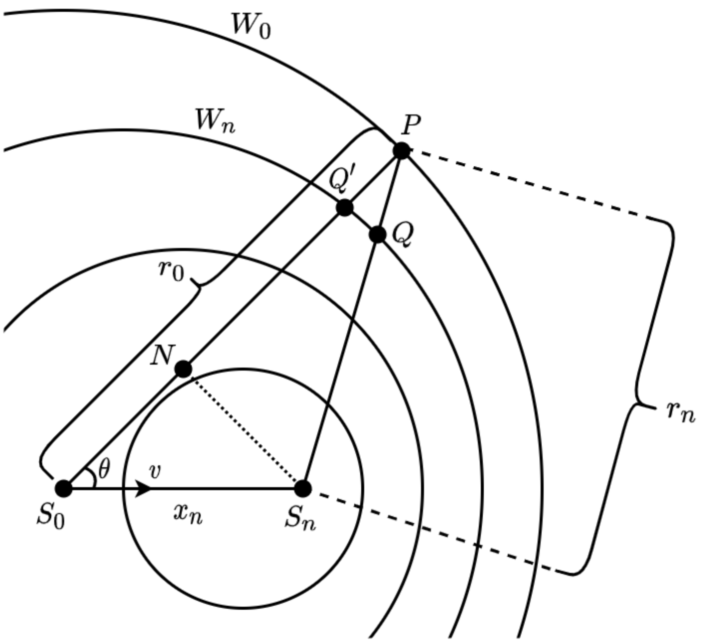

Figure 14: Waves arriving at a distant point P from a source moving from S_0 to S_n

As usual, v is the speed of the source, c is the speed of the wave propagation, and T is the time between consecutive wave emissions. The points marked S_0 and S_n represent the positions of the source at $t = 0$ and at $t = nT$ (n periods later). Since the speed of the source is v, we have

$$\overline{S_0 S_n} = x_n = vnT \tag{83}$$

Since the point of observation, P, is assumed to be far away, the angle $\angle S_0 P S_n$ is very small. This means that the wavefronts arriving at P from S_0 and S_n (and all intermediate source points) are almost parallel. Suppose that the wave W_0 from S_0 has just reached P. This defines a time

$$t_P \equiv \frac{r_0}{c} \tag{84}$$

The wave from S_n started out at $t = nT$; thus at time t_P it has been traveling only for a time $t_P - nT$; its wavefront is at W_n, and we have

$$\begin{aligned} \overline{S_n Q} &= c(t_P - nT) \\ &= ct_P - cnT \\ &= r_0 - cnT \end{aligned} \tag{85}$$

The distance between the wavefronts can be taken to be equal to either $\overline{QP}$ or $\overline{Q'P}$ (the difference between them is insignificant). If we put $\overline{S_n P} = r_n$, we have

$$\begin{aligned} \overline{QP} &= r_n - \overline{S_n Q} \\ &= r_n - r_0 + cnT \\ &= cnT - (r_0 - r_n) \end{aligned} \tag{86}$$

But if we drop a perpendicular from S_n onto the line $\overline{S_0P}$, we also have $\overline{NP} \approx r_n$ (again because of the smallness of the angle $\angle S_0PS_n$), so that

$$\begin{aligned} r_0 - r_n &\approx \overline{S_0N} \\ &= x_n \cos\theta \\ &= vnT\cos\theta \end{aligned} \tag{87}$$

Substituting this in the preceding expression for $\overline{QP}$, we have

$$\begin{aligned} \overline{Q'P} \approx \overline{QP} &\approx cnT - vnT\cos\theta \\ &= cnT\left(1 - \frac{v\cos\theta}{c}\right) \\ &= n\lambda\left(1 - \frac{v\cos\theta}{c}\right) \end{aligned} \tag{88}$$

