A Barrier to the Vaidya Event Horizon

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For distant observers stationary black holes are objects existing in the infinite future. This stems from asymptotic behaviour of inbound null geodesics at their critical radii. However, these objects either exist in a vacuum or are the source of an electromagnetic field if charged. By contrast, astrophysical black holes emit radiation [1] and exist in an environment containing accreting matter. More realistic models of black holes due to Vaidya [2] exchange mass-energy with their environment and consequently these solutions are non-stationary. Recently the Vaidya metric was presented in diagonal form [3], and this exposes features that may not be apparent in the traditional Eddington-Finkelstein form. One of these features appears to be an abrupt change in the direction of gravitational acceleration close to the critical radius, leading to a diverging potential barrier halting further collapse. It must be noted however, that this may only be a feature of the specific solution considered here, and such behaviour may not be generic. The purpose of this letter is to show how this comes about, and to suggest that, if true generally, would imply a resolution of the information paradox.

The Vaidya metric is traditionally represented in the Eddington-Finkelstein form

$$ds^2 = \left(1 - \frac{2m(z)}{r}\right) dz^2 + 2dzdr - r^2 d\Omega^2 \tag{1}$$

where, given null time variables u or v, $z = -v$ for an accreting black hole, or $z = u$ in the evaporating case. However, this form of the metric hides important features such as the ability of falling particles to reach the horizon in a finite time relative to asymptotic observers. This feature is far from apparent in the form of equation (1). Moreover, care must be taken to include energy-momentum tensor fields of the surrounding environment, and these fields are sufficient to perturb the solution in such a way as to allow inbound null geodesics to intersect the critical radius before final evaporation. That freefalling matter close to the critical radius is best modelled as null radiation makes equation (1) a natural choice of coordinates for this application.

However, to elucidate important features of the Vaidya solution we diagonalise equation (1). This has the benefit of making the coordinates time-symmetric and more meaningful as far as physical measurement is concerned. More recently Berezin *et al* [3] have recast this metric in diagonal form. A diagonal Vaidya metric may be written

$$ds^2 = f_0(t, r; \alpha) dt^2 - \left(1 - \frac{2M(t)}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 \tag{2}$$

where f_0 is given by

$$f_0(t,r;\alpha)=\left(1-\frac{2M(t)}{r}\right)^{-1}\left|\frac{1}{2}-\frac{2M(t)}{r}+\frac{\sqrt{1-8\alpha}}{2}\right|^{1+\sqrt{1-8\alpha}}\left|\frac{1}{2}-\frac{2M(t)}{r}-\frac{\sqrt{1-8\alpha}}{2}\right|^{1-\sqrt{1-8\alpha}}. \tag{3}$$

The parameter, α, is accretion or evaporation rate as appropriate, which at any instant Berezin *at al* made linear in the null time variable. This simplifies things by setting $\alpha\,(=-dm/dz)$ as a constant. It is immediately seen that equation (3) is valid for $\alpha\in[0,1/8]$.

Inspection of equation (3) shows that space-time is partitioned into four regions bounded by three concentric timelike hyper-tubes, whose radii vary with *t*. Here we label these radii as $r_c<r_1<r_2$. The critical radius, r_c, is where the event horizon appears in the Schwarzschild solution. The function, f_0, vanishes at r_1 and r_2, and we necessarily see divergent time dilation at these points. In the Schwarzschild limit $r_1\to r_c$ and $r_2\to\infty$. At sufficiently large distance the modelling of accreting matter as null radiation is inaccurate and outgoing Hawking-radiation is relatively weak, therefore it is difficult to see the physical significance of the zero at r_2. Moreover, the exponent on the corresponding factor in equation (3) is small. So we confine our interest to the range (r_c,R), $R<r_2$.

The radial freefall time, Δt_F, from a specified radius, $R>r_1$ to r_1 is represented by

$$\Delta t_F=\int_{r_1}^{R}\sqrt{\frac{-g_{rr}}{g_{00}}}dr \qquad r_1<R<r_2.$$

where $g_{rr}=-\left(1-2M(t)/r\right)^{-1}$. In the Schwarzschild limit, this integral is known to diverge. Moreover similar freefall times also diverge for Kerr-Newman metrics. However, taking $g_{00}=f_0(r,t;\alpha)$ from equation (3) we see convergence because the exponent in the second factor $1+\sqrt{1-8\alpha}<2$. Therefore an inbound null trajectory will exhibit an inflection point at r_1 instead of approaching it asymptotically as in stationary cases. Beyond r_1, the null cones open out again, allowing inbound null trajectories to reach r_c for $t<\infty$.

