Twin primes and other related prime generalization

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Abstract

I developed a very special summation function by using the floor function, which provides a characterization of twin primes and other related prime generalization.

\[ \pi(x) = \sum_{k=1}^{\lfloor x/2 \rfloor} \left( \left\lfloor \frac{x+1}{k} \right\rfloor - \frac{x}{k} \right) + \left( \left\lfloor \frac{x-1}{k} \right\rfloor - \frac{x-2}{k} \right) \]

Where \( x \geq 4 \)

the initial idea was to use my function to prove the twin primes conjecture but then I started to look into other stuff for the twin primes conjecture the idea was to check this:

when \( \pi(x) = 2 \) then \( x - 1 \) and \( x + 1 \) are (twin) primes
when \( \pi(x) \) is an odd value then those cases (of the function getting odd values) are rarer then the twin prime case

Let \( d \) be an odd value and \( \pi(6a) = d \), \( \pi(6b) = 2 \), \( \pi(6c) = d \) where \( a < b < c \)

meaning there is a twin prime case between two odd values cases of the function

so you just need to show that one of the odd value cases will continue to infinity and this automatically will prove the twin prime conjecture

http://myzeta.125mb.com/100000_first_values.html
http://myzeta.125mb.com/100000_odd_only.html
http://myzeta.125mb.com/100000_odd_twin.html
http://myzeta.125mb.com/1000000_odd_twin.html

other then that, this function has a very nice, weird and sometimes gives totally unexpected results

for instance, look what happens when \( \pi(x) = 7 \)

If \( p \) is a prime (bigger then 3) such that \( \sqrt{9p - 2} \) is also a prime then \( \left(1 + \sqrt{p}\right)/2 \) is a continued fraction with a period 6

If \( p \) is a prime such that \( \sqrt{49p + 2} \) is also a prime then \( \left(1 + \sqrt{p}\right)/2 \) is a continued fraction with a period 8
\[ \pi(x) = 2 \]

\[ x = 4, 6, 12, 18, 30, 42, 60, 72, 102, 108, 138, 150, 180, 192, 198, 228, 240, 270, 282, 312, 348, 420, 432, 462, 522, \ldots \]

https://oeis.org/A014574  
Side Note: [A014574] [A129297] [A167777]

\[ n = 4, 6, 12, 18, 30, 42, 60, 72, 102, 108, 138, 150, 180, 192, 198, 228, 240, 270, 282, 312, 348, 420, 432, 462, 522, \ldots \]

Average of twin prime pairs. \( \pi(n) = 2 \)

\[ \pi(x) = 3 \]

\[ x = 8, 10, 24, 48, 168, 360, 840, 1368, 1848, 2208, 3720, 5040, 7920, 10608, 11448, 16128, 17160, 19320, 29928, 36480, 44520, 49728, 54288, 57120, 66048, 85848, 97968, \ldots \]

Exceptions: \( \pi(10) \)

https://oeis.org/A062326  
Side Note: [A062326] [A049002]

\[ p = \sqrt{x+1} = 2, 3, 5, 7, 13, 19, 29, 37, 43, 47, 61, 71, 89, 103, 107, 127, 131, 139, 173, 191, 211, 223, 233, 239, 257, \ldots \]

Primes \( p \) such that \( p^2 - 2 \) is also prime. if \( p > 2 \) then \( \pi(p^2 - 1) = 3 \)

\[ \pi(x) = 4 \]


Exceptions: \( \pi(28) \), \( \pi(126) \), \ldots, This is for later: \( \pi(6858) = \pi(19^3 - 1) \)

https://oeis.org/A176686


Numbers \( n \) such that \( n^2 - 1 \) are products of 3 distinct primes. \( \pi(n) = 4 \)

\[ \pi(x) = 5 \]

\[ x = 5, 26, 50, 80, 82, 120, 122, 288, 290, 528, 842, 960, 1370, 1680, 1850, 2400, 2808, 2810, 4488, 5328, 5330, 6240, 6242, 6888, 6890, 9408, 9410, \ldots, 14640, \ldots, 28560, \ldots, 707280, \ldots \]

Exceptions: \( \pi(26) \), \( \pi(82) \)

https://oeis.org/A109953 \( p_1 = \sqrt{x-1} = 2, 7, 11, 17, 29, 37, 43, 53, 73, 79, 83, 97, \ldots \)

https://oeis.org/A242260 \( p_2 = \sqrt{x+1} = 11, 17, 23, 31, 41, 53, 67, 73, 79, 83, 97, \ldots \)

https://oeis.org/A154832 \( p_3 = \frac{\sqrt{x+1}}{2} = 3, 7, 11, 13, 29, \ldots \)

