Derivation of finite temperature ”hot absolute zero” in pure state form

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Abstract

So far, we have proposed a method of expressing finite temperature in the form of ground state and pure state by changing the shape of the distribution function and using the step function. In this paper, we calculate the Hamiltonian and its eigenvalues in the second quantized system in this format. Here, the Hamiltonian in the ground state is operated by the creation and annihilation operator in which the matrix elements are changed according to the change in the statistics. It also shows the basics of commutation relations of parastatistics and discusses generalized mathematics.

1 Introduction

In our previous report [1], we expressed finite temperature energy in the statistics of the form of absolute zero by changing the shape of the distribution function.

In this report, we introduce a creation and annihilation operator that incorporates changes in this distribution function.

By acting it on the Hamiltonian in the ground state and performing a second quantization, we try to express a finite temperature field in the form of a pure state.

2 Second quantization of finite temperature field by statistical change

Various physical quantities in a finite temperature field are most commonly calculated using a mixed-state density matrix.

Here, we present another new method.

We use a method in which the Hamiltonian in the ground state, which is represented in the pure state, is acted on by the field operator using the creation and annihilation operator, which represents the state in which the statistics are changed.

First, we will reprint the method of expressing finite temperature by changing statistics, which was shown in the previous report.
Now, we assume the following ground state temperatures.

\[ T \to T_0 \simeq 0 \]  

Then assume the following distribution function.

\[ \frac{1}{\exp(E - \mu)/k_B T_0 + \kappa} \]  

Here, in order to express the temperature of this system depending on \( \kappa \) in this function, a function of the following shape is prepared.

\[ \kappa(\mu, T) = \frac{1}{\pi} \tan^{-1} \left( f \left( \frac{\mu}{k_B T} \right) \right) + \frac{1}{2} \]  

FIG. 1 shows an image of the temperature change of the conventional distribution function represented by this and the temperature change of the distribution function introduced this time.

\[ \kappa(\mu, T) \] is calculated by matching the integrals of each distribution function shown on the left and right of this figure.

\[ n_0 = \int_0^\infty \frac{1}{\exp(\alpha + \beta \epsilon) + 1} \frac{4\pi p^2}{\hbar^3} dp = \int_0^\infty \frac{1}{\exp(\epsilon - \mu)/T_0 + \kappa} \frac{g4\pi p^2}{\hbar^3} dp = \frac{8\pi}{3\hbar^3 \kappa} \]  

\[ \alpha = -\frac{\mu}{k_B T}, \beta = \frac{1}{k_B T} \]  

However, here as well, the change in Fermi energy due to temperature was ignored for the sake of simplicity.

We would like to express the eigenvalues of any energy level according to this statistic by using the Hamiltonian in the following ground state.

\[ \hat{H} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \]
Here, in the following distribution function,
\[
\frac{1}{\exp(E - \mu)/k_B T_0 + \kappa}, \quad T = \text{Actual Temperature}, T_0 = 0
\]  
(7)

when the value of the energy eigenvalue is \( E < \mu \), the number of quantum states that can be occupied by one particle is \( 1/\kappa \), which is a new quantum system. In order to perform the second quantization on this, we consider a change in the creation and annihilation operator displayed in a matrix as shown below.
\[
\hat{a}_1^\dagger = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \hat{a}_0 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \implies \hat{a}_1^\dagger = \begin{pmatrix} 0 & 1/\sqrt{\kappa} \\ 0 & 0 \end{pmatrix}, \quad \hat{a}_0 = \begin{pmatrix} 0 & 0 \\ 1/\sqrt{\kappa} & 0 \end{pmatrix}
\]  
(8)

Here, the number density operator is as follows.
\[
N = \hat{a}_1^\dagger \hat{a}_1 = \begin{pmatrix} 0 & 1/\sqrt{\kappa} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1/\sqrt{\kappa} & 0 \end{pmatrix} = \begin{pmatrix} 1/\kappa & 0 \\ 0 & 0 \end{pmatrix}
\]  
(9)

The commutation relation is given as follows.
\[
\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_1 \hat{a}_1^\dagger = \begin{pmatrix} 1/\kappa & 0 \\ 0 & 1/\kappa \end{pmatrix}
\]  
(10)

In addition, the following relational expression holds.
\[
\hat{a}_0 \hat{a}_0^\dagger = \hat{a}_1^\dagger \hat{a}_1 = 0
\]  
(11)

As a result, a system in which the minimum unit of the number of particles generated and extinguished is \( 1/\kappa \) is expressed.

