The Extremal Higher Dimensional Constructions of Fundamental Brane–Antibrane Systems

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Abstract

We calculate the extremal higher dimensional effective actions of fundamental brane-antibrane systems elegantly presented in the theoretical framework of advanced membrane theory constructions. Detailed study of brane-antibrane systems reveals when brane separation is smaller than the superstring length scale, spectrum of this system has different tachyonic modes and interaction regimes in the moduli superspace. The higher dimensional effective actions should then include these modes because they are the most important ones which rule the extremal dynamics of the fundamental brane systems. In this regard, it has been shown that an effective action of Born-Infeld type proposed in the current literature can capture many properties of the decay of non-BPS Dp-branes in superstring and membrane theory. The effective actions of brane-antibrane systems in Types IIA and IIB superstring theories should be given by some extension of the DBI action and the WZ terms which include the tachyon field configurations. The DBI part may be given by the projection of the effective action of two non-BPS Dp-branes in Type IIB theory. We are interested in this paper in the appearance of tachyon, gauge field and the RR field in these extremal higher dimensional actions. Using the consistency of the present constructions, we have also found the first higher derivative corrections to the exceptional part of the extremal effective actions with brane-antibrane systems.

1 Introduction

In the presented doctrina dominum article, we will focus our efforts and concentrate on building a perfect complete picture of the extremal higher-dimensional fundamental interactions of brane-antibrane systems with the exclusive inclusion of superstring theory principles plus magical set of supergravity consistency conditions with the dream aiming to place an advanced degree in the formation of a realistic membrane theory. We examine the low energy effective field theory following from the tachyon effective action in the background of static brane-antibrane systems, neglecting the gravitational and Ramond-Ramond fields. In the limit that the thickness of the branes tends to zero, and the tachyon field between them approaches the superstring vacua, we should expect interactions between the branes to vanish and the resulting spectrum should represent the effective field theory around isolated branes. It has recently been realized that brane-antibrane annihilation may result in defect formation, due to the dynamics of the tachyon field. Studies of this possibility have generally ignored the interaction of the brane fields with fields in the bulk, recently it has been argued that interactions with bulk fields suppress or even eliminate defect formation. The usual approach to defect formation during tachyon condensation is to take into account only the fields that live in the worldvolume of the decaying non-BPS brane or brane-anti-brane pair. This approach seems to be motivated by the fact that the resulting lower-dimensional branes formed during the decay are localized inside the worldvolume of the parent brane. However, the final state defects are themselves D-branes, and therefore couple to bulk RR-fields. One should include the effects of these fields in the defect formation process. The open string tachyon condensation on non-BPS brane systems has attracted much interest recently. One framework of analysis is level truncation of the open string field theory (SFT) which lead to very good numerical agreements with expected values of vacuum energy and lower-dimensional D-branes tensions. Another exceptional theoretical framework is the boundary SFT (BSFT). It was argued that while in the SFT approach an infinite number of massive fields are involved in the condensation process, in the BSFT one can restrict to the tachyon field and study some aspects of the condensation, such as the tensions of the lower-dimensional D-branes, exactly. Recently, it has been realized that the spectrum in some vacua of type II string theories contains not only BPS D-branes but also unstable non-BPS D-branes. And the in- stability of these non-BPS D-branes has been interpreted in the stringy context in terms of the tachyonic mode arising in the spectrum of open strings ending on the non-BPS D- branes. Indeed, there could be several ways in which one can argue why the tachyonic mode should develop in the spectrum of open strings ending on unstable D-branes such as brane-antibrane systems or non-BPS D-branes with unidentified worldvolume dimensions in IIA/IIB superstring theories. In this note we will use the notion of superconnections, which when considering the branes-antibranes system and non-BPS Dp-branes appears naturally via the Chan-Paton factors. We will make the assumption that the effective action of tachyon and gauge fields for the brane-antibrane system and non-BPS Dp-branes can be written in a Quillen-like framework in terms of the supercurvature. The tachyon potential that arises in this framework is exponential in the tachyon field. We will propose a form of the effective action and use it to study the process of tachyon condensation. Kink solutions that we will find, with infinite constant value of the gauge field strength, reproduce the exact tensions of the lower-dimensional D-branes at the minimum of the tachyon potential. The effective action is different from the BSFT proposal. It can be related by field redefinitions, in some cases, to the extremal effective action proposed in the article. In superstring theory a pair of parallel brane-antibrane system constitutes an unstable object. To study the dynamics of unstable D-branes, the BSFT is a useful tool and it has provided a good understanding of tachyon condensation at the classical level. It describes the off-shell dynamics of open strings in a fixed on-shell background of closed strings in which an open string field configuration corresponds to a boundary term in the world-sheet action of the string. Therefore, specifying a boundary term means giving the background values of the various modes of the open string. It is based on the Batalin-Vilkovisky formalism whose master equation provides the effective action of the theory. The Tachyon-Dirac-Born-Infeld (TDBI) action captures some aspects of the dynamics of non-BPS D-branes in type II string theory. We show that it can also be used to study the classical interactions of BPS branes and antibranes. Our analysis sheds light on real time tachyon condensation, on the proposal that the tachyon field can be thought of as an extra spatial dimension whose role is similar to the radial direction in holography, and on A. Sen's open string completeness conjecture. However, it is easy to see that the numerical coefficients of the higher derivative terms do not match those of the BSFT action. Indeed, in the current literature one studies the tachyon condensation with only the tachyon field excited and one gets the precise tension of the lower-dimensional D(p-1)-brane. In our variables, we needed a nonzero configuration of the gauge field strength in order to derive the precise tension of D(p-1)-brane from the kink solution. Upon addition of the gauge fields in the BSFT formalism there is still a difference between the actions. We try to analyze the mechanism of defect formation when bulk fields are involved. We first construct the extremal model which captures the essential features of the full 10-dimensional model that are responsible for the formation of the topological defects. Using this special model we study the classical evolution of the brane and bulk fields, so the deSitter quantum fluctuations are responsible only for seeding perturbations which grow during the subsequent evolution. The final configuration of the bulk field gradients is a result of this classical evolution. The energy of the system comes from tension of the brane-antibrane pair, which can be modeled as the potential energy of the tachyon field sitting at the top of the potential. In our extremal model this corresponds to the potential energy of a complex scalar living in the worldvolume of the fundamental brane-antibrane system. We will present the supergravity Lagrangian for the fields which live both inside the worldvolume of the brane-antibrane system and the bulk, and we will construct a extremal model with the same physical features. We examine the low energy effective field theory following from the tachyon effective action in the background of static brane-antibrane systems, neglecting the gravitational and Ramond-Ramond fields. Moreover, recently the interest in the nature of this tachyon potential has been multipled by the idea of unstable brane decay via the open string tachyon condensation on these unstable Dp- branes. The conjecture according to which the tachyon potential has a minimum and the negative energy density contribution from the tachyon potential at the minimum should exactly cancel the sum of the tensions of the brane-antibrane system. Thus the endpoint of the unstable brane-antibrane annihilation should be a closed superstring vacua without any D-brane. In the present work, we shall attempt to read off the supergravity analogue of tachyon potential from the semi-classical supergravity description for brane-antibrane interaction. To this end, we begin with the brief review of the conjecture for tachyon condensation in the brane-antibrane annihilation process which can be stated as follows. We start this study here with the examination of the equations of motion on parallel branes which are possible for fundamental test of brane-antibrane systems. These are among the simplest to use in applications, partly because of the triviality of the various consistency conditions which would otherwise arise when trying to construct brane configurations inside compact extra dimensions. The dynamic process of unstable D-branes decaying into stable ones with one dimension lower can be described by a tachyon field with a Dirac-Born-Infeld effective action. We investigate the fluctuation modes of the tachyon field around a two-parameter family of static solutions representing an array of brane-antibrane systems. We extended the effective action of a Dp-brane via its tangential and transverse dynamics. In fact, there are various extensions for the brane effective action: derivative corrections, tachyonic extension and curvature corrections. Accordingly, for a given setup of a brane-antibrane systems one may combine some of these modifications to construct a suitable higher-dimensional effective interaction. For a dynamical brane-antibrane system with the inclusion of tachyon fields in a curved supergravity background one should add an appropriate tachyon potential and the supercurvature improvements to the specific action. The presented research work suggests further investigations in many different directions in establishing a unified framework for fundamental membrane theory.

2 The Equations Relevant to Motion of Parallel Branes

We start by setting up and solving the equations which are relevant to the motion of a probe antibrane moving within the background of N parallel branes. To do so we first examine the fields which the N source branes set up, and then examine the equations of motion which they imply for a test brane or antibrane. We show how these equations may be solved for the motion of the test-brane centre of mass when all other brane modes are frozen. Let us first consider the gravitational, dilaton and electromagnetic fields which are set up by a set of N parallel p-branes in D-dimensional spacetime. We take the action for the fields to be given in Einstein Frame by

$$S_s = -\int d^D x\,\sqrt{-g}\left[\frac{1}{2}\,g^{MN}\Big(R_{MN} + \partial_M\,\phi\,\partial_N\,\phi\Big) + \frac{1}{2\,n!}\,e^{\alpha\phi}\,F_{M_1\dots M_n}\,F^{M_1\dots M_n}\right], \tag{2.1}$$

where the n-form field strength is related to its $(n-1)$-form gauge potential in the usual way $F_{[n]} = dA_{[n-1]}$. Here n is related to the spacetime dimension d of the N p-branes by $d = p+1 = n-1$. We denote the d coordinates parallel to the branes by x^μ and the $D-d$ transverse coordinates by y^m.

The constant α depends on which kind of brane is being considered. For instance, if $A_{[n-1]}$ were to arise from a string-frame action

$$S_{\rm SF} = -\frac{1}{2\,n!}\int d^D x\,\sqrt{-\hat{g}}\;e^{\alpha_s\phi}\,F_{\widehat{M}_1\dots\widehat{M}_n}\,F^{\widehat{M}_1\dots\widehat{M}_n}\,, \tag{2.2}$$

with the string-frame metric given by $\hat{g}_{MN} = e^{\lambda\phi}\,g_{MN}$ for $\lambda = 4/(D-2)$, then $\alpha = \alpha_s + 2 - \lambda(n-1)$. Two cases of particular interest are: (i) NS-NS fields (like the fields rising in the gravity supermultiplet in various dimensions), for which $\alpha_s = -2$ and so $\alpha = \alpha_{NS} = -4(n-1)/(D-2)$; (ii) R-R fields, for which $\alpha_s = 0$ and so $\alpha = \alpha_R = 2(D-2n)/(D-2)$. In the special case $D = 10$ we have $\lambda = 12$ and so these two cases become $\alpha_{NS} = -12(n-1) = -12(p+1)$ or $\alpha_R = 12(5-n) = 12(3-p)$, respectively.

In the above background let us now follow the motion of another single parallel p-brane, displaced from the original N by the radial coordinate-distance r. This motion is described by the brane action, which can be decomposed as the sum of two pieces: the Born-Infeld (BI) and the Wess-Zumino (WZ) parts. The dilaton and graviton couplings are given by the Born-Infeld contribution, which is (in the String Frame):

$$S_{BI} = -T_p\int d^d\xi\;e^{-\phi}\sqrt{-\det(\hat{g}_{MN}\,\partial_\mu x^M\partial_\nu x^N + 2\,\ell_s^2\mathcal{F}_{\mu\nu})}\,, \tag{2.3}$$

where x^M are the coordinates of the embedding, ξ^μ are the world-volume coordinates and T_p denotes the brane tension. $\mathcal{F}_{\mu\nu}$ denotes the field strength for any open-string gauge modes confined to the brane, although for the moment we put these fields to zero.

The coupling to the bulk gauge field, $A_{[p+1]}$, is given by the Wess-Zumino part of the brane action:

$$S_{WZ} = -q\,T_p\int A_{[p+1]}\,, \tag{2.4}$$

where q represents the brane charge, with $q = 1$ representing a probe brane and $q = -1$ representing a probe antibrane.

Since our branes are straight and parallel it is convenient to choose the following coordinate gauge:

$$x^\mu = \xi^\mu\,, \tag{2.5}$$

where as before x^μ are the coordinates parallel to the world volume. To the extent that we follow only the brane's overall centre-of-mass motion – we return to the validity of this assumption in the next consideration. We have seen that probe brane orbits decay due to radiation into bulk modes, and but this radiation could also be accompanied by radiation onto the brane. We next perform an estimate of this rate, along the lines of the one done earlier for bulk radiation, with the conclusion that radiation into brane modes is subdominant to bulk radiation.

We use for these purposes the coupling of the brane-bound gauge field, given by the (String-Frame) Born-Infeld action:

$$S_{BI} = -T_p \int d^d\xi \; e^{-\phi} \sqrt{-\det(_{MN})} \; [1 + \ell_s^4{}^{\mu\lambda\nu\rho} \mathcal{F}_{\mu\nu}\mathcal{F}_{\lambda\rho} + \cdots] \,, \tag{2.6}$$

where $_{\mu\nu} = \hat{g}_{MN}\partial_\mu x^M \partial_\nu x^N$ is the brane's induced metric and $\mathcal{F}_{\mu\nu}$ is the gauge field strength. We see that the relevant coupling is in this case $\kappa_B = T_p \, \ell_s^4 \, e^{-\phi_b}$, where $\phi_b = \phi(y_b)$ is the dilaton field evaluated at the position of the brane.

The effective action of a $(Dp, \overline{Dp})$ brane system in Type IIA(B) theory should be given by some extension of the DBI action and the WZ terms which include the tachyon fields. The DBI part may be given by the projection of the effective action of two non-BPS D_p-branes in Type IIB(A) theory with $(-1)^{F_L}$ projection. We are interested in this paper in the appearance of tachyon, gauge field and the RR field in these actions. These fields appear in the DBI part as the following:

$$S_{DBI} \;\; = \;\; -T_p \int d^{p+1}\sigma Tr \; \left(V(\mathcal{T}) \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} + 2\pi\alpha' D_a\mathcal{T} D_b\mathcal{T})} \right) \;, \tag{2.7}$$

where T_p is the p-brane tension. The trace in the above action should be completely symmetric between all matrices of the form $F_{ab}, D_a\mathcal{T}$, and individual $\mathcal{T}$ of the tachyon potential. These matrices are

$$F_{ab} = \begin{pmatrix} F_{ab}^{(1)} & 0 \\ 0 & F_{ab}^{(2)} \end{pmatrix}, \; D_a\mathcal{T} = \begin{pmatrix} 0 & D_a T \\ (D_a T)^* & 0 \end{pmatrix}, \; \mathcal{T} = \begin{pmatrix} 0 & T \\ T^* & 0 \end{pmatrix} \tag{2.8}$$

where $F_{ab}^{(i)} = \partial_a A_b^{(i)} - \partial_b A_a^{(i)}$ and $D_a T = \partial_a T - i(A_a^{(1)} - A_a^{(2)})T$. The tachyon potential which is consistent with S-matrix element calculations has the following expansion:

$$V(\mathcal{T}) \;\; = \;\; 1 + \pi\alpha' m^2 \mathcal{T}^2 + \frac{1}{2}(\pi\alpha' m^2 \mathcal{T}^2)^2 + \cdots$$

where m^2 is the mass squared of tachyon, $i.e. m^2 = -1/(2\alpha')$. The above expansion is consistent with the potential $V(\mathcal{T}) = e^{\pi\alpha' m^2 \mathcal{T}^2}$ which is the tachyon potential of BSFT. This action has the following expansion:

$$\mathcal{L}_{DBI} = -2T_p - T_p(2\pi\alpha') \left(m^2|T|^2 + DT \cdot (DT)^* - \frac{\pi\alpha'}{2}\left(F^{(1)} \cdot F^{(1)} + F^{(2)} \cdot F^{(2)} \right) \right) + \cdots \tag{2.9}$$

where dots refers to the terms which have more than two fields.