By holding *t* constant, it is useful to plot f_0 against *r* to see the relevant features and to compare with the Schwarzschild limit ($\alpha\to 0$), see figure 1. The most obvious feature is a sign change in the gradient seen at r_1, which becomes negative in the interval (r_c,r_1). The geodesic equation for a test particle, where $U^r=0$ momentarily, gives the radial component of four-acceleration as

$$\dot{U}^r=\frac{1}{2}g^{rr}\left(g_{00,r}\right)=\frac{1}{2}g^{rr}f_{0,r}.$$

From equation (2) we notice that g_{rr} (and therefore g^{rr}) is negative for $r>r_c$. The positive gradient of f_0 above r_1 makes $\dot{U}^r$ negative thus representing attraction as expected. The change in direction of $f_{0,r}$ at r_1 implies gravitational repulsion from the centre in (r_c,r_1) with f_0 as an effective potential. The question is, does the actual potential diverge at r_c ?

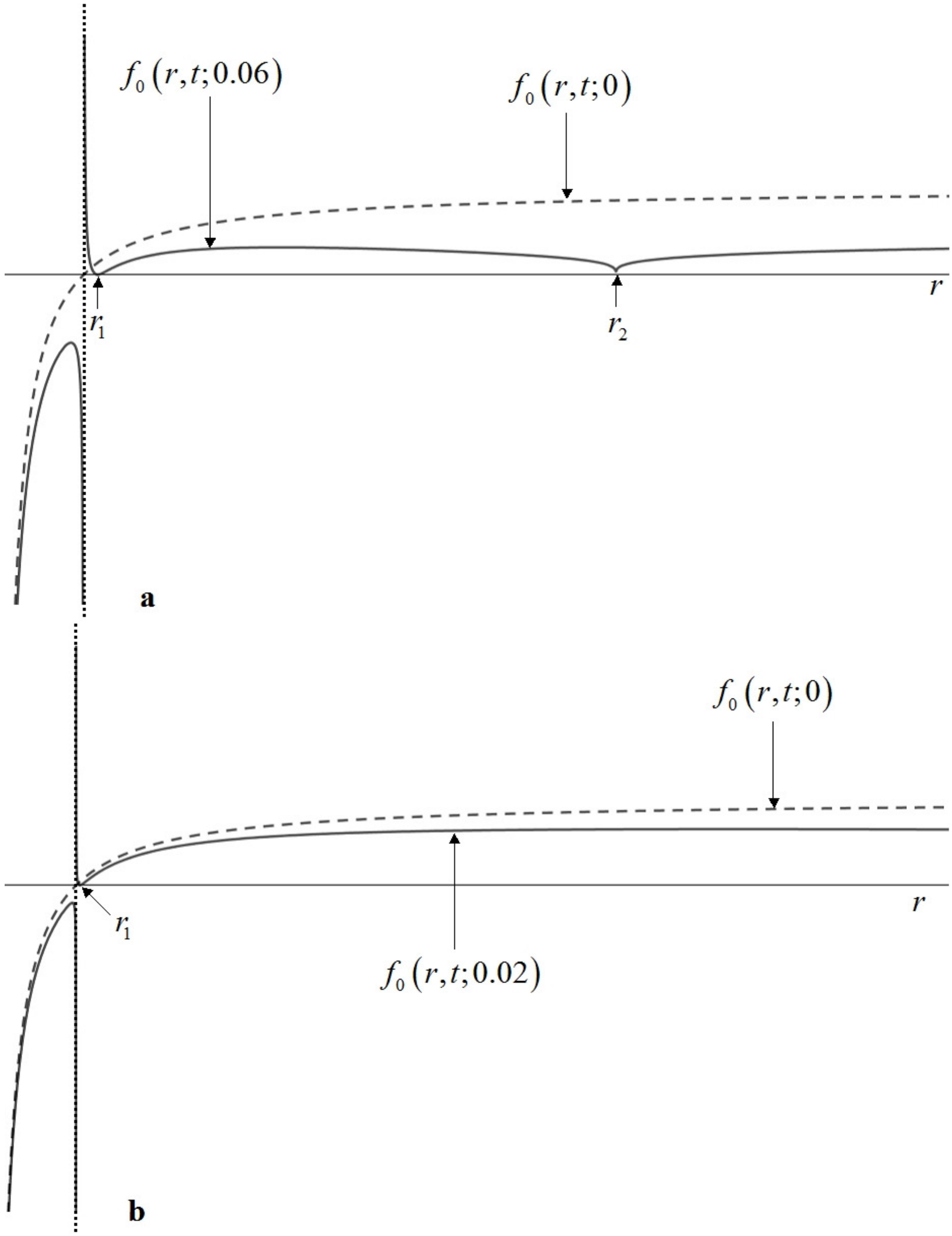

Figure 1: Graph of $f_0(r,t;\alpha)$ from equation (3) (solid line). The dashed line is the corresponding graph in the Schwarzschild case ($\alpha = 0$). The vertical dotted line marks the position of the critical radius, r_c. **a** $\alpha = 0.06$, **b** $\alpha = 0.02$.