Here, the field operator represented by the wave function \( \psi_i \) in an arbitrary ground state is expressed as follows using the new creation and annihilation operator shown above.
\[
\hat{\psi}(x) = \sum_i \psi_i(x) \hat{a}_i \implies \hat{\psi}^\dagger(x) = \sum_i \psi_i(x) \hat{a}_i^\dagger
\]  
(12)

\[
\hat{\psi}^\dagger(x) = \sum_i \psi_i^*(x) \hat{a}_i^\dagger \implies \hat{\psi}^\dagger(x) = \sum_i \psi_i^*(x) \hat{a}_i^\dagger
\]  
(13)

Using this, the eigenvalue of the Hamiltonian shown in the equation (6) at temperature \( T \) is that the same number of \( 1/\kappa \) particles are occupied by all the levels up to the Fermi level. Based on the approximation that it is, it is calculated as follows.
\[
\tilde{F}_1 = \int \hat{\psi}^\dagger \hat{H}_1(x) \hat{\psi}^\dagger(x) dx = \sum_{ij} \left[ \psi_i^*(x) \hat{H}_1(x) \psi_j(x) \right] \hat{a}_i^\dagger \hat{a}_j
\]  
(14)

\[
= \frac{1}{\kappa} \left[ \psi_i^*(x) \hat{H}_1(x) \psi_j(x) \right] \hat{a}_i^\dagger \hat{a}_j
\]  
(15)

Therefore, we were able to express the Hamiltonian eigenvalues at finite temperature in the form of pure and ground states.

I would like to call this form "hot absolute zero".

3
3 Summary of parastatistics commutation relations

In the discussion so far, we have expressed statistics in which the basic unit of the number of particles created and annihilated by operators exceeds one, so I would like to touch on the basics of "parastatistics" systematized for such statistics. [2] [3].

The general commutation relation of parastatistics is expressed as follows.

\[
\begin{align*}
[a^\dagger_i, a_j] &\pm a_k = -2\delta_{ik} a_j, \quad [[a^\dagger_i, a_j] \pm a_k^\dagger] = 2\delta_{jk} a_i^\dagger \\
[a_i, a_j] &\pm a_k = 2(\delta_{jk} a_i \pm \delta_{ik} a_j) \quad (16) \\
[[a_i^\dagger, a_j^\dagger] \pm a_k] &\pm = -2(\delta_{jk} a_i^\dagger \pm \delta_{ik} a_j^\dagger) \quad (17) \\
[[a_i^\dagger, a_j] \pm a_k] &\pm = -2(\delta_{jk} a_i \pm \delta_{ik} a_j) \quad (18) \\
[[a_i, a_j] \pm a_k] &\pm = 0, \quad[[a_i^\dagger, a_j^\dagger] \pm a_k^\dagger] = 0 \quad (19)
\end{align*}
\]

Then, the conditions for the basis vector are as follows.

\[
a_j |0\rangle = 0, \quad a_j^\dagger a_k |0\rangle = p\delta_{jk} |0\rangle \quad (20)
\]

Here, \( p \) takes a natural number, and when \( p = 1 \), the system follows the general Bose statistics in the case of the upper sign and the Fermi statistics in the case of the lower sign.

In the case of \( p \neq 1 \), the system follows the parabose statistics of the order \( p \) when the case of the upper sign, and the parafermi statistics of the order \( p \) when the case of the lower sign.

In this case of parabose statistics of order \( p \), any number of state vector subscripts can be symmetric, but only \( p \) can be antisymmetric. On the contrary, in the parafermi statistics of order \( p \), there can be any number in antisymmetric relations, but only \( p \) can be symmetric.

In the statistical method used in this paper, \( p \) appearing in the equation (20) is extended to all positive numbers, or Kronecker delta is extended to an integrable general function.

I would like to discuss the mathematical structure of this area in the future.

4 Conclusion and Discussion

In this paper, we have presented a method for expressing a finite temperature in the form of ground state and pure state by using a step function, which can also be expressed in a second quantized system. We did this by letting the ground-state Hamiltonian act on the creation and annihilation operators with altered matrix elements as the statistics changed.

We are studying a method to further extend this and express the perturbation of the interacting system as the excitation of temperature change.

References

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