Using the expansion for the exponential term in the WZ action (6.123), one finds many different terms. The terms which involve at most three open string fields are the following:

$$\begin{aligned} \mu_p(2\pi\alpha')C \wedge \mathrm{STr}\, i\mathcal{F} &= \mu_p(2\pi\alpha')C_{p-1} \wedge (F^{(1)} - F^{(2)}) \\ \frac{\mu_p}{2!}(2\pi\alpha')^2 C \wedge \mathrm{STr}\, i\mathcal{F} \wedge i\mathcal{F} &= \frac{\mu_p}{2!}(2\pi\alpha')^2 C_{p-3} \wedge \left\{ F^{(1)} \wedge F^{(1)} - F^{(2)} \wedge F^{(2)} \right\} \\ &\quad + C_{p-1} \wedge \left\{ -2\beta^2|T|^2(F^{(1)} - F^{(2)}) + 2i\beta^2 DT \wedge (DT)^* \right\} \\ \frac{\mu_p}{3!}(2\pi\alpha')^3 C \wedge \mathrm{STr}\, i\mathcal{F} \wedge i\mathcal{F} \wedge i\mathcal{F} &= \frac{\mu_p}{3!}(2\pi\alpha')^3 C_{p-3} \left\{ 3i\beta^2(F^{(1)} + F^{(2)}) \wedge DT \wedge (DT)^* \right\} \end{aligned} \tag{2.10}$$

The coupling of one RR field C_{p-1}, two tachyons and one gauge field in the above terms can be combined into the following form:

$$\begin{aligned} -\beta^2\mu_p(2\pi\alpha')^2 \int_{\Sigma_{(p+1)}} C_{(p-1)} &\wedge \left\{ d(A^{(1)} - A^{(2)})TT^* - (A^{(1)} - A^{(2)})d(TT^*) \right\} \\ &= -\beta^2\mu_p(2\pi\alpha')^2 \int_{\Sigma_{p+1}} H_{(p)} \wedge (A^{(1)} - A^{(2)})TT^* \end{aligned} \tag{2.11}$$

We want to create the higher dimensional model which captures the important features of the formation of defects with bulk fields included, and at the same time is amenable to a lattice regularization. This will allow us to follow the evolution to see whether defects form, and how the interactions with the bulk fields affect that formation. We want to include the minimal field content that will allow us to study the formation of the defects, so we will not include the metric and dilaton fields present in the full 10-dimensional model. We can extend the extremal model we used to a higher dimensional one which will have fewer differences with respect to the full superstring theory model. The most straightforward generalization would be to consider a $3+1$-dimensional brane and a $9+1$ dimensional bulk. The field content of the worldvolume theory would still be the Abelian Higgs model, but the bulk field will now be a rank-2 antisymmetric tensor field, which has the appropriate rank to couple to the 2-dimensional world-volume of a string. The Abelian Higgs model in $3+1$ dimensions admits stable string-like defects which in our model will be charged under the bulk field. The Lagrangian of the model is:

$$\mathcal{L} = \int_{\mathcal{M}_4} d^3x\, dt \left[-\frac{1}{4g^2_{\rm brane}} F^2 - D_\mu\phi D^\mu\phi^* - V(\phi) \right] - \frac{c_{cs}}{2}\int_{\mathcal{M}_4} F\wedge C + \int_{\mathcal{M}_{10}} d^9x\, dt \left[-\frac{1}{12g^2_{\rm bulk}} H^2 \right], \tag{2.12}$$

where as before we denote by $F = dA$ the field strength of the field A and by $H = dC$ the field strength of the field C. The equations of motion now become:

$$\begin{aligned} \partial_\mu F^{\mu\nu} + i g^2_{\rm brane}\left(\phi D^\nu\phi^* - \phi^* D^\nu\phi\right) + c_{cs} g^2_{\rm brane}\frac{\epsilon^{\nu\alpha\beta\gamma}}{2} H_{\alpha\beta\gamma} &= 0\,, && (2.13)\\ \partial_\mu H^{\mu\nu\lambda} - c_{cs} g^2_{\rm bulk}\frac{\epsilon^{\alpha\beta\nu\lambda}}{2} F_{\alpha\beta}\,\delta(z) &= 0\,. && (2.14) \end{aligned}$$

If we now want to study how the solution for a Nielsen-Olesen string is modified by the presence of the bulk field, for the brane fields we make the usual ansatz for a string placed along the z-direction: far from the string the scalar and gauge fields. Since we want to study the formation of codimension 2 defects the brane must have at least 2 spatial dimensions, so we will choose a 2+1 dimensional brane. The defects that form are 0+1-dimensional and the corresponding bulk field that couples to their worldvolume must carry a single index. Therefore we will have a vector field living in the bulk. This field corresponds to the C_{p-1} RR field in the 10-dimensional model. We choose the bulk to have the minimal space dimensionality, 1 space dimension more than the brane. Inside the brane we put the same field content as in the full 10-dimensional model, an Abelian gauge field corresponding to the linear combination $F^+_{\mu\nu} - F^-_{\mu\nu}$ and a complex scalar field corresponding to the complex tachyon T.

In Type II superstring theories, there are two kinds of branes: stable or BPS branes, which are supersymmetric and charged, and unstable or non-BPS branes, which are non-supersymmetric and uncharged. The p-dimensional unstable D (Dp) branes eventually decay into stable D$(p-1)$ ones. This decay process is described by the dynamics of a tachyon field T. The effective action of this unstable brane system in low energy approximation is conjectured to be of the Dirac-Born-Infeld (DBI) type form, as derived from superstring theory:

$$S = -\int d^{p+1}x\sqrt{-g}V(T)\sqrt{1+\partial_\mu T\partial^\mu T}, \tag{2.15}$$

where the effective potential $V(T)$ takes its maximal value at $T=0$ and vanishes asymptotically as T tends to infinity. The homogeneous decay of the tachyon field involves the field rolling to its vacua at $T=\pm\infty$, towards a state which can be characterised as a pressureless fluid without propagating modes. This is consistent with the notion that the open string states disappear from the spectrum as the brane decays.

The equation of motion derived from the DBI action is:

$$\Box T + (g^{\mu\nu}\partial T\cdot\partial T - \partial^\mu T\partial^\nu T)\partial_\mu\partial_\nu T - \frac{V'}{V}(1+\partial T\cdot\partial T) = 0. \tag{2.16}$$

We are interested in time-independent solutions, neglecting the gravitational and Ramond-Ramond fields of the brane. It is known that there are solutions depending on only one space coordinate, which we denote x.

The energy of this system is

$$E = \int d^p x V(T)\sqrt{1+(\partial_x T)^2} = \frac{1}{V_0}\int d^{p-1}x \int dx V^2. \quad (2.17)$$

The energy per unit $(p-1)$-dimensional volume in one period of the tachyon field is $\sigma_1 = \pi V_m/\beta$ with the specific potential adopted in this paper. The tension of the parent Dp-brane is $\mathbf{T}_p = V_m$, so this is the correct tension for a D$p-1$-brane, $\mathbf{T}_{p-1}$.

We will briefly recall the form of the effective TDBI action obtained in superstrings that we introduced earlier. We will first see the case of the non-BPS brane then that of the $(Dp,\overline{Dp})$ brane systems. We will add the contribution of Ramond-Ramond fields in the Wess-Zumino term. We will start with branes of maximum dimension, of dimension $9+1$ because the expressions are simpler and capture the physics of any other brane. Indeed, the actions on the lower dimensional branes are obtained by T-duality along the directions made transverse. For the non-BPS brane, the effective action of a D9-brane in type IIA superstring theory is

$$S_{nonBPS} = \sqrt{2}T_9 \int \mathrm{d}^{10}\sigma\, e^{\Phi} V(T)\, \sqrt{\det G_{ab} + B_{ab} + 2\pi\alpha' F_{ab} + \partial_a T\partial_b T} + S_{WZ} \quad (2.18)$$

The expression is abelian, since there is only one brane. We discuss its scope in chapter refcc chap: mot. It extends apriori only along spatial condensations. To discuss the temporal condensations the basis of study is the tachyon action in the static gauge :

$$S_T = \sqrt{2}T_p \int \mathrm{d}^{p+1}\sigma\, V(T)\, \sqrt{\eta_{ab} + \partial_a T\partial_b T} \quad (2.19)$$

the Wess-Zumino term is known to be of the form

$$S_{WZ} = \mu_p \int_{p+1} W(T) dT \wedge \sum_{m\in\ IIA} C_{(m)} \wedge e^{B+2\pi\alpha' F} \quad (2.20)$$

where $W(T) \propto V(T)$ and B and F respectively the pull-backs on the volume of the brane of the Kalb-Ramond field and the Maxwell tensor of the open superstring gauge field. The charge μ_p is proportional to the voltage of the brane. The R-R gauge fields are type IIA differential forms of even index. In type IIB, they have an odd index. Note that the tachyon is not minimally coupled to the gauge fields of the open strings, which requires their confinement during condensation.

3 The Dynamics in BSFT of the $(Dp,\overline{Dp})$ Brane System

In superstring theory a pair of parallel brane-antibrane system constitutes an unstable object in the theoretical construction. To study the dynamics of unstable D-branes, the BSFT is a useful tool and it has provided a good understanding of tachyon condensation at the classical level. It describes the off-shell dynamics of open strings in a fixed on-shell background of closed strings in which an open string field configuration corresponds to a boundary term in the world-sheet action of the string. Therefore, specifying a boundary term means giving the background values of the various modes of the open string. It is based on the Batalin-Vilkovisky formalism whose master equation provides the effective action of the theory. In the bosonic string theory, the disk partition function of the open string theory Z and the BSFT action are related by the master equation

$$S = \left(1 + \beta^i \frac{\partial}{\partial g^i}\right) Z \quad (3.21)$$

where g^i are the couplings of the boundary interactions and β^i are the corresponding world-sheet β-functions. Given a specific form of the tachyon profile, the BSFT action reduces to the effective action for the tachyon field allowing us to compute the tree level tachyon potential. For superstrings, the tachyon β-function is zero and (3.21) reduces to

$$S = Z \tag{3.22}$$

The partition function Z was computed in current research article. The disc partition function is formally defined as

$$Z = \int \mathcal{D}X\mathcal{D}\psi\, e^{-\left(S_{bulk}+S_{bndy}\right)} \tag{3.23}$$

where

$$S_{bulk} = \frac{1}{4\pi}\int d^2z\left(\frac{2}{\alpha'}\partial X^\mu\overline{\partial}X_\mu + \psi^\mu\overline{\partial}\psi_\mu + \overline{\psi}^\mu\partial\overline{\psi}_\mu\right) \tag{3.24}$$

is the bulk action for the NSR superstring.

The boundary term of the system is computed introducing auxiliary boundary fermion superfields $\Gamma^I = \eta^I + \theta F^I$ where $I = 1,\ 2^m$, and, $N = 2^{m-1}$ is the number of pairs. Consider, for example, the case where we have 2^m branes. The $2^m \times 2^m$ matrices of the gauge group $U(2^m)$, generated by the branes, can be expanded in terms of $SO(2m)$ gamma matrices. Now, instead of gamma matrices, one can introduce $2m$ boundary fermion superfields Γ^I with action

$$S = -\int d\tau d\theta \frac{1}{4}\Gamma^I D\Gamma^I \tag{3.25}$$

and, after canonically quantizing, one arrives at the anti-commutation relations $\{\eta^I, \eta^J\} = 2\delta^{IJ}$. Thus, η^I can represent the Clifford algebra needed for the expansion of the $2^m \times 2^m$ matrices. In the case of a single brane-antibrane system, expanding the resulting action in terms of the component fields one has

$$\begin{aligned} S_{bnry} \;=\; -\int\Big[&-\frac{\alpha'}{4}T^IT^I + \frac{1}{4}\dot{\eta}^I\eta^I + \frac{\alpha'}{2}D_\mu T^I\psi^\mu\eta^I + \frac{i}{2}\left(\dot{X}^\mu A_\mu + \tfrac{1}{2}F_{\mu\nu}\psi^\mu\psi^\nu\right) \\ &+ \frac{i}{4}\left(\dot{X}^\mu A^{IJ}_\mu + \tfrac{1}{2}\alpha' F^{IJ}_{\mu\nu}\psi^\mu\psi^\nu\right)\eta^I\eta^J\Big]\, d\tau \end{aligned} \tag{3.26}$$

Here $I, J = 1, 2,$

$$A^\pm_\mu \;=\; \tfrac{1}{2}\left(A_\mu \pm iA^{12}_\mu\right) \tag{3.27}$$

$$D_\mu T^I \;=\; \partial_\mu T^I - iA^{IJ}_\mu T^J \tag{3.28}$$

and the gauge fields $A^\pm_\mu$ on the brane and anti-brane, respectively, have been expressed in terms of the abelian gauge fields A^{IJ}_μ, (anti-symmetrized in I, J) and A_μ. Moreover, $F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$ and ${F^{IJ}}_{\mu\nu} = \partial_{[\mu}{A^{IJ}}_{\nu]}$. In the case of a constant tachyon field and zero gauge fields, the boundary action reduces to

$$S_{bdry} = \frac{\alpha'}{4}\int d\tau\, T^IT^I \tag{3.29}$$

Since there is no other dependence on the tachyon field in the bulk action, we learn that in this case the tachyon potential for the system is

$$V_0(T) = 2T_9\, e^{-2\pi\alpha'|T|^2} \tag{3.30}$$

where we defined $T = \frac{1}{2}(T^1 + iT^2)$, whereas T_9 denotes the tension of a D_9-brane which is defined, for general p, by

$$T_p = \frac{1}{(2\pi)^{p\frac{p+1}{2}}g_s} \tag{3.31}$$

where g_s is the string coupling constant. The stable vacua is at $T = \infty$, where the vacua energy vanishes. Since the potential (3.30) is exact, it gives a proof of Sen's conjecture that the negative energy contribution from the tachyon precisely cancels the D-brane tension: under tachyon condensation, the D-brane will decay into the closed string vacuum without any D-branes, therefore, excitations are described by closed strings alone.