Primes \( p_1 \) such that \( p_1^2 + 2 \) is a semiprime. \( \pi(p_1^2 + 1) = 5 \)

Primes \( p_2 \) such that \( p_2^2 - 2 \) is a semiprime. \( \pi(p_2^2 - 1) = 5 \)

Primes \( p_3 \) such that \( p_3^4 - 2 \) is also prime. \( \pi(p_3^4 - 1) = 5 \)
\( \pi(x) = 6 \)


Exceptions: \( \pi(44), \pi(46), \ldots \) alot more! (i did not pursue this matter because i am more interested when the function is getting an odd value)

https://oeis.org/A189974


Numbers \( n \) such that \( d(n - 1) = d(n + 1) = 4 \), where \( d(k) \) is the number of divisors of \( k \) (A000005). \( \pi(n) = 6 \)

\[ \pi(x) = 7 \]

\[ x = 15, 170, 362, 530, 624, 728, 962, 3480, 4490, 5042, 17162, 18768, 22202, 24648, 28562, 37250, 52442, 83520, 85850, 113570, 124608, 130320, 130322, 139128, 167282, 175562, 214370, 279842, 368450, 380690, 418608, \ldots \]

\[ p_1 = \sqrt{x - 1} = 13, 19, 23, 31, 67, 71, 131, 149, 169, 193, 229, 293, 337, 361, 409, 419, 463, 529, 607, 617, \ldots \]

\( p_1 \) is not from [A109953] but what about the number 5 ?

( **Note: there is no \( p_1 = 5 \) because \( x = 26 \) was one of the exceptions at \( \pi(x) = 5 \) ) ( maybe also because of the 3 x 3 x 3 )

\[ 56^2 + 2 = 27 = 3 \times 3 \times 3 \]
\[ 13^2 + 2 = 171 = 3 \times 3 \times 19 \]
\[ 19^2 + 2 = 363 = 3 \times 11 \times 11 \]
\[ 23^2 + 2 = 531 = 3 \times 3 \times 5 \]
\[ 31^2 + 2 = 963 = 3 \times 3 \times 107 \]
\[ 67^2 + 2 = 4491 = 3 \times 3 \times 499 \]
\[ 71^2 + 2 = 5043 = 3 \times 41 \times 41 \]
\[ 131^2 + 2 = 17163 = 3 \times 3 \times 1907 \]
\[ 149^2 + 2 = 22203 = 3 \times 3 \times 2467 \]
\[ 169^2 + 2 = 28563 = 3 \times 9521 \]
\[ 193^2 + 2 = 37251 = 3 \times 3 \times 4139 \]
\[ 229^2 + 2 = 52443 = 3 \times 3 \times 5827 \]
\[ 239^2 + 2 = 85851 = 3 \times 3 \times 9539 \]
\[ 337^2 + 2 = 113571 = 3 \times 3 \times 12619 \]
\[ 361^2 + 2 = 130323 = 3 \times 43441 \]
\[ 409^2 + 2 = 167283 = 3 \times 3 \times 18587 \]
\[ 419^2 + 2 = 175563 = 3 \times 3 \times 19507 \]
\[ 463^2 + 2 = 214371 = 3 \times 3 \times 23819 \]
\[ 529^2 + 2 = 279843 = 3 \times 93281 \]
\[ 607^2 + 2 = 368451 = 3 \times 3 \times 40939 \]
\[ 617^2 + 2 = 380691 = 3 \times 3 \times 42299 \]

3691^2 + 2 = 13623483 = 3 \times 2131 \times 2131

there is no oeis for: Primes p3 such that \((p3^2+2)/9\) is a prime.

But it seems to be a subset of [A146351]

there is no oeis for: Primes p4 such that \((p4^2+2)/3\) is a prime.

But it seems to be a subset of [A122430]

there is no oeis for: Primes p5 such that \((p5^2+2)/3\) is a square of a prime.