The best way to answer this is to consider the Hamiltonian for a low mass particle of intrinsic mass μ in a general gravitational field. Here we start by representing the intrinsic mass in terms of four-momentum, p_a, in a general space-time: this is given by $\mu^2 = p^a p_a$. The derivation is as follows; decomposing the right hand side into temporal and spatial components we have

$$\mu^2 = p_a p_b g^{ab}$$
$$= p_0 p_0 g^{00} + 2 p_0 p_i g^{0i} + p_i p_j g^{ij}, \quad i, j = r, \theta, \phi$$

Solving the quadratic for the Hamiltonian, $H = p_0$, gives

$$H = -\frac{p_i g^{0i}}{g^{00}} + \sqrt{\left(\frac{p_i g^{0i}}{g^{00}}\right)^2 - \frac{p_i p_j g^{ij} - \mu^2}{g^{00}}}. \tag{4}$$

Now considering only the radial component of momentum in a diagonal metric, the Hamiltonian becomes

$$H = \sqrt{\frac{\mu^2 - p_r^{\ 2} g^{rr}}{g^{00}}}. \tag{5}$$

As $r \to r_c$, $g^{00} \to 0$ for $\mu > 0$ therefore $H \to \infty$. For $\mu = 0$, we have $g^{rr} \to 0$ implying that $p^r \to 0$ as $r \to r_c$. For material particles ($\mu > 0$) and photons ($\mu = 0$) this indicates a divergent potential barrier at the critical radius.

For a realistic black hole that has reached equilibrium with its environment, the interval $r_1 - r_c$ is very small, most likely less than a Planck length. The potential barrier is therefore *harder* than during the initial collapse phase when the accretion rate is high, and all the mass of a star is yet to be consumed. But even there, the energy required to reach the increasing critical radius still diverges in the spherically symmetric case. If this is correct then what are the implications? Most significant, if this mechanism extends to asymmetric collapse, it is an immediate resolution of the information paradox. It appears that a void forms in a collapsing star with all the consumed matter being squeezed onto its inner surface. Inside this surface, at $r = r_c(t)$, there would be no matter present.

However, this is where we need to be cautious. If there is no mass-energy flux at r_c then the Vaidya solution is not valid, and this would limit the otherwise divergent potential, and allow material to leak across the boundary and enter the void. At present it is difficult to calculate the proportion of material crossing this boundary. If the total mass entering the void is low enough to have a negligible effect then it may be attracted back towards the potential minimum at r_1. Other mechanisms for leakage may also include quantum tunnelling, particularly if momenta are reduced to levels where associated wavelengths are comparable to r_c.

That said there is the possibility that initially inbound material would bounce back across the potential minimum and oscillate back and forth in a damped fashion, though such turbulence is likely to be short lived [4]. During periods when the system is in equilibrium, mass-energy influx and outgoing Hawking radiation are at a minimum. When this minimum approaches zero, we are back to the Schwarzschild solution. Taking a pragmatic view, all we know about black holes is outside the critical radius. So, notwithstanding all of the consistent mathematical models of black hole interiors, they remain in the realms of speculation. An object with all its mass concentrated at its critical radius instead of a central singularity would, to an outside observer, appear similar to any black hole satisfying more traditional models. In this idealized model, the main attractor is not a central (or a ring) singularity at

$r = 0$, but a sphere at or just outside the critical radius. When the system approaches the Schwarzschild limit, the potential barrier is replaced by the asymptotic behaviour of inbound null geodesics at r_c.

Apart from an extended version of this work [5] the author is currently unaware of a similar model being proposed elsewhere or if so, whether it has been rigorously tested in the theoretical arena. Unless what is being suggested here is mistaken then it is tempting to propose the following conjecture

Conjecture: Given a spacelike Cauchy hypersurface, *S*, with no regions of radius, *R*, containing mass, $M > \frac{1}{2}\left(R + a^2 + Q^2 / (4\pi\varepsilon_0)\right)$ for specific angular momentum, *a*, and charge, *Q*, then the laws of physics combine to prevent such regions forming to the future of *S*.

In other words, if no fully formed black holes exist at present then none can form in the future.

If this is correct then it raises questions as to whether it can be tested. If true then most of the consumed material is confined to a uniform layer just above the critical radius with a thickness of no more than a few Planck lengths. Moreover it is possible that some scattered photons can be emitted normal to this surface and escape to infinity. Whether such photons carry information characterising this layer is an open question. If detectable, the image of a black hole would appear with a relatively bright spot in the centre of the image. Initial steps toward imaging black holes have already been taken with the Event Horizon Telescope (EHT) [6-8], but these observations are to date, in their infancy. It is hoped that similar observations in the future, employing some descendent of the EHT, possibly with a much larger baseline, would make this kind of testing possible.

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