Let us turn now to the case of a spatially dependent tachyon. In this case, by a combination of spacetime and gauge rotations one can bring T^I to the following form:

$$\sqrt{\alpha'}T^I = u^I X^I \tag{3.32}$$

where u^I are constants. When the gauge fields are zero, one can compute the partition function (3.23) using eqs. (3.24), (3.26) and (3.32). The result is [?]

$$Z = 2T_9 \int d^{10}X_0\, e^{-2\pi\alpha' T\overline{T}} \prod_{I=1}^{2} F(\pi\alpha'^2(\partial_I T^I)^2) \tag{3.33}$$

where

$$F(x) = \frac{4^x x\Gamma(x)^2}{2\Gamma(2x)} \tag{3.34}$$

The partition function allows us to have an expression for the action of the tachyon at all orders in derivatives. However, there is an ambiguity in the expansion, because any term with at least two derivatives acting on T can be added. At quadratic order the result is unambiguous

$$S \approx 2T_9 \int d^{10}x\, e^{-2\pi\alpha' T\overline{T}} \left[1 + 8\pi\alpha'^2 \ln(2)\, \partial^\mu \overline{T} \partial_\mu T + \ldots\right] \tag{3.35}$$

where has been used the expansion

$$F(x) = 1 + 2\ln(2)\, x + \mathcal{O}(x^2), \qquad x \to 0 \tag{3.36}$$

Now, consider the case where one of the spatial directions, y, is wrapped on a circle of radius $\widetilde{R} \leq \sqrt{}$ and that we have a constant Wilson line A wrapping the compact direction on say the D_9 brane. The gauge field strength in (3.26) vanishes and the only dependence on the gauge field comes from the covariant derivative. We can lift the above expression to include the covariant derivative by simply changing the argument of the function F.
Applying a T-duality transformation along y, the gauge field is mapped to the Higgs field which measures the distance d between a pair, separated along the dual coordinate $\widetilde{y}$ with $d \sim |A|$.

Adopting the normalization of the tachyon field the action (3.35) becomes

$$S = 2T_9 \int d^9x\, dy\, e^{-|T|^2} \left[1 + 2\alpha' |\partial_\mu T|^2 + 2\alpha' A^2 |T|^2\right] \tag{3.37}$$

The potential term is

$$V_0(T) = 2T_9\, e^{-|T|^2} \left[1 + 2\alpha' A^2 |T|^2\right] \tag{3.38}$$

The extrema are given by

$$\frac{\partial V_0(T)}{\partial |T|} = 2T_9\, |T| e^{-|T|^2} \left(4A^2\alpha' - 2\left(2A^2\alpha'|T|^2 + 1\right)\right) = 0 \tag{3.39}$$

i.e.,

$$|T| = 0, \quad |T| = +\infty, \quad \text{and} \quad |T| = \frac{\sqrt{2A^2\alpha' - 1}}{\sqrt{2\alpha'}A} \tag{3.40}$$

To study the nature of these extrema, we need to compute the second derivative: around $|T| = 0$ we have

$$\frac{\partial^2 V_0(T)}{\partial^2 |T|}|_{|T|=0} = m^2 = 4T_9 \left(2\alpha' A^2 - 1\right) \tag{3.41}$$

Therefore, we see that this potential has a minimum at $|T| = 0$ if $A > \sqrt{\frac{1}{2\alpha'}}$ or it has a true tachyonic instability if $A < \sqrt{\frac{1}{2\alpha'}}$. This behavior has a clear physical interpretation: recall that our model is equivalent to the case of a parallel brane-antibrane system separated by a distance d. If the distance d is large enough, then the tachyon mode between the two should go away, since the tachyon field comes from the open string suspended between the two branes and thus that string acquires a mass lift when two branes are distant. Notice also that in order to get a canonical kinetic term in the BSFT action we must perform the following redefinition of the tachyon field: $T = T(\phi)$ with

$$\phi = \sqrt{8\alpha' T_9} \int_0^{|T|} ds\, e^{-s^2/2} \tag{3.42}$$

With this redefinition, the action (3.37) becomes

$$S = \int d^9x\, dy \left(\tfrac{1}{2}(\partial\phi)^2 + V_0(T(\phi))\right) \tag{3.43}$$

and the tachyon vacuum at infinity is placed at a finite value of the new field ϕ. Indeed, the two local minima are

$$\phi_0 = 0\,, \quad \phi_1 = \sqrt{4\pi\alpha' T_9}. \tag{3.44}$$

This redefinition allows us to compute the mass of the tachyon: in the presence of a Wilson line A it is given by

$$M^2 = \frac{\partial^2 V(\phi)}{\partial\phi^2} = \frac{1}{\alpha'}\left[\frac{|T|^2 - 1}{2} + \alpha' A^2\left(|T|^4 - 4|T|^2 + 1\right)\right] \tag{3.45}$$

whereas if $A = 0$ we have

$$M^2_{A=0} = \frac{1}{\alpha'}\frac{|T|^2 - 1}{2} \tag{3.46}$$

Notice that the same results were found in the current literature but with different methods. Henceforth, we will consider only the real part of the tachyon field: this is consistent with the tachyon equations of motion and it is also a natural setup since we are not interested in lower dimensional D-brane left after the tachyon condensation which needs complex tachyon configurations.

The $\mathcal{N} = (1,1)$ superspace action on the disk was written in research article, including the coupling to background gauge and tachyon fields. In the present context one considers non-trivial Wilson Lines along the circle, T-dual to the brane positions x_1 and x_2 along X, the T-dual of Y. They naturally appear in the form $x^{(\pm)} = x_1 \pm x_2$. Setting aside the 'spectator' dimensions, one considers a pair of $\mathcal{N} = (1,1)$ superfields on the disk, one time-like (X_0) and the other compactified on a circle (Y), with *e.g.* $X_0 = X_0 + \frac{i}{\sqrt{2}}(\theta\psi_0 + \overline{\theta\psi}_0) + \theta\overline{\theta}F_0$. The superspace coordinates are denoted as $\widehat{z} = (z, \theta, \overline{\theta})$. At the boundary of the disk, the Grassmann coordinates satisfy the boundary condition $\theta = \pm\overline{\theta}$. The algebra of the Chan-Patton factors for the brane-antibrane system is conveniently implemented by the canonical quantization of boundary fermions [**?**], see below. These boundary fermions are the bottom components of fermionic superfields of the boundary $\mathcal{N} = 1$ superspace. For the brane-antibrane system one needs a complex superfield

$$\Gamma^{\pm} = \eta^{\pm} + \theta F^{\pm}\,. \tag{3.47}$$

with $\Gamma^- = (\Gamma^+)^*$.

Then the worldsheet action on the disk, including the tachyon background as well as Wilson lines around the circle, reads:

$$\begin{aligned} S_{BCFT}(\lambda^+, \lambda^-) &= \frac{1}{2\pi}\int_{D^2} \mathrm{d}^2z\, \mathrm{d}^2\theta\, -DX^0\overline{D}X^0 + DY\overline{D}Y + i\oint_{S^1} \mathrm{d}u\, \mathrm{d}\theta\, \frac{x^{(+)}}{4\pi} D_u Y \\ &\quad - \oint_{S^1} \mathrm{d}u\, \mathrm{d}\theta \Gamma^+ D_u + i\frac{x^{(-)}}{2\pi} D_u Y \Gamma^- - \Gamma^+ T^+ - \Gamma^- T^-\,, \end{aligned} \tag{3.48}$$

with the measure ${}^2\theta = \theta\,\overline{\theta}$, the superspace holomorphic derivative $D = \partial_\theta + \theta\partial$ and the superspace boundary derivative $D_u = \partial_\theta + \theta\partial_u$, with the boundary coordinate u on S^1.

We consider simple rolling tachyon profiles of the form:

$$T^{\pm} = \frac{\lambda^{\pm}}{2\pi} e^{\omega X^0} , \tag{3.49}$$

with $0 < \omega 1\sqrt{2}$. In order to get a real action, one chooses $(\lambda^+)^* = \lambda^-$. These are actually the tachyons that we are expecting to be solutions of the spacetime effective action. It is understood in this expression that the superfield X is taken on the superboundary of the disk.

The space-time gauge field $A^{(-)} = -\frac{x^{(-)}}{4\pi}\, dy$ being locally pure gauge, its minimal coupling to the fermionic superfields can be absorbed by the gauge transformation. One has to be careful with this transformation if Y-dependent insertions appear in the path-integral; a prescription must be chosen.

$$\Gamma^{\pm} \to \Gamma^{\pm} e^{\pm i \frac{x^{(-)}}{2\pi} Y} . \tag{3.50}$$

After this field redefinition, the boundary fermionic superfields are free, with the propagator on the real axis:

$$\Gamma^+(\widehat{z})\Gamma^-(\widehat{w}) = \widehat{\epsilon}(\widehat{z} - \widehat{w}) = \epsilon(z - w) - 2\,\theta_z\theta_w\delta(z - w) , \tag{3.51}$$

with the sign function $\epsilon(z) = \Theta(z) - \Theta(-z)$. This implies that $\Delta(\Gamma^{\pm}) = 0$, *i.e.* vanishing conformal dimension.

In terms of these new variables the worldsheet action reads:

$$\begin{aligned} S_{BCFT}(\lambda^+,\lambda^-) &= \frac{1}{2\pi}\int_{D^2} \mathrm{d}^2z\,{}^2\theta\; -DX^0\overline{D}X^0 + DY\overline{D}Y + i\oint_{S^1} \mathrm{d}u\,\theta\frac{x^{(+)}}{4\pi}D_uY \\ &\quad - \oint_{S^1} \mathrm{d}u\,\theta\,\Gamma^+D\Gamma^- - \Gamma^+T^+ - \Gamma^-T^- \end{aligned} \tag{3.52}$$

where the tachyon fields have now the expression:

$$T^{\pm} = \frac{\lambda^{\pm}}{2\pi} e^{\pm i \frac{x^{(-)}}{2\pi} Y + \omega X^0} . \tag{3.53}$$

Starting from the action eq:action, renaming Y as $\widetilde{X}$, and integrating over the fermionic coordinates one gets the action:

$$\begin{aligned} S_{BCFT}(\lambda^+,\lambda^-) &= \frac{1}{2\pi}\int_{D^2} \mathrm{d}^2z\; -\partial X^0\overline{\partial}X^0 + \partial X\overline{\partial}X + i\oint_{S^1} \mathrm{d}u\,\frac{x^{(+)}}{4\pi}\partial_u\widetilde{X} \\ &\quad + \oint_{S^1} \mathrm{d}u\eta^+\partial_u\eta^- - \frac{\lambda^+}{2\pi}\,\eta^+\psi^+T^+ - \frac{\lambda^-}{2\pi}\,\eta^-\psi^-T^- \\ &\quad - \oint_{S^1} \mathrm{d}uF^+F^- - F^+\,T^+ - F^-\,T^- , \end{aligned} \tag{3.54}$$

with:

$$\begin{aligned} \psi^{\pm} &= \pm ir\sqrt{2}\widetilde{\psi}^x + \omega\sqrt{2}\psi^0 \\ T^{\pm} &= e^{\pm ir\widetilde{X} + \omega X^0} . \end{aligned} \tag{3.55}$$

Auxiliary fields $F^{\pm}$ are then integrated to give:

$$\begin{aligned} S_{BCFT}(\lambda^+,\lambda^-) &= \frac{1}{2\pi}\int_{D^2} \mathrm{d}^2z\; -\partial X^0\overline{\partial}X^0 + \partial X\overline{\partial}X + i\oint_{S^1} \mathrm{d}u\,\frac{x^{(+)}}{4\pi}\partial_u\widetilde{X} \\ &\quad + \oint_{S^1} \mathrm{d}u\eta^+\partial_u\eta^- - \frac{\lambda^+}{2\pi}\eta^+\psi^+T^+ - \frac{\lambda^-}{2\pi}\eta^-\psi^-T^- + \varepsilon^{1-4r^2}\frac{\lambda^+\lambda^-}{4\pi^2}T^+T^- \end{aligned} \tag{3.56}$$

The contact term at the end of second line shows up, with a UV cutoff ε. This term, that does not follow from the equations of motion contributes nevertheless to correlation functions when $1/2 < |r| < 1/\sqrt{2}$. Finally, as the center-of-mass perturbation completely factorizes and commutes with any operators, one can set $x^{(+)} = 0$ without loss of generality.

Upon quantizing canonically the boundary fermions $\eta^{\pm}$, one recovers the Chan-Patton algebra corresponding to the brane-antibrane system, where now the prescription for the path integral is

$$Z = Tr \int \mathcal{D}X^i \mathcal{D}\psi^i \mathcal{P} e^{-S[X^i,\psi^i]} \tag{3.57}$$

which includes a path ordering for the operator insertions and a trace over the CP factors. In this context the tachyon becomes a boundary changing operator and when inserted on the boundary of the disk, it interpolates between the two distinct boundary conditions corresponding to the brane and to the antibrane.

The worldsheet action on the disk takes finally the form

$$S = S_{bulk} - \oint_{S^1} \mathrm{d}u \frac{\lambda^+}{2\pi} \sigma^+ \otimes \psi^+ e^{ir\widetilde{X} + \omega X^0} + \frac{\lambda^-}{2\pi} \sigma^- \otimes \psi^- e^{-ir\widetilde{X} + \omega X^0} - \frac{\lambda^+\lambda^-}{4\pi^2} \varepsilon^{1-4r^2} e^{2\omega X^0} \,. \tag{3.58}$$

Coming back to the brane-antibrane system, we consider the following worldsheet action on the upper half-plane, as a function of the boundary couplings. So now we take the boundary variable to be $u \in R$. For convenience, we rescale the coupling according to $\lambda^{\pm} \to 2\pi\lambda^{\pm}$.

$$S = S_{bulk} - \int \mathrm{d}x \Big(\lambda^+ \sigma^+ \otimes \psi^+ e^{ir\widetilde{X} + \omega X^0} + \lambda^- \sigma^- \otimes \psi^- e^{-ir\widetilde{X} + \omega X^0} - i\frac{\delta r}{2} \sigma^3 \otimes \partial_u \widetilde{X} \Big) \tag{3.59}$$

We omitted for the moment the contact term, which will enter later on in the discussion.