But $\overline{Q'P}$ or $\overline{QP}$ spans n wavelengths of the disturbance as observed at the direction θ to the moving source. Thus we have

$$\lambda' = \lambda\left(1 - \frac{v\cos\theta}{c}\right) \tag{89}$$

A.2 Discussion

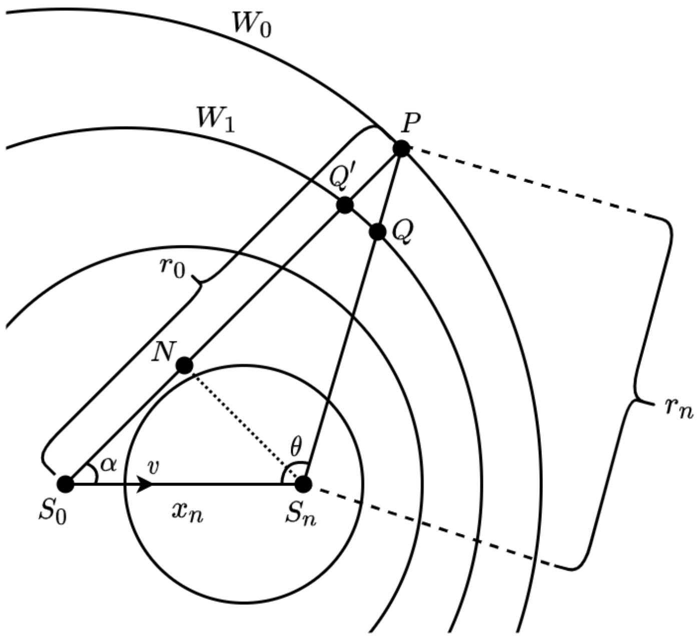

Figure 15: Waves arriving at a distant point P from a source moving from S_0 to S_n

First, we note that figure (14) is not labeled appropriately. W_n should be located much closer to S_n since it was the most recently emitted wavefront; thus, it is more sensible to relabel W_n as W_1 and to substitute $n = 1$ in all places where it occurs in the previous derivation. We have relabeled W_n as W_1 in figure (15) to illustrate the correction. The rest of the derivation can then proceed as before.

Second, we note that the derivation makes clear that it is an *approximation* in several places; it treats $\overline{Q'P}$ and $\overline{QP}$ interchangeably, and it treats $\overline{NP}$ as equal to r_n, when $\overline{NP}$ is clearly shorter[24].

Thus, it is perhaps an open secret that equation (89) is an approximation, not a precise equation. Why then, is it treated as a precise equation when applied to the Michelson-Morley experiment? It should raise suspicion that the analysis of the longitudinal path of the interferometer includes longitudinal Doppler shift, yet no Doppler shift is applied to the transverse path. Equation (89), which is an approximation, is applied to the Michelson-Morley experiment as if it is a precise equation–yet it is not.

Third, we note that the angle θ in the original figure is not the correct angle for the calculation of Doppler shift. In figure (15) we have relabeled this angle α and labeled the correct angle θ. This distinction is important; the angle needs to be placed at the current location of the source when the wavefront reaches P, not the location of the source when the wavefront was emitted. Otherwise, we won't obtain the correct expression for Doppler shift.

To derive the precise expression for Doppler shift from figure (15), we can use the law of cosines to write:

$$r_0^2 = x_n^2 + r_n^2 - 2x_n r_n \cos\theta \tag{90}$$

Then, substituting $r_0 = n\lambda$, $r_n = n\lambda'$, $x_n = vnT$, and $T = \dfrac{\lambda}{c}$, we have:

$$(n\lambda)^2 = \left(\frac{n\lambda v}{c}\right)^2 + (n\lambda')^2 - 2\left(\frac{n\lambda v}{c}\right)(n\lambda')\cos\theta \tag{91}$$

After canceling n's, this becomes

$$\lambda^2 = \left(\frac{\lambda v}{c}\right)^2 + \lambda'^2 - 2\left(\frac{\lambda v}{c}\right)\lambda'\cos\theta \tag{92}$$

We leave it as an exercise for the reader to verify that the general equation for Doppler shift (equation (30)) can be derived from equation (92).

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