But it seems to be a subset of [A046184]

Primes \( p_3 \) such that \((p_3^2 + 2)/9\) is a prime. \( \pi(p_3^2 + 1) = 7 \)

Primes \( p_4 \) such that \((p_4^2 + 2)/3\) is a prime. \( \pi(p_4^2 + 1) = 7 \)

Primes \( p_5 \) such that \(\sqrt{(p_5^2 + 2)/3}\) is a prime. \( \pi(p_5^2 + 1) = 7 \)
Primes $p$ such that continued fraction of $\frac{1 + \sqrt{p}}{2}$ has period 6: primes in A146331.

$$\frac{1 + \sqrt{19}}{2} = 2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{8 + \frac{1}{2 + \frac{1}{3 + \frac{1}{\ldots}}}}}}$$

$$\frac{1 + \sqrt{59}}{2} = 4 + \frac{1}{1 + \frac{1}{14 + \frac{1}{2 + \frac{1}{7 + \frac{1}{\ldots}}}}$$

Indices of octagonal numbers which are also square.

Primes of the form $1 + 2n + 3n^2$ where $n$ is A086285.
Primes p such that continued fraction of \((1 + \sqrt{p})/2\) has period 8; primes in A146333.

\[
\frac{1 + \sqrt{71}}{2} = 4 + \cfrac{1}{2 + \cfrac{1}{16 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{7 + \ddots}}}}}
\]

\[
\frac{1 + \sqrt{383}}{2} = 10 + \cfrac{1}{3 + \cfrac{1}{38 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{19 + \ddots}}}}}
\]

Primes p such that continued fraction of \((1 + \sqrt{p})/2\) has period 12: primes in A146336.

\[
\frac{1 + \sqrt{263}}{2} = [8;1,1,1,4,32,4,1,1,1,1,15]
\]

\[
\frac{1 + \sqrt{14831}}{2} = [61;2,1,1,3,1,242,1,3,1,1,2,121]
\]

\[
\frac{1 + \sqrt{453983}}{2} = [337;2,1,1,3,1,1346,1,3,1,1,2,673]
\]

Primes p such that \(p^6 - 2\) is also prime.

\[
\pi(p^6 - 1) = 7
\]

Primes \(p\) such that \(p^1 - 2\) is also prime. \(\pi(p^1 - 1) = 2\) first at: \(\pi(5^1 - 1) = \pi(5) = 2\) [Twin Prime Case!]

Primes \(p\) such that \(p^2 - 2\) is also prime. \(\pi(p^2 - 1) = 3\) first at: \(\pi(3^2 - 1) = \pi(8) = 3\) (because \(p^n - 1 \geq 4\))

Primes \(p\) such that \(p^3 - 2\) is also prime. \(\pi(p^3 - 1) = 4\) first at: \(\pi(19^3 - 1) = \pi(6858) = 4\)

Primes \(p\) such that \(p^4 - 2\) is also prime. \(\pi(p^4 - 1) = 5\) first at: \(\pi(3^4 - 1) = \pi(80) = 5\)

Primes \(p\) such that \(p^5 - 2\) is also prime. \(\pi(p^5 - 1) = 6\) first at: \(\pi(3^5 - 1) = \pi(242) = 6\)

Primes \(p\) such that \(p^6 - 2\) is also prime. \(\pi(p^6 - 1) = 7\) first at: \(\pi(3^6 - 1) = \pi(728) = 7\)

Generalization for \(p^n - 2\): Primes \(p\) such that \(p^n - 2\) is also prime. \(\pi(p^n - 1) = n + 1\)

\[
\pi(p^n - 1) = \sum_{k=1}^{\lfloor \frac{p^n - 1 + 1}{k} \rfloor} \left( \left\lfloor \frac{p^n - 1}{k} \right\rfloor - \left\lfloor \frac{p^n - 1 - 1}{k} \right\rfloor - \left\lfloor \frac{p^n - 1 - 2}{k} \right\rfloor \right)
\]

Where \(p^n - 1 \geq 4\)
Primes $p$ such that $p^n - 2$ is also prime.

$$\pi(p^n - 1) = \sum_{k=1}^{p^n - 3} \left( \left\lfloor \frac{p^n}{k} \right\rfloor - \left\lfloor \frac{p^n - 1}{k} \right\rfloor + \left\lfloor \frac{p^n - 2}{k} \right\rfloor - \left\lfloor \frac{p^n - 3}{k} \right\rfloor \right) = n + 1 \quad \text{where } p^n \geq 5$$

now that is interesting …

I am stopping here (for now anyway) because I don’t have a lot of free time on my hands and I am working all alone with no help …

I started 2 months ago … wanted to try and check the twin prime conjecture … I wasn’t expecting it to be such a time consuming lol

I have other materials on the twin primes conjecture that maybe I will upload soon as well

(I hope someone will read this and will continue from were I stopped)

yours truly,
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