In BSFT of open strings we have a relation between the action of BSFT - on the space of field theories - on-shell and the partition function calculated on the disk D^2 which in supersymmetric theory appears to be particularly simple apriori if we assume that matter and ghosts are decoupled:

$$S[\phi^i_{on}] = Z_{D^2}[\phi^i_{on}] \tag{3.60}$$

where the expression of the partition function is explicitly

$$Z_{D^2}[\phi^i_{on}] = \mathrm{tr}\, \mathcal{P} \int [\mathrm{d}X][\mathrm{d}\psi] e^{-S_{bulk}[G_{ab},B_{ab},\Phi] - \sum_i \phi^i_{on} \oint_{S1} V_i} \tag{3.61}$$

In this context, ϕ^i_{on} is a constant value corresponding to a fixed point of the renormalization group for the coupling associated with the vertex operator V_i. The equality suggests by extraction of the zero mode of the bosonic fields X^a that we can express an action density, the lagrangian as a function of the partition function density noted Z' according to:

$$\int \mathrm{d}^{p+1}x \; \mathcal{L}[\varphi^i_{on}(x)] = \int \mathrm{d}^{p+1}x \; Z'_{D^2}[\varphi^i_{on}(x)] \tag{3.62}$$

The expression of $\varphi^i_{on}(x)$ is simply given by the identity $\varphi^i_{on}(x^a) = \phi^i_{on} V_i(X^a)_0$ with $\ldots_0$ the correlator calculated in free theory. Therefore, if we want to obtain the on-shell action along a rolling tachyon which we know to be a solution of the equations of motion in the case of a only non-BPS brane or a pair of separate brane-antibrane system, we must calculate the partition function on the disk for which we add a supersymmetric deformation:

$$\delta S = \oint_{S^1} T(X^0) \tag{3.63}$$

We are therefore looking at a fixed distance brane-antibrane system which we have shown to be for all r a BCFT. We need to calculate the partition function for this system and we will start from the action defined on the following disk superspace:

$$S = S_{bulk} - \oint \Gamma^+ D\Gamma^- - i\oint \frac{\lambda^+}{2\pi}\Gamma^+ e^{ir\widetilde{X}+\omega X^0} - i\oint \frac{\lambda^-}{2\pi}\Gamma^- e^{-ir\widetilde{X}+\omega X^0} \tag{3.64}$$

Here the tachyon is therefore chosen on-shell at a constant distance. In the bulk the background is trivial, that the space is flat and that $B_{ab} = 0$ and $\Phi = \phi$ constant. On the unit disk we will use that the variables are $z = \rho e^{it}$ with $\rho < 1$ and in particular on the edge, so along the unit circle we will have $z = e^{it}$. So the formula we use to calculate the partition function is:

$$Z_{D^2}[\lambda^\pm, r] = \int [\mathrm{d}\Gamma^+\mathrm{d}\Gamma^-][\mathrm{d}X][\mathrm{d}X^0] e^{-S_{bulk} - \oint \Gamma^+ D\Gamma^- - i\oint_{S^1} \frac{\mathrm{d}_z \mathrm{d}_\theta}{2\pi}\lambda^+\Gamma^+ e^{ir\widetilde{X}+\omega X^0} - i\oint_{S^1}\frac{\mathrm{d}_z \mathrm{d}_\theta}{2\pi}\lambda^-\Gamma^- e^{-ir\widetilde{X}+\omega X^0}} \tag{3.65}$$

Nevertheless, there have been several attempts to generalize the BSFT to the one-loop amplitude in the $(Dp, \overline{Dp})$ brane systems. All of them assume that the relation (3.22) is still true at one loop. Then, they construct the partition function at one loop by keeping fixed the boundary of the disk and the tachyon profile on it and adding more boundaries and handles to the string world-sheet diagram. In particular, on the annulus one has

$$S[\mathcal{U}] = \int_{annulus} \mathcal{Z}[\mathcal{A}_{fixed}, \mathcal{B}, \mathcal{U}] \tag{3.66}$$

where $\mathcal{U}$ is the coefficient of the linear tachyon profile (3.32), $\mathcal{A}_{fixed}$ is the boundary of the disk and $\mathcal{B}$ is the inner boundary of the annulus. Similarly, the cylinder amplitude can be computed in the closed string channel using the boundary state formalism. It is well known that the partition functions obtained in the two different schemes agree on-shell thanks to the open-closed string duality. However, the presence of the tachyon takes the theory off-shell and it is not clear, a priori, that the two different schemes yield the same result. In particular, since the boundary interactions are due to non-primary fields, the use of conformal maps to transform one worldsheet into another one is not helpful because the transformation laws of the fields are unknown. However, it seems that at one-loop at least, the two results are equivalent.

4 World-volume Fermions and κ-symmetry

For definiteness we shall restrict our analysis to D-branes in type IIA superstring theory, but generalization to type IIB theory is straightforward following the analysis of in the current literature. On a non-BPS Dp-brane world-volume in type IIA superstring theory, we have a 32 component anti-commuting field Θ which transforms as a Majorana spinor of the 10 dimensional Lorentz group. We shall denote by Γ_M the ten dimensional γ-matrices, and take the indices M, N to run from 0 to 9. In order to construct the world-volume action involving the fields A_μ, Y^I, Θ and T ($0 \le \mu \le p$, $(p+1) \le I \le 9$) in static gauge, we first define:

$$\Pi^\nu_\mu = \delta^\nu_\mu - \overline{\Theta}\Gamma^\nu \phi_\mu \Theta, \qquad \Pi^I_\mu = \phi_\mu Y^I - \overline{\Theta}\Gamma^I \phi_\mu \Theta, \tag{4.67}$$

$$\mathbf{G}_{\mu\nu} = \eta_{MN}\Pi^M_\mu \Pi^N_\nu + \phi_\mu T \phi_\nu T\,, \tag{4.68}$$

and

$$\mathbf{F}_{\mu\nu} = F_{\mu\nu} - \left[\{\overline{\Theta}\Gamma_{11}\Gamma_\nu \phi_\mu\Theta + \overline{\Theta}\Gamma_{11}\Gamma_I \phi_\mu \Theta \phi_\nu Y^I - \frac{1}{2}\overline{\Theta}\Gamma_{11}\Gamma_M \phi_\mu \Theta \overline{\Theta}\Gamma^M \phi_\nu \Theta\} - \{\mu \leftrightarrow \nu\}\right], \tag{4.69}$$

where

$$F_{\mu\nu} = \phi_\mu A_\nu - \phi_\nu A_\mu\,. \tag{4.70}$$

In terms of these variables, the DBI part of the world-volume action is given by

$$S_{DBI} = -\int d^{p+1}x\, V(T)\sqrt{-\det(\mathbf{G}+\mathbf{F})}\,. \tag{4.71}$$

The action is invariant under the supersymmetry transformation parametrized by a ten dimensional Majorana spinor ϵ. In the static gauge in which we are working, the infinitesimal supersymmetry transformation laws are given by

$$\begin{aligned}
&\delta_p\Theta = \epsilon - (\bar\epsilon\Gamma^\mu\Theta)\phi_\mu\Theta, \qquad \delta_p Y^I = \bar\epsilon\Gamma^I\Theta - (\bar\epsilon\Gamma^\mu\Theta)\phi_\mu Y^I\,, \qquad \delta_p T = -(\bar\epsilon\Gamma^\mu\Theta)\phi_\mu T\,,\\
&\delta_p A_\nu = \bar\epsilon\Gamma_{11}\Gamma_\nu\Theta + \bar\epsilon\Gamma_{11}\Gamma_I\Theta\phi_\nu Y^I - \frac{1}{6}(\bar\epsilon\Gamma_{11}\Gamma_M\Theta\,\overline\Theta\Gamma^M\phi_\nu\Theta + \bar\epsilon\Gamma_M\Theta\,\overline\Theta\Gamma_{11}\Gamma^M\phi_\nu\Theta)\\
&\qquad\quad - (\bar\epsilon\Gamma^\mu\Theta)\phi_\mu A_\nu - \phi_\nu(\bar\epsilon\Gamma^\mu\Theta)A_\mu\,. \qquad (4.72)
\end{aligned}$$

The subscript p in δ_p denotes that these are the supersymmetry transformation laws on the D-p-brane world-volume. The supersymmetry transformation parameter ϵ is a Majorana spinor of the ten dimensional Lorentz group.

Besides the DBI term, the world-volume action also contains a Wess-Zumino term. In the bosonic sector this term is important only for non-vanishing RR background field, but once we take into account the world-volume fermions, this term survives even for zero RR background. The structure of this term is

$$S_{WZ} = \int W(T)\,dT\wedge\mathbf{C}\wedge e^{\mathbf{F}}\,, \qquad (4.73)$$

where $\mathbf{F} = \mathbf{F}_{\mu\nu}dx^\mu\wedge dx^\nu$, $W(T)$ is an even function of T which vanishes as $T\to\pm\infty$, and $\mathbf{C}$ is a specific combination of background RR fields and the world-volume fields Y^I, Θ on the D-brane[2]. In particular, the bosonic part of $\mathbf{C}$ is given by $\sum_{q\ge0}C^{(p-2q)}$ where $C^{(p-2q)}$ denotes the pull-back of the RR $(p-2q)$-form field on the D-p-brane world-volume. This vanishes for vanishing RR background, but there is a part of $\mathbf{C}$ involving the world-volume fermion fields that survives even in the absence of any RR background. Since we shall not need the explicit form of $\mathbf{C}$ for our analysis, we shall not give it here. The Wess-Zumino term is also invariant under the supersymmetry transformations (4.72). Later we shall see that consistency requires:

$$\int_{-\infty}^{\infty}W(T)dT = \int_{-\infty}^{\infty}V(T)dT = \mathcal{T}_{p-1}\,, \qquad (4.74)$$

Since we want to compare the world-volume action on a kink solution with that on the BPS D-$(p-1)$-brane, we need to first know the form of the world-volume action on a BPS D-$(p-1)$-brane. The world-volume fields in this case consist of a vector field $a_\alpha(\xi)$ ($0\le\alpha\le(p-1)$), a set of $(9-p+1)$ scalar fields which we shall denote by $y^I(\xi)$ ($(p+1)\le I\le 9$) and $y^p(\xi)\equiv t(\xi)$ respectively in the convention, and a Majorana spinor $\theta(\xi)$ of the ten dimensional Lorentz group. Here $\{\xi^\alpha\}$ denote the world-volume coordinate on the D-$(p-1)$-brane. The DBI part of the action is given by

$$S_{dbi} = -\mathcal{T}_{p-1}\int d^p\xi\,\sqrt{-\det(\mathbf{g}+\mathbf{f})}\,, \qquad (4.75)$$

where

$$\mathbf{g}_{\alpha\beta} = \eta_{MN}\pi^M_\alpha\pi^N_\beta\,, \qquad (4.76)$$

$$\pi^\beta_\alpha = \delta^\beta_\alpha - \bar\theta\Gamma^\beta\phi_\alpha\theta, \qquad \pi^I_\alpha = \phi_\alpha y^I - \bar\theta\Gamma^I\phi_\alpha\theta, \qquad \pi^p_\alpha = \phi_\alpha t - \bar\theta\Gamma^p\phi_\alpha\theta, \qquad (4.77)$$

$$\mathbf{f}_{\alpha\beta} = -\Big[\{\bar\theta\Gamma_{11}\Gamma_\beta\phi_\alpha\theta + \bar\theta\Gamma_{11}\Gamma_I\phi_\alpha\theta\phi_\beta y^I + \bar\theta\Gamma_{11}\Gamma_p\phi_\alpha\theta\phi_\beta t - \frac{1}{2}\bar\theta\Gamma_{11}\Gamma_M\phi_\alpha\theta\bar\theta\Gamma^M\phi_\beta\theta\}\Big]\,, \qquad (4.78)$$

The Wess-Zumino term, on the other hand, has the form:

$$S_{wz} = \mathcal{T}_{p-1}\int\mathbf{c}\wedge e^{\mathbf{f}}\,, \qquad (4.79)$$

where $\mathbf{f} = \mathbf{f}_{\alpha\beta}d\xi^\alpha\wedge d\xi^\beta$, and $\mathbf{c}$ is an expression containing the RR background and the world-volume fields y^I, t, θ. The bosonic part of $\mathbf{c}$ is given by $\sum_{q\ge0}C^{(p-2q)}$ where $C^{(p-2q)}$ denotes the pull-back of the RR $(p-2q)$-form field on the D-$(p-1)$-brane world-volume. Like $\mathbf{C}$, $\mathbf{c}$ also contains a term involving y^I and θ which survive even for trivial RR background. If we think of the world-volume of the D-$(p-1)$-brane as sitting inside that of a D-p-brane along the surface $x^p = t(\xi)$, then $\mathbf{c}$ is in fact the pullback of $\mathbf{C}$ appearing in (4.73) provided we identify θ and y^I as the restriction of Θ and Y^I along the surface $x^p = t(\xi)$.

Both S_{dbi} and S_{wz} are separately invariant under the infinitesimal supersymmetry transformation:

$$\begin{aligned}
&\delta_{p-1}\theta = \epsilon - (\bar\epsilon\Gamma^\alpha\theta)\phi_\alpha\theta, \qquad \delta_{p-1}y^I = \bar\epsilon\Gamma^I\theta - (\bar\epsilon\Gamma^\alpha\theta)\phi_\alpha y^I\,, \qquad \delta_{p-1}t = \bar\epsilon\Gamma^p\theta - (\bar\epsilon\Gamma^\alpha\theta)\phi_\alpha t\,,\\
&\delta_{p-1}a_\beta = \bar\epsilon\Gamma_{11}\Gamma_\beta\theta + \bar\epsilon\Gamma_{11}\Gamma_I\theta\phi_\beta y^I + \bar\epsilon\Gamma_{11}\Gamma_p\theta\phi_\beta t - \frac{1}{6}(\bar\epsilon\Gamma_{11}\Gamma_M\theta\,\bar\theta\Gamma^M\phi_\beta\theta + \bar\epsilon\Gamma_M\theta\,\bar\theta\Gamma_{11}\Gamma^M\phi_\beta\theta)\\
&\qquad\qquad - (\bar\epsilon\Gamma^\alpha\theta)\phi_\alpha a_\beta - \phi_\beta(\bar\epsilon\Gamma^\alpha\theta)a_\alpha\,.
\end{aligned} \tag{4.80}$$

The subscript $(p-1)$ on δ_{p-1} indicates that these represent supersymmetry transformation laws on the world-volume of a BPS D-$(p-1)$-brane.

In order to show that the world-volume action $S_{dbi} + S_{wz}$ on the BPS D-$(p-1)$-brane arises from the world-volume action on the tachyon kink solution, we need to first propose an ansatz relating the fields $T(x,\xi)$, $A_\mu(x,\xi)$, $Y^I(x,\xi)$ and $\Theta(x,\xi)$ to the fields $a_\alpha(\xi)$, $y^I(\xi)$, $t(\xi)$ and $\theta(\xi)$ on the BPS D-brane. For this we propose the following ansatz:

$$\begin{aligned}
&T(x,\xi) = f\Big(a(x-t(\xi))\Big), \qquad Y^I(x,\xi) = y^I(\xi), \qquad \Theta(x,\xi) = \theta(\xi),\\
&A_x(x,\xi) = 0 \qquad A_\alpha(x,\xi) = a_\alpha(\xi)\,.
\end{aligned} \tag{4.81}$$

We can now compute $\mathbf{G}_{\mu\nu}$ and $\mathbf{F}_{\mu\nu}$ in terms of the variables a_α, y^I, t and θ using (4.67)-(4.70) and (4.81). The result is:

$$\begin{aligned}
&\mathbf{G}_{xx} = 1 + a^2(f')^2\,, \qquad \mathbf{G}_{\alpha x} = \mathbf{G}_{x\alpha} = -a^2(f')^2\phi_\alpha t - \bar\theta\Gamma^p\phi_\alpha\theta\,,\\
&\mathbf{G}_{\alpha\beta} = \mathbf{g}_{\alpha\beta} + \phi_\alpha t\,\bar\theta\Gamma^p\phi_\beta\theta + \phi_\beta t\,\bar\theta\Gamma^p\phi_\alpha\theta + (a^2(f')^2 - 1)\phi_\alpha t\phi_\beta t\,,\\
&\mathbf{F}_{\alpha x} = -\mathbf{F}_{x\alpha} = -\bar\theta\Gamma_{11}\Gamma^p\phi_\alpha\theta\,,\\
&\mathbf{F}_{\alpha\beta} = \mathbf{f}_{\alpha\beta} - \phi_\alpha t\,\bar\theta\Gamma_{11}\Gamma^p\phi_\beta\theta + \phi_\beta t\,\bar\theta\Gamma_{11}\Gamma^p\phi_\alpha\theta\,,
\end{aligned} \tag{4.82}$$

with $\mathbf{g}_{\alpha\beta}$ and $\mathbf{f}_{\alpha\beta}$ defined as in (4.76). Using manipulations we can now show that

$$\det(\mathbf{G}+\mathbf{F}) = a^2(f')^2\{\det(\mathbf{g}+\mathbf{f}) + \mathcal{O}(a^{-2})\}\,, \tag{4.83}$$

and

$$S_{DBI} = -\int d^{p+1}x\, V(T)\,\sqrt{-\det(\mathbf{G}+\mathbf{F})} = -\mathcal{T}_{p-1}\int d^p\xi\,\sqrt{-\det(\mathbf{g}+\mathbf{f})} = S_{dbi}\,. \tag{4.84}$$

The analysis for S_{WZ} is even simpler; – indeed this term was designed to reproduce the Wess-Zumino term on the world-volume of a kink solution. For this let us define $u = x - t(\xi)$ Then from (4.82) we get

$$\begin{aligned}
\mathbf{F} \equiv \mathbf{F}_{\mu\nu}\,dx^\mu\wedge dx^\nu &= 2\mathbf{F}_{x\beta}\,dx\wedge d\xi^\beta + \mathbf{F}_{\alpha\beta}\,d\xi^\alpha\wedge d\xi^\beta\\
&= 2\bar\theta\Gamma_{11}\Gamma_p\phi_\alpha\theta du\wedge d\xi^\alpha + \mathbf{f}_{\alpha\beta}\,d\xi^\alpha\wedge d\xi^\beta\,.
\end{aligned} \tag{4.85}$$

Since we have $dT = af'(au)\,du$ only the second term on the right hand side of (4.85) will contribute to S_{WZ} given in (4.73). Thus we can replace $\mathbf{F}$ by $\mathbf{f}$ in (4.73). On the other hand, we can analyze $\mathbf{C}$ by writing it as

$$\begin{aligned}
\mathbf{C} &= \sum_q \mathbf{C}^{(q)}_{\mu_1\cdots\mu_q}dx^{\mu_1}\wedge\cdots dx^{\mu_q}\\
&= \sum_q (q\,\mathbf{C}^{(q)}_{x\alpha_2\cdots\alpha_q}dx\wedge d\xi^{\alpha_2}\wedge\cdots d\xi^{\alpha_q} + \mathbf{C}^{(q)}_{\alpha_1\cdots\alpha_q}d\xi^{\alpha_1}\wedge\cdots d\xi^{\alpha_q})\\
&= \sum_q \Big[q\,\mathbf{C}^{(q)}_{x\alpha_2\cdots\alpha_q}du\wedge d\xi^{\alpha_2}\wedge\cdots d\xi^{\alpha_q} + (q\,\mathbf{C}^{(q)}_{x\alpha_2\cdots\alpha_q}\phi_{\alpha_1}t + \mathbf{C}^{(q)}_{\alpha_1\cdots\alpha_q})\,d\xi^{\alpha_1}\wedge\cdots d\xi^{\alpha_q}\Big]\,,
\end{aligned} \tag{4.86}$$

where in the last step we have used $dx = du + \phi_\alpha t d\xi^\alpha$. The term proportional to du does not contribute to (4.73), whereas the term proportional to $d\xi^{\alpha_1} \wedge \cdots d\xi^{\alpha_q}$, after being summer over q, is precisely the pull-back of $\mathbf{C}$ on the kink world-volume along $x = t(\xi)$ and hence can be identified with $\mathbf{c}$. Thus using (4.74) we get

$$S_{WZ} = \int W(f(au))\, a\, f'(au)\, du \wedge \mathbf{c} \wedge e^{\mathbf{f}} = \mathcal{T}_{p-1} \int \mathbf{c} \wedge e^{\mathbf{f}} = S_{wz}\,, \tag{4.87}$$

This shows that $S_{DBI} + S_{WZ}$ reduces to $S_{dbi} + S_{wz}$ under the identification (4.81). In principle we also need to check that any solution of the equations of motion derived from $S_{dbi} + S_{wz}$ is automatically a solution of the equations of motion derived from $S_{DBI} + S_{WZ}$. Presumably this can be done following the structural analysis, but we have not worked out all the details.

Finally, we need to check that the supersymmetry transformations (4.80) are compatible with the supersymmetry transformations (4.72). For this we need to calculate $\delta_{p-1} A_\mu$, $\delta_{p-1} Y^I$ and $\delta_{p-1} T$ using (4.80), (4.81) and compare them with (4.72). The calculation is straightforward, and we get:

$$\begin{aligned} &\delta_p A_x = \delta_{p-1} A_x + \bar{\epsilon}\Gamma_{11}\Gamma_p\theta, \qquad \delta_p A_\alpha = \delta_{p-1} A_\alpha - \bar{\epsilon}\Gamma_{11}\Gamma_p\theta\phi_\alpha t, \\ &\delta_p Y^I = \delta_{p-1} Y^I, \qquad \delta_p T = \delta_{p-1} T\,. \end{aligned} \tag{4.88}$$

Thus we see that δ_p and δ_{p-1} differ for the transformation laws of A_x and A_α. This difference, however, is precisely of the form induced by the function $\phi(x,\xi)$ with $\phi(x,\xi) = \bar{\epsilon}\Gamma_{11}\Gamma_p\theta(\xi)$. As was argued below, this is a gauge transformation. Thus we see that the action of δ_p and δ_{p-1} differ by a gauge transformation in the world-volume theory on the D-p-brane.

This establishes that the world volume action on the kink reduces to that on a D-$(p-1)$-brane. The latter has a local κ-symmetry which can be used to gauge away half of the world-volume fermion superfields. This leads to a puzzle. Whereas on a BPS D-brane the local κ-symmetry is postulated to be a gauge symmetry, i.e. different configurations related by κ-transformation are identified, on a kink solution the appearance of the κ-symmetry seems accidental and a priori there is no reason to identify field configurations which are related by κ-symmetry. We believe the resolution of this puzzle lies in the general principle advocated below that any local transformation of the fields which does not change the action must be a gauge symmetry. This will automatically imply that the κ-transformation is a gauge transformation and we should identify the configurations related by κ-transformation. This κ-symmetry can now be used to gauge away half of the fermion fields on the world-volume of the kink.

5 The Tachyon Condensation with Bulk Fields

Study of various aspects of tachyon dynamics on a non-BPS D-brane of type IIA or IIB superstring field theory has led to some understanding of the tachyon dynamics near the tachyon vacua. The proposed tachyon effective action, describing the dynamics of the tachyon field on a non-BPS Dp-brane of type IIA or IIB superstring theory, is given by:

$$\begin{aligned} S &= \int d^{p+1}x\, \mathcal{L}\,, \\ \mathcal{L} &= -V(T)\sqrt{-\det \mathbf{A}}\,, \end{aligned} \tag{5.89}$$

where

$$\mathbf{A}_{\mu\nu} = \eta_{\mu\nu} + \phi_\mu T \phi_\nu T + \phi_\mu Y^I \phi_\nu Y^I + F_{\mu\nu}\,, \tag{5.90}$$

$$F_{\mu\nu} = \phi_\mu A_\nu - \phi_\nu A_\mu\,. \tag{5.91}$$

A_μ and Y^I for $0 \le \mu, \nu \le p$, $(p+1) \le I \le 9$ are the gauge and the transverse scalar fields on the world-volume of the non-BPS brane, and T is the tachyon field. $V(T)$ is the tachyon potential which is symmetric under $T \to -T$, has a maximum at $T = 0$, and has its minimum at $T = \pm\infty$ where it vanishes. We are using the convention where $\eta = diag(-1, 1, \dots 1)$ and the fundamental string tension has been set equal to $(2\pi)^{-1}$ ($i.e.$ $\alpha' = 1$).

The usual approach to defect formation during tachyon condensation is to take into account only the fields that live in the worldvolume of the decaying non-BPS brane or brane-anti-brane pair. This approach seems to be motivated by the fact that the resulting lower-dimensional branes formed during the decay are localized inside the worldvolume of the parent brane. However, the final state defects are themselves D-branes, and therefore couple to bulk RR-fields. One should include the effects of these fields in the defect formation process.

When studying the formation of topological defects by the brane worldvolume fields only, one usually considers a single non-BPS brane with the action:

$$S = -T_p \int d^{p+1}x \, e^{-\phi} \, V(T) \sqrt{\det \left(P\left[G_{ab} + B_{ab}\right] + 2\pi\alpha' \left[F_{ab} + \partial_a T \partial_b T\right]\right)} \,, \tag{5.92}$$

where $P\left[G_{ab} + B_{ab}\right]$ represents the pull-back of the bulk metric and NS-NS two-form field on the brane and F_{ab} is the field strength of the Abelian world-volume gauge superfield. This action was used extensively to study the evolution of the tachyon field in various special settings. The most common one is the spatially uniform field in a Friedmann-Robertson-Walker universe. In this case one usually sets the NS-NS two-form field and the brane gauge field to vanish everywhere, and study the time evolution of the tachyon field and scale factor of the universe. The uniform field does not lead to the formation of defects, it behaves like a pressureless fluid known as "tachyon matter".

When studying the formation of topological defects during tachyon condensation, one usually chooses a flat metric, sets the NS-NS field and the brane gauge field to vanish everywhere, but chooses a tachyon field that is both time and space dependent. The equation of motion for the tachyon field

$$\frac{\partial_\alpha \left(\sqrt{-g} g^{\alpha\beta} \partial_\beta T\right)}{\sqrt{1 + \partial_\alpha T \partial^\alpha T}} - \frac{\sqrt{-g} g^{\alpha\beta} \partial_\beta T \partial_\alpha \left(\partial_\mu T \partial^\mu T\right)}{2\left(1 + \partial_\alpha T \partial^\alpha T\right)^{3/2}} - \frac{V'\left(T\right)}{V\left(T\right)} \frac{\sqrt{-g}}{\sqrt{1 + \partial_\alpha T \partial^\alpha T}} = 0 \,, \tag{5.93}$$

is non-linear, and one usually solves the equation around a point where $T = 0$. There the space and time dependence can be approximated by a linear space profile with a time-dependent slope, $T\left(t, x\right) \simeq u\left(t\right) x$, and the resulting equation for $u\left(t\right)$ can be solved. The solution becomes singular in finite time, the occurrence of the singularity marking the formation of the topological defect. This result confirms the String Theory calculation in which a linear tachyon profile $T\left(x\right) = ux$ reproduces the correct tension of a codimension one brane in the limit $u \to \infty$.

The brane gauge superfield is usually included in the form of a uniform background electric field or a constant gauge potential (Wilson line). The case where both the brane gauge superfield and a worldvolume scalar superfield other than the tachyon are included. This action is highly non-linear and in order to perform a lattice regularization we prefer an expression in which the square-root is expanded to quadratic order in the field strengths. Also for a single non-BPS brane the tachyon is a real scalar which cannot be minimally coupled to the world-volume gauge superfield.

It is therefore more convenient to consider the action for a brane-anti-brane pair. In the case of a $(D9, \overline{D9})$ brane system the expanded action involving the complex tachyon coupled to the gauge superfields living inside each brane is given

$$\begin{aligned} S &= 2T_{D9} \int d^{10}x \, e^{-\phi} e^{-2\pi\alpha' T\overline{T}} \left[1 + 8\pi\alpha' \ln\left(2\right) D^\mu \overline{T} D_\mu T + \right. \\ & \left. \frac{\left(2\pi\alpha'\right)^2}{8} \left(F^+_{\mu\nu}\right)^2 + \frac{\left(2\pi\alpha'\right)^2}{8} \left(F^-_{\mu\nu}\right)^2 + \frac{\beta\alpha'^2}{8} \left(F^+_{\mu\nu} - F^-_{\mu\nu}\right)^2\right] . \end{aligned} \tag{5.94}$$

The two gauge fields live in the worldvolume of each brane and the tachyon field couples only with one linear combination:

$$D_\mu T = \partial_\mu T - \left(A^+_\mu - A^-_\mu\right) T\,. \tag{5.95}$$

The brane fields couple with the Ramond-Ramond (RR) bulk fields through the Chern-Simons coupling also given in Ref. [?]:

$$S^{D\overline{D}}_{RR} = T_{D9} \int C \wedge Str\; e^{2\pi i \alpha' \mathcal{F}}\,. \tag{5.96}$$

One can expand the exponential above, and the leading order coupling between the brane fields and the bulk RR field involves the same linear combination of brane gauge fields that couples to the tachyon:

$$S^{D\overline{D}}_{RR} = 2\pi\alpha' T_{D9} \int C_{p-1} \wedge \left(F^+ - F^-\right) \tag{5.97}$$

We see that the orthogonal linear combination, $A^+_\mu + A^-_\mu$ does not couple to any other fields, so we will drop it from the action. Including the kinetic terms for the RR field, the dilaton and the metric, the 10-dimensional action describing the decay of the brane-anti-brane pair is:

$$\begin{aligned} S = &\frac{1}{2\kappa^2_{10}} \int \sqrt{-G}\, d^{10}x \left[e^{-2\phi}\left(R + 2\left(\nabla\phi\right)^2\right) - \frac{1}{2\,(p)!} F^2_p\right] \\ &-2T_{D9} \int d^{10}x\, e^{-\phi}\, e^{-2\pi\alpha' T\overline{T}} \left[1 + 8\pi\alpha' \ln\left(2\right) D^\mu \overline{T} D_\mu T + \frac{\left(2\pi^2 + \beta\right)\alpha'^2}{8} \left(F^+_{\mu\nu} - F^-_{\mu\nu}\right)^2\right], \end{aligned} \tag{5.98}$$

where F_p is the corresponding field strength for the potential C_{p-1}, $F_p = dC_{p-1}$. The C_{p-1} will be the only field we consider here, we will not include the dilaton and the metric in the extremal model we consider. Here we expand the Chern-Simons coupling of the brane and bulk fields to second order in α' where

$$C = \sum_{p=odd} \frac{(-i)^{\frac{9-p}{2}}}{(p+1)!} C_{\mu_0 \ldots \mu_p} dx^{\mu_0} \wedge \ldots \wedge dx^{\mu_p}\,. \tag{5.99}$$

The supertrace of the matrix is defined as,

$$Str\; M = Tr \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} M\,, \tag{5.100}$$

and $\mathcal{F}$ is the curvature of the superconnection, given by

$$i\mathcal{F} = \begin{pmatrix} iF^+ - T\overline{T} & D\overline{T} \\ DT & iF^- - T\overline{T} \end{pmatrix}. \tag{5.101}$$

Here we are interested in the particular case of codimension defects, so the relevant coupling will be with the C_{p-1} RR-field that couples with the defects. We want to keep both the tachyon and the gauge fields non-zero, so we will expand the exponential inside the supertrace in powers of α':

$$\begin{aligned} Str\; e^{2\pi i\alpha'\mathcal{F}} \;=\; & Tr \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \left(2\pi\alpha'\right) Tr \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} iF^+ - T\overline{T} & D\overline{T} \\ DT & iF^- - T\overline{T} \end{pmatrix} \\ & + \frac{\left(2\pi\alpha'\right)^2}{2} Tr \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} iF^+ - T\overline{T} & D\overline{T} \\ DT & iF^- - T\overline{T} \end{pmatrix} \wedge \begin{pmatrix} iF^+ - T\overline{T} & D\overline{T} \\ DT & iF^- - T\overline{T} \end{pmatrix} \\ & + \ldots \\ =\; & \left(2\pi i\alpha'\right)\left[F^+ - F^-\right] + \\ & \left(2\pi\alpha'\right)^2 \left[D\overline{T} \wedge DT - iT\overline{T}\left(F^+ - F^-\right) - \frac{1}{2}(F^+ \wedge F^+ - F^- \wedge F^-)\right] + \ldots\,. \end{aligned} \tag{5.102}$$

The terms of the type $F \wedge F$ couple with C_{p-3} and count only in the formation of codimension defects. Therefore, in the case of a $(D9, \overline{D9})$ brane system, the important couplings between the brane and the bulk fields are

$$T_{D9} \int -iC_8 \wedge \left\{ (2\pi i \alpha') \left[F^+ - F^- \right] + (2\pi\alpha')^2 \left[D\overline{T} \wedge DT - iT\overline{T} \left(F^+ - F^- \right) \right] \right\} . \tag{5.103}$$

Since we are interested in the simplest model which involves such a coupling, when we construct the toy model we keep only the interaction that corresponds to the leading term in α',

$$(2\pi\alpha')\, T_{D9} \int C_8 \wedge \left[F^+ - F^- \right] . \tag{5.104}$$

The easiest setup to study the formation of defects in the original theory is to consider the action for a brane-anti-brane pair for only the tachyon field in flat space-time, with all the other fields turned off, since the lower-dimensional branes are in fact vortices of the complex tachyon field. The action is simply:

$$S = 2T_{D9} \int d^{10}X \, e^{-2\pi\alpha' T\overline{T}} \left[1 + 8\pi\alpha' \ln(2)\, D^\mu \overline{T} D_\mu T \right] . \tag{5.105}$$

One can also consider the action for lower-dimensional brane-anti-brane pairs. The equation of motion derived from the acion above is:

$$\partial_\mu \partial^\mu T - 2\pi\alpha' \overline{T} \partial_\mu T \partial^\mu T + \frac{T}{4\ln 2} = 0 . \tag{5.106}$$

As in the case of the real tachyon field, one can approximate the profile of the field with a linear one as the vortex will form at the place where $T = 0$, and solve the resulting equation for the slope of the profile. The resulting defect formed in the decay of a $(Dp, \overline{Dp})$ brane system is a $Dp - 2$ brane.

In order to undestand this we have to go back to the calculation of the space-time action calculated on linear tachyon profiles and estimate the action in the limit of infinite slope. The calculation was done and we reproduce the important points here. In the case of a $(D9, \overline{D9})$ brane pair the space-time action for a linear tachyon profile is

$$S\left(y^I\right) = 2T_{D9} \int dX^{10} \, e^{-2\pi\alpha' T\overline{T}} \prod_{I=1}^{2} F\left(\pi\alpha' y^I\right) \tag{5.107}$$

where $T^I = u^I X^I / \sqrt{\alpha'}$ and $y^I = \left(u^I\right)^2$. The function F has the expression and in the large argument limit it takes the form:

$$F(x) \simeq \sqrt{\pi x} . \tag{5.108}$$

Calculating the action on the profile $y^1 \to \infty$ and $y^2 \to \infty$ the authors of Ref. [?] obtain:

$$\begin{aligned} S\left(y^I\right) &= 2T_{D9} \int dX^{10} e^{-\frac{\pi}{2}\left[y^1 \left(X^1\right)^2 + y^2 \left(X^2\right)^2\right]} F\left(\pi\alpha' y^1\right) F\left(\pi\alpha' y^2\right) \\ &= 2T_{D9} \int dX^8 \sqrt{\frac{2}{y^1}} \sqrt{\frac{2}{y^2}} \sqrt{\pi^2\alpha' y^1} \sqrt{\pi^2\alpha' y^2} \;\to\; 4\pi^2\alpha' T_{D9} \int dX^8 . \end{aligned} \tag{5.109}$$

The result gives the correct tension for a $D7$ brane, $T_{D7} = \left(2\pi\sqrt{\alpha'}\right)^2 T_{D9}$. Regarding the RR charge of the vortex, we can estimate it by using the result (5.102) in the expression of the coupling between brane fields and the bulk RR fields,

$$S_{RR} = T_{D9} \int C_8 \wedge e^{-2\pi\alpha' T\overline{T}} (2\pi\alpha')^2 \, dT \wedge d\overline{T} = \frac{(2\pi\alpha')^2}{\alpha'} T_{D9} \int C_8 , \tag{5.110}$$

which again reproduces the correct result for the $D7$ brane RR charge.

These results allow us to obtain an upper limit for the density of defects formed, based only on energetic considerations. The brane tension is equal to the mass per unit volume for the brane and the result

$$T_{Dp-2} = \left(2\pi\sqrt{\alpha'}\right)^2 T_{Dp} \tag{5.111}$$

tells us that we can have at most one defect on each patch of area $\left(2\pi\sqrt{\alpha'}\right)^2 = (2\pi l_s)^2$ where l_s is the string length. This is a very large density and it shows that constraints other than the energetic ones are more important in determining the final density of defects. In a realistic model we expect that a very important role will be that of the other fields that we have neglected so far, namely the dilaton and the graviton. These two fields have universally attractive interactions and their presence allows for the existence of BPS states in which there is no interaction between identical, parallel, branes. We expect the presence of these fields to also change the evolution of the resulting network of defects, since our toy model allows for repulsive interactions between same-charge defects, while no repulsive interactions are possible in the full Superstring Theory model. The reduction of the number of fermionic degrees of freedom due to κ-symmetry for a brane stretched in the T direction occurs here as well. The D-branes used in the construction of are BPS in $9+1$ dimensions. Thus, a D-brane localized on the circle transverse to the fivebranes has a sixteen component fermionic field living on its worldvolume, while for a D-brane wrapping the circle the analog of the κ-symmetry discussed above eliminates half of the fermions, and leaves a dynamical eight component spinor on the brane. This agrees with spacetime expectations. It is well known from the Hanany-Witten construction that a D-brane ending on a stack of $NS5$-branes has the property that the massless fields on it are the worldvolume gauge field and fields related to it by supersymmetry. Since the D/NS system preserves eight supercharges, those fields form a vector multiplet of $N = 1$ supersymmetry in $5 + 1$ dimensions, reduced to the p-dimensional worldvolume of the D-brane. This multiplet contains an eight-component fermion, in agreement with our discussion above.

6 The Effective Actions of $(Dp, \overline{Dp})$ Brane Systems

In the scheme of effective field theory, instability of $(Dp, \overline{Dp})$ brane-antibrane system is represented by a complex tachyon field $(T, \overline{T})$, and this tachyon is indispensable for the generation of topological defects. Since this system possesses U(1)×U(1) gauge symmetry, we need two gauge fields, A^a_μ, $a = 1, 2$. Separation of the brane and the antibrane is described by scalar fields X^I_a corresponding to the transverse coordinates of individual branes. When the U(1)×U(1) gauge fields in (6.114) are rewritten as $A_\mu = (A^1_\mu + A^2_\mu)/2$ and $C_\mu = (A^1_\mu - A^2_\mu)/2$, the former A_μ remains to be massless in symmetric phase and the latter C_μ becomes massive in broken phase. It is easily read by the form of the action

$$S = -\mathcal{T}_p \int d^{p+1}x\, V(T) \left[\sqrt{-\det(X^+_{\mu\nu})} + \sqrt{-\det(X^-_{\mu\nu})} \,\right], \tag{6.112}$$

where $C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$, $D_\mu T = (\partial_\mu - 2iC_\mu)T$, and

$$X^\pm_{\mu\nu} = g_{\mu\nu} + F_{\mu\nu} \pm C_{\mu\nu} + (\overline{D_\mu T} D_\nu T + \overline{D_\nu T} D_\mu T)/2. \tag{6.113}$$

Note that the charge of T is 2 in the unit system of consideration. In this coincidence limit, the D-brane is distinguished from the $\overline{\text{D}}$-brane by coupling to the gauge field C_μ.

For $F_{\mu\nu}$, we employ the same configuration, i.e., all the components vanish for singular local D-vortex and F_{0r} is turned on for regular local D-vortex as has been done for global D-vortices.

Among various proposed tachyon effective actions, we shall deal with Dirac-Born-Infeld (DBI) type effective action of the $(Dp, \overline{Dp})$ brane system

$$S = -\mathcal{T}_p \int d^{p+1}x\, V(T, X^I_1 - X^I_2)$$

$$\times \left\{ \sqrt{-\det\left[g_{\mu\nu} + F^1_{\mu\nu} + \partial_\mu X^I_1 \partial_\nu X^I_1 + (\overline{D_\mu T} D_\nu T + \overline{D_\nu T} D_\mu T)/2\right]} \right.$$
$$\left. + \sqrt{-\det\left[g_{\mu\nu} + F^2_{\mu\nu} + \partial_\mu X^I_2 \partial_\nu X^I_2 + (\overline{D_\mu T} D_\nu T + \overline{D_\nu T} D_\mu T)/2\right]} \right\}, \tag{6.114}$$

where $\mathcal{T}_p$ is the tension of the Dp-brane, $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$, and $D_\mu T = (\partial_\mu - iA^1_\mu + iA^2_\mu)T$. For small tachyon amplitude τ from $T = \tau e^{i\chi}$, behavior of the tachyon potential is

$$V(T, X^I_1 - X^I_2) = 1 - \left[\frac{1}{R^2} - \sum_I (X^I_1 - X^I_2)^2\right] \tau^2 + \mathcal{O}(\tau^4), \tag{6.115}$$

where R is $\sqrt{2}$ in superstring theory.

In the context of DBI-type effective action describing D-brane systems with instability, there have been much study on codimension-one solitons (codimension-one branes), particularly tachyon kinks. For vortices (codimension-two branes), only the singular local vortex solution with finite energy was constructed from the DBI action, and regular tachyon vortex solutions were obtained in local field theory action with quadratic derivative terms and polynomial tachyon potential.

Let us consider $(Dp, \overline{Dp})$ brane system in the coincidence limit of two branes, $X^I_1 = X^I_2$, with fundamental strings. Then the macroscopic fundamental strings in fluid state are represented by electric fluxes along their directions. Vortex-like codimension-two objects of our interest could be interpreted as D$(p-2)$-branes. For description of the global vortex-like objects, the gauge fields and their field strengths should behave as $A^1_\mu = A^2_\mu = A_\mu$ and $F^1_{\mu\nu} = F^2_{\mu\nu} = F_{\mu\nu}$ in the action (6.114). Then the action (6.114) in (1+p)-dimensions becomes

$$S = -2\mathcal{T}_p \int d^{p+1}x \, V(\tau)\sqrt{-\det X}\,, \tag{6.116}$$

where

$$X_{\mu\nu} = g_{\mu\nu} + F_{\mu\nu} + (\partial_\mu \overline{T} \partial_\nu T + \partial_\nu \overline{T} \partial_\mu T)/2. \tag{6.117}$$

Additionally we assume that the produced D$(p-2)$-branes are flat and all the transverse degrees are frozen. Then we can neglect dependence of the transverse coordinates and it is enough to find D0-branes from flat $(Dp, \overline{Dp})$. In the context of solitons in the effective theory, it is translated as point-like vortices on a plane. We will call this vortex as D-vortex in what follows.

For unstable D-branes, the coupling to the bulk RR fields can be read off from the Wess-Zumino term and, for $(Dp, \overline{Dp})$, it is possibly be extended as

$$\begin{aligned} S_{\rm WZ} &= \mu \, {\rm Str} \int_{\Sigma_3} C_{\rm RR} \wedge \exp \begin{pmatrix} F^1 - T\overline{T} & i^{3/2}\, DT \\ -i^{3/2}\, \overline{DT} & F^2 - \overline{T}T \end{pmatrix} \\ &= \mu \int_{\Sigma_3} e^{-T\overline{T}} C_{\rm RR} \wedge (2C + iDT \wedge \overline{DT}) \\ &= 2\mu \int_{\Sigma_3} C_{\rm RR} \wedge dr \wedge d\theta \, \frac{d}{dr}\left[e^{-\tau^2}\left(C_\theta - \frac{n}{2}\right)\right], \end{aligned} \tag{6.118}$$

where μ is a real constant and the supertrace Str is defined to be a trace with insertion of σ_3. Note that $C \equiv C_{\mu\nu} dx^\mu \wedge dx^\nu$ and $C_{\mu\nu} = (F^1_{\mu\nu} - F^2_{\mu\nu})/2$. For the $(Dp, \overline{Dp})$ system, it is obvious that all the singular and regular, global ($C_\theta = 0$) and local D-vortices on the $(Dp, \overline{Dp})$ carry only D0 charge. In addition, the total D0 charge ($= 4\pi\mu \int dr \frac{d}{dr}[e^{-\tau^2}\left(C_\theta - \frac{n}{2}\right)] = 2\pi\mu n$) is exactly proportional to the vorticity n (or the quantized magnetic flux (**??**) for the local vortex) irrespective of the nature of the D-vortices, global or local, as expected.

The effective action of a $(Dp, \overline{Dp})$ brane system in Type IIA(B) theory should be given by some extension of the DBI action and the WZ terms which include the tachyon fields. The DBI part may be given by the projection of the effective action of two non-BPS D_p-branes in Type IIB(A) theory with $(-1)^{F_L}$. We are interested in this paper in the appearance of tachyon, gauge field and the RR field in these actions. These fields appear in the DBI part as the following

$$S_{DBI} = -\int d^{p+1}\sigma Tr \left(V(\mathcal{T})\sqrt{-\det(\eta_{ab}+2\pi\alpha' F_{ab}+2\pi\alpha' D_a\mathcal{T}D_b\mathcal{T})}\right) , \qquad (6.119)$$

The trace in the above action should be completely symmetric between all matrices of the form $F_{ab}, D_a\mathcal{T}$, and individual $\mathcal{T}$ of the tachyon potential. These matrices are

$$F_{ab} = \begin{pmatrix} F^{(1)}_{ab} & 0 \\ 0 & F^{(2)}_{ab} \end{pmatrix}, \; D_a\mathcal{T} = \begin{pmatrix} 0 & D_aT \\ (D_aT)^* & 0 \end{pmatrix}, \; \mathcal{T} = \begin{pmatrix} 0 & T \\ T^* & 0 \end{pmatrix} \qquad (6.120)$$

where $F^{(i)}_{ab} = \partial_a A^{(i)}_b - \partial_b A^{(i)}_a$ and $D_aT = \partial_a T - i(A^{(1)}_a - A^{(2)}_a)T$. If one uses ordinary trace, instead, the above action reduces to the action was proposed after making the kinetic term symmetric and performing the trace. This latter action is not consistent with S-matrix calculation. The tachyon potential which is consistent with S-matrix element calculations has the following expansion:

$$V(|T|) = 1 + \pi\alpha' m^2|T|^2 + \frac{1}{2}(\pi\alpha' m^2|T|^2)^2 + \cdots$$

where T_p is the p-brane tension , m^2 is the mass squared of tachyon, $i.e.m^2 = -1/(2\alpha')$. The above expansion is consistent with the potential $V(|T|) = e^{\pi\alpha' m^2|T|^2}$ which is the tachyon potential of BSFT.

The terms of the above action which has contribution to the S-matrix element of one gauge field and two tachyons in which we are interested in this paper are the following

$$\begin{aligned} \mathcal{L}_{DBI} = & -T_p(2\pi\alpha')\left(m^2|T|^2 + DT\cdot(DT)^* - \frac{\pi\alpha'}{2}\left(F^{(1)}\cdot F^{(1)} + F^{(2)}\cdot F^{(2)}\right)\right) + T_p(\pi\alpha')^3 \\ & \times\left(\frac{2}{3}DT\cdot(DT)^*\left(F^{(1)}\cdot F^{(1)} + F^{(1)}\cdot F^{(2)} + F^{(2)}\cdot F^{(2)}\right)\right. \qquad (6.121) \\ & +\frac{2m^2}{3}|\tau|^2\left(F^{(1)}\cdot F^{(1)} + F^{(1)}\cdot F^{(2)} + F^{(2)}\cdot F^{(2)}\right) \\ & \left.-\frac{4}{3}\left((D^\mu T)^* D_\beta T + D^\mu T(D_\beta T)^*\right)\left(F^{(1)\mu\alpha}F^{(1)}_{\alpha\beta} + F^{(1)\mu\alpha}F^{(2)}_{\alpha\beta} + F^{(2)\mu\alpha}F^{(2)}_{\alpha\beta}\right)\right) \end{aligned}$$

Note that if one uses the on-shell value for the tachyon mass $m^2 = -1/(2\alpha')$, the above terms would not be ordered in terms of power of α'.

The WZ term describing the coupling of RR field to gauge superfield of brane-antibrane system is given by

$$S = \mu_p \int_{\Sigma_{(p+1)}} C \wedge \left(e^{i2\pi\alpha' F^{(1)}} - e^{i2\pi\alpha' F^{(2)}}\right) , \qquad (6.122)$$

where $\Sigma_{(p+1)}$ is the world volume and μ_p is the RR charge of the branes. In above equation, C is a formal sum of the RR potentials $C = \sum_n (-i)^{\frac{p-m+1}{2}} C_m$. Note that the factors of i disappear in each term of (6.122). The inclusion of the tachyon fields into this action has been proposed in the literature using the superconnection of noncommutative geometry

$$S_{WZ} = \mu_p \int_{\Sigma_{(p+1)}} C \wedge \mathrm{STr}\, e^{i2\pi\alpha'\mathcal{F}} \qquad (6.123)$$

where the curvature of the superconnection is defined as:

$$\mathcal{F} = d\mathcal{A} - i\mathcal{A}\wedge\mathcal{A} \qquad (6.124)$$

the superconnection is

$$i\mathcal{A} = \begin{pmatrix} iA^{(1)} & \beta T^* \\ \beta T & iA^{(2)} \end{pmatrix} ,$$

where β is a normalization constant with dimension $1/\sqrt{\alpha'}$ which we shall find it later, and a "supertrace" is defined by

$$\mathrm{STr}\begin{pmatrix} A & B \\ C & D \end{pmatrix} = Tr\ A - Tr\ D\ .$$

Using the multiplication rule of the supermatrices [?]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} = \begin{pmatrix} AA' + (-)^{c'}BC' & AB' + (-)^{d'}BD' \\ DC' + (-)^{a'}CA' & DD' + (-)^{b'}CB' \end{pmatrix} \tag{6.125}$$

where x' is 0 if X is an even form or 1 if X is an odd form, one finds that the curvature is

$$i\mathcal{F} = \begin{pmatrix} iF^{(1)} - \beta^2|T|^2 & \beta(DT)^* \\ \beta DT & iF^{(2)} - \beta^2|T|^2 \end{pmatrix} ,$$

where $F^{(i)} = \frac{1}{2}F^{(i)}_{ab}dx^a \wedge dx^b$ and $DT = [\partial_a T - i(A^{(1)}_a - A^{(2)}_a)T]dx^a$. The WZ action (6.123) has the following terms:

$$\begin{aligned} C \wedge \mathrm{STr}\, i\mathcal{F} &= C_{p-1} \wedge (F^{(1)} - F^{(2)}) \\ C \wedge \mathrm{STr}\, i\mathcal{F} \wedge i\mathcal{F} &= C_{p-3} \wedge \left\{F^{(1)} \wedge F^{(1)} - F^{(2)} \wedge F^{(2)}\right\} \\ &\quad + C_{p-1} \wedge \left\{-2\beta^2|T|^2(F^{(1)} - F^{(2)}) + 2i\beta^2 DT \wedge (DT)^*\right\} \\ C \wedge \mathrm{STr}\, i\mathcal{F} \wedge i\mathcal{F} \wedge i\mathcal{F} &= C_{p-5} \wedge \left\{F^{(1)} \wedge F^{(1)} \wedge F^{(1)} - F^{(2)} \wedge F^{(2)} \wedge F^{(2)}\right\} \\ &\quad + C_{p-3}\left\{-3\beta^2|T|^2(F^{(1)} \wedge F^{(1)} - F^{(2)} \wedge F^{(2)}) \right. \\ &\qquad \left. + 3i\beta^2(F^{(1)} + F^{(2)}) \wedge DT \wedge (DT)^*\right\} \\ &\quad + C_{p-1}\left\{3\beta^4|T|^4 \wedge (F^{(1)} - F^{(2)}) - 6i\beta^4|T|^2 DT \wedge (DT)^*\right\} \end{aligned} \tag{6.126}$$

The appearance of $C_{p-1} \wedge dT \wedge dT^*$ has been checked by studying the disk level S-matrix element of one RR field and two tachyons. In the present paper we will check, among other things, the appearance of $C_{p-1} \wedge DT \wedge (DT)^*$ and $C_{p-1} \wedge |T|^2(F^{(1)} - F^{(2)})$ terms and fix their coefficients using the S-matrix element of one RR field, two tachyons and one gauge superfield.

One can rewrite the supercurvature in complete form using the Clifford algebra. We replace $dx^{\mu_1}...dx^{\mu_n} \to \frac{1}{n!}\gamma^{\mu_1}...\gamma^{\mu_n}$, where γ^μ satisfy the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.

The supercurvature reads now

$$\mathcal{F} = \begin{pmatrix} \frac{1}{2}\gamma^{\mu\nu}F^1_{\mu\nu} - (T\overline{T} - m\overline{m}) & i\gamma^\mu D_\mu T \\ i\gamma^\mu \overline{D_\mu T} & \frac{1}{2}\gamma^{\mu\nu}F^2_{\mu\nu} - (T\overline{T} - m\overline{m}) \end{pmatrix} , \tag{6.127}$$

where $\gamma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$, namely $dx^\mu \wedge dx^\nu \to \gamma^{\mu\nu}$. Note that in (6.127) we used the freedom to add a constant part, represented by $m\overline{m}$. In the non-commutative formalism with algebras $\mathcal{C}^\infty(\mathcal{R}^4) \otimes (\mathbb{C} \oplus \mathbb{C})$, $m, \overline{m}$ correspond to the $\mathbb{C} \oplus \mathbb{C}$ part.

There are two natural trace operations we can take over the Clifford algebra. We denote by tr the one simply taken over the Clifford algebra elements, e.g. $\mathrm{tr}(\gamma^\mu\gamma^\nu) = 2^{[(p+2)/2]}g^{\mu\nu}$ in a $(p+1)$-dimensional space. We denote by atr the antisymmetric trace over the Clifford algebra, e.g. $\mathrm{atr}(\gamma^\mu\gamma^\nu) = \mathrm{tr}\gamma^{\mu\nu}$, which leads naturally to the wedge-product structure. We denote by Tr the one taken over the matrix structure of $\mathcal{F}$.

Since the superconnection appears naturally in the description of the branes-antibranes system it is natural to ask whether we can write the effective action in terms of the supercurvature. The first hint is the $(Dp, \overline{Dp})$ higher-dimensional effective action up to second order, as computed in perturbative string theory

$$S_2 = T_p \int d^{p+1}x \left(\frac{1}{4} F^{1\mu\nu} F^1_{\mu\nu} + \frac{1}{4} F^{2\mu\nu} F^2_{\mu\nu} - D^\mu T \overline{D_\mu T} - (T\overline{T} - m\overline{m})^2 \right) , \tag{6.128}$$

where by T_p we denote the tension of a BPS Dp-brane. This action can be written as

$$S_2 = -\frac{T_p}{2^{[(p+2)/2]}} \int d^{p+1}x \mathrm{Tr}(\mathrm{tr}\mathcal{F}^2) . \tag{6.129}$$

One may suspect then that the higher order terms in the effective action, in the slowly varying fields approximation, where we neglect terms like $\partial^k F$ and $\partial^l T, l > 1$, could be of the form $\mathcal{F}^n, n > 2$. We will work in the slowly varying fields approximation in the following. We will see in the next section that this approximation is sufficient for the analysis of some exact properties of the tachyon condensation.

The second hint comes from the form of the Wess-Zumino (WZ) term of the branes-antibranes system. It can be written as

$$S_{WZ} = \tau \int d^{p+1}x \mathrm{Tr_s}(\mathrm{atr}(\Gamma\, \mathcal{C}\, e^{\mathcal{F}})) , \tag{6.130}$$

where τ is a normalisation constant, $\tau = \frac{e^{-m\overline{m}}\, \mu_p}{2^{[(p+2)/2]}}$ and $\mu_p = g_s T_p$. Γ is given by

$$\Gamma = i^{[\frac{p-1}{2}]} \begin{pmatrix} \tilde{\gamma} & 0 \\ 0 & \tilde{\gamma} \end{pmatrix}, \quad \tilde{\gamma} = i^{[\frac{p-1}{2}]} \gamma^0 ... \gamma^p , \tag{6.131}$$

and

$$\mathcal{C} = \sum \frac{1}{n!} \gamma^{\mu_1, \cdots \mu_n} C_{\mu_1, \cdots \mu_n} \tag{6.132}$$

where $C_{\mu_1, \cdots \mu_n}$ is an n-form corresponding to the RR n-form field.

In the language of differential forms (6.130) reads

$$S_{WZ} = \mu_p e^{-m\overline{m}} \int \mathcal{C} \wedge \mathrm{Tr_s}(\, e^{\mathcal{F}}) . \tag{6.133}$$

This WZ action was proposed, and is expected in view the fact that D-branes charge is measured by the K-theory class.

The supercurvature $\mathcal{F}$ can be decomposed as

$$\begin{aligned} \mathcal{F} &= \begin{pmatrix} \frac{1}{2}\gamma^{\mu\nu} F^1_{\mu\nu} & i\gamma^\mu D_\mu T \\ i\gamma^\mu \overline{D_\mu T} & \frac{1}{2}\gamma^{\mu\nu} F^2_{\mu\nu} \end{pmatrix} - (T\overline{T} - m\overline{m}) \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} \\ &= \overline{\mathcal{F}} - (T\overline{T} - m\overline{m})\mathbb{1} . \end{aligned} \tag{6.134}$$

Using this form of the curvature the WZ action (6.133) can be written as

$$S_{WZ} = \mu_p \int d^{p+1}x e^{-T\overline{T}} \mathcal{C} \wedge \mathrm{Tr_s} \left(\sum_{n \leq p+1} \frac{\overline{\mathcal{F}}^n}{n!} \right) . \tag{6.135}$$

The WZ action (6.135) suggests that the tachyon potential is

$$V(T, \overline{T}) \sim e^{-T\overline{T}} . \tag{6.136}$$

We now turn to the non-topological part of the branes-antibranes action, which we will denote by DBI. We expect to get the same tachyon potential in the DBI part. We now make the assumption that we can write it via the supercurvature. Since the superconnection and supercurvature appear as part of the structure of the system via the Chan-Paton factors one may expect this to be the case. However, it is also possible that only the topological part of the branes-antibranes action can be written using the supercurvature. This is related to the question whether the superbundle structure is indeed a structure of the brane-antibrane system or only of its topological part. We will continue with the assumption, bearing in mind that we do not have a proof for it.

The requirement of being able to write the DBI part using the supercurvature, together with the requirement of getting the same tachyon potential in the DBI part, uniquely fixes the DBI action to

$$S_{DBI} = -\tau_0 \int d^{p+1}x \mathrm{Tr}(\mathrm{tr} e^{\mathcal{F}}) \ . \tag{6.137}$$

τ_0 is a normalisation constant given by $\frac{T_p}{2^{[(p+1)/2]}} = \tau_0 e^{m\overline{m}}$.

The order $\mathcal{F}^2$ of (6.137) is precisely (6.128). Using the form of the curvature (6.134) we have

$$S_{DBI} = -\frac{T_p}{2^{[(p+2)/2]}} \int d^{(p+1)}x \ e^{-T\overline{T}} \ \mathrm{Tr}\left(\mathrm{tr} \ e^{\overline{\mathcal{F}}}\right) \ . \tag{6.138}$$

Thus, the proposed effective action of the branes-antibranes system, written in terms of the supercurvature (6.134,) is

$$S = S_{DBI} + S_{WZ} \tag{6.139}$$

with S_{DBI} given by (6.138) and S_{WZ} by (6.135).

One can rewrite the supercurvature using Clifford algebra as

$$\begin{aligned} \mathcal{F} &= \begin{pmatrix} \frac{1}{2}\gamma^{\mu\nu}F_{\mu\nu} & i\gamma^\mu\partial_\mu T \\ i\gamma^\mu\partial_\mu T & \frac{1}{2}\gamma^{\mu\nu}F_{\mu\nu} \end{pmatrix} - (T^2 - m^2)\begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} \\ &= \overline{\mathcal{F}} - (T^2 - m^2)\mathbb{1} \ , \end{aligned} \tag{6.140}$$

where we added a constant part.

Consider the WZ part of the non-BPS Dp-brane action. The above discussion of the D-branes charges associated with the non-BPS Dp-brane imply that the WZ-term is

$$S_{WZ} = \frac{-i\mu_p}{\sqrt{2}} \int d^{p+1}x e^{-T^2} \mathcal{C} \wedge \mathrm{Tr}_\sigma \left(\sum_{n \le p+1} \frac{\overline{\mathcal{F}}^n}{n!} \right) \ . \tag{6.141}$$

That is, the Chern characters of the superbundle encode the Dp-brane charges of condensates on the non-BPS branes. The WZ action suggests that the tachyon potential is $V(T) \sim e^{-T^2}$. Following the same reasoning as for the $(Dp, \overline{Dp})$ brane system we propose a form of the DBI part of the non-BPS Dp-brane action, written in terms of the supercurvature as

$$S_{DBI} = -\frac{T_p}{\sqrt{2}2^{[(p+2)/2]}} \int d^{(p+1)}x \ e^{-T^2} \ \mathrm{Tr}\left(\mathrm{tr} \ e^{\overline{\mathcal{F}}}\right) \ , \tag{6.142}$$

with $\overline{\mathcal{F}}$ given by (6.140). The factor $1/\sqrt{2}$ in (6.142) is due to the fact that the tension of a non-BPS Dp-brane is $\sqrt{2}T_p$, and the matrix structure of $\mathcal{F}$.

To summarize, the proposed effective action of the non-BPS Dp-brane is $S = S_{DBI} + S_{WZ}$, with S_{DBI} given by (6.142) and S_{WZ} by (6.141).

Consider a non-BPS Dp-brane that carries a D(p-1)-brane charge. Upon tachyon condensation we expect to get a BPS D(p-1)-brane. The tachyon T is a function of one coordinate transverse to the expected D(p-1)-brane world volume. Denote this coordinate by $x_1 = x$. We take the tachyon configuration to be $T = \alpha x$ where α is constant. For such a configuration higher than two derivatives of the tachyon vanish and we do not have to worry about not including them in the effective action.

Using the WZ action (6.141) we get the coupling of RR p-form to the Non-BPS-brane. It reads

$$\begin{aligned} S_{WZ} &= \sqrt{2}\mu_p \int d^{p+1}x\, C_p\, \partial T\, e^{-T^2} \\ &= \mu_{p-1} \int d^p x C_p \;, \end{aligned} \quad (6.143)$$

with $\mu_{p-1} = \mu_p\, 2\pi l_s$, and we have rescaled to restore the appropriate dimensions. We see that we get the charge corresponding to the D(p-1)-brane, independently of the form of the gauge field strength.

Plugging the $\alpha \to \infty$ solution into the effective action, we get

$$S_{\text{kink}} = -\sqrt{2}T_p \int d^{p+1}x e^{-\alpha^2 x^2}\sqrt{1+\alpha^2}|_{\alpha\to\infty} = (-\sqrt{2}T_p)\sqrt{\pi}(\int d^p y) \;. \quad (6.144)$$

Therefore the tension of the kink is $T_{\text{kink}} = \sqrt{2\pi}\; T_p$. After restoring the appropriate units we have

$$T_{\text{kink}} = (2\pi\sqrt{\alpha'})\; T_p \equiv T_{p-1} \;, \quad (6.145)$$

as the exact value.

As we noted, finite changes of the value of F do not affect the tension of the kink, but infinite changes will. All the other configurations will not satisfy the BPS relation between the charge and the tension.

It is worth exploring the kink profile in another set of variables. Consider the DBI action for the non-BPS Dp-brane proposed

$$S = -T_p \int d^{p+1}x V(\widetilde{T})\sqrt{-\det(\eta_{\mu\nu} + \widetilde{F}_{\mu\nu} + \partial_\mu \widetilde{T}\partial_\nu \widetilde{T})} \;. \quad (6.146)$$

One can study the kink solutions via this action.

Consider tachyon condensation on a $(Dp, \overline{Dp})$ brane system carrying a D(p-2)-brane charge. The tachyon should form a vortex-like configuration, with the topological charge of the vortex encoding the $D(p-2)$ brane charge.

We take the tachyon configuration $T = \alpha z$, $\overline{T} = \overline{\alpha z}$, where $z = x^1 + ix^2$. Inserting into the WZ action (6.135) we get the coupling of RR p-form to the BPS-brane. It reads

$$\begin{aligned} S^{(2)}_{WZ} &= \mu_p \int d^{p+1}x \frac{1}{2\; p!}\epsilon^{\mu_0,\ldots\mu_{p-1}\alpha\beta} C_{\mu_0\ldots\mu_{p-1}} \left((F^1 - F^2)_{\alpha\beta} + 2D_\alpha T\overline{D_\beta T}\right) e^{-T\overline{T}} \\ &= \mu_p\; (2\pi)(1+\Delta F) \int d^{p-1}x \frac{1}{p!}\epsilon^{\mu_0\ldots\mu_{p-1}} C_{\mu_0\ldots\mu_{p-1}} \;, \end{aligned} \quad (6.147)$$

where $\Delta F = F^1 - F^2$. Reinstalling $2\pi\alpha'$ one thus finds $\mu_{cond} = 2\pi\; \mu_{p-2}(1+\Delta F)$. Assume that only $F^i_{12}, i = 1,2$ is different from zero. In order to find the exact charge the vortex-like solution should have $F^1_{12} - F^2_{12} = 0$.

The correct scaling for the field strength F can be found from calculating the tension of the vortex. Plugging the $\alpha \to \infty$ solution into the action, we get

$$S|_{\text{vortex}} = -2T_p \int d^{p+1}x e^{-|\alpha|^2|z|^2}\sqrt{1+\beta^2} = -2\pi T_p\; \frac{\sqrt{1+\beta^2}}{|\alpha|^2} \int d^{p-1}x \;. \quad (6.148)$$

Scaling F such that $|\beta| \to |\alpha|^2$ the tension of the vortex is $T_{p-2,cond} = 2\pi\; T_{p-2}$. After reinstalling $2\pi\alpha'$ one finds

$$T_{p-2} = (2\pi)^2\alpha'\; T_{p-2} \;, \quad (6.149)$$

which is the correct value of the D(p-2)-brane tension.

Consider, for instance, non-BPS Dp-brane case where the tachyon is real $T = \overline{T}$. Up to two derivatives the action has the structure familiar from BSFT and σ-model perturbation theory

$$S = -2T_p \int d^{(p+1)}x e^{-T^2}(1 + \frac{1}{2}\partial_\mu T \partial^\mu T) \ . \tag{6.150}$$

However, it is easy to see that the numerical coefficients of the higher derivative terms do not match those of the BSFT action. Indeed, one studies the tachyon condensation with only the tachyon field excited and one gets the precise tension of the lower-dimensional D(p-1)-brane. In our variables, we needed a nonzero configuration of the gauge field strength in order to derive the precise tension of D(p-1)-brane from the kink solution. Upon addition of the gauge fields in the BSFT formalism there is still a difference between the effective actions.

One can also set $T = \overline{T} = A^2 = 0$ in the action (6.138), which leads to

$$S = -\frac{T_p}{2^{(p+1)/2}} \int d^{(p+1)}x \ \mathrm{tr} \ e^{\frac{1}{2}\gamma^{\mu\nu}F_{\mu\nu}} \ , \tag{6.151}$$

which one can map to the DBI action

$$S = -T_p \int d^{p+1}x \sqrt{-\det(\eta_{\mu\nu} + \widetilde{F}_{\mu\nu})} \ , \tag{6.152}$$

by relating $\widetilde{F}$ to a formal expansion of $\sinh(F_{\mu\nu})$. However, as above, one would expect a change of variables to include the higher derivative terms which were neglected in the slowly varying fields approximation.

We shall generalize the construction to a vortex solution on a brane-antibrane system. For this we need to begin with a tachyon effective action on a brane-antibrane system pair. In this case we have a complex tachyon field T, besides the massless gauge fields $A_\mu^{(1)}$, $A_\mu^{(2)}$ and scalar fields $Y^I_{(1)}$, $Y^I_{(2)}$ corresponding to the transverse coordinates of individual branes. We shall work with the following effective action that generalizes (5.89):

There have been various other proposals for the tachyon effective action and / or vortex solutions on brane-antibrane system. The action is

$$S = -\int d^{p+1}x \, V(T, Y^I_{(1)} - Y^I_{(2)}) \left(\sqrt{-\det \mathbf{A}_{(1)}} + \sqrt{-\det \mathbf{A}_{(2)}}\right) , \tag{6.153}$$

where

$$\mathbf{A}_{(i)\mu\nu} = \eta_{\mu\nu} + F^{(i)}_{\mu\nu} + \phi_\mu Y^I_{(i)} \phi_\nu Y^I_{(i)} + \frac{1}{2}(D_\mu T)^*(D_\nu T) + \frac{1}{2}(D_\nu T)^*(D_\mu T) \, , \tag{6.154}$$

$$F^{(i)}_{\mu\nu} = \phi_\mu A^{(i)}_\nu - \phi_\nu A^{(i)}_\mu \, , \qquad D_\mu T = (\phi_\mu - iA^{(1)}_\mu + iA^{(2)}_\mu)T \, , \tag{6.155}$$

and the potential $V(T)$ depends on $|T|$ and $\sum_I (Y^I_{(1)} - Y^I_{(2)})^2$ only. For small T, V behaves as

$$V(T, Y^I_{(1)} - Y^I_{(2)}) = \mathcal{T}_p \left[1 + \frac{1}{2} \left\{ \sum_I \left(\frac{Y^I_{(1)} - Y^I_{(2)}}{2\pi} \right)^2 - \frac{1}{2} \right\} |T|^2 + \mathcal{O}(|T|^4) \right] . \tag{6.156}$$

$\mathcal{T}_p$ denotes the tension of the individual D-p-branes. Although this action has not been derived from first principles, we note that this obeys the following consistency conditions:

1. The action has the required invariance under the gauge transformation:

$$T \to e^{2i\alpha(x)} \, T, \qquad A^{(1)}_\mu \to A^{(1)}_\mu + \phi_\mu \alpha(x), \qquad A^{(2)}_\mu \to A^{(2)}_\mu - \phi_\mu \alpha(x) \, . \tag{6.157}$$

2. For $T = 0$ the action reduces to the sum of the usual DBI action on the individual branes.

3. If we require the fields to be invaiant under the symmetry $(-1)^{F_L}$ that exchanges the brane and the antibrane, we get the restriction:

$$T = \text{real}, \qquad A^{(1)}_\mu = A^{(2)}_\mu \equiv A_\mu \,, \qquad Y^I_{(1)} = Y^I_{(2)} \equiv Y^I \,. \tag{6.158}$$

Under this restriction the action becomes proportionl to that on a non-BPS D-p-brane, as given in (5.89). This is a necessary consistency check, as modding out a brane-antibrane configuration by $(-1)^{F_L}$ is supposed to produce a non-BPS D-p-brane[39].

We should keep in mind however that these constraints do not fix the form of the action uniquely. Nevertheless we shall make the specific choice given in (6.153) and proceed to study the vortex solution in this theory.

We expect our analysis to be valid for a more general action of the form:

$$\begin{aligned} S &= -\int d^{p+1}x\, V(T, Y^I_{(1)} - Y^I_{(2)}) \left[\sqrt{-\det(g^{(1)}_{\mu\nu} + F^{(1)}_{\mu\nu})} F(G^{\mu\nu}_{(1)} D_\mu T^* D_\nu T) \right. \\ & \quad \left. + \sqrt{-\det(g^{(2)}_{\mu\nu} + F^{(2)}_{\mu\nu})} F(G^{\mu\nu}_{(2)} D_\mu T^* D_\nu T)\right] \end{aligned} \tag{6.159}$$

where $g^{(i)}_{\mu\nu} = \eta_{\mu\nu} + \phi_\mu Y^I_{(i)} \phi_\nu Y^I_{(i)}$ is the induced closed string metric on the ith brane, $G^{\mu\nu}_{(i)}$ is the open string metric on the ith brane and the function $F(u)$ grows as $u^{1/2}$ for large u.

The energy momentum tensor $T^{\mu\nu}$ associated with this action is given by:

$$T^{\mu\nu} = -V(T, Y^I_{(1)} - Y^I_{(2)}) \left[\sqrt{-\det(\mathbf{A}_{(1)})} (\mathbf{A}^{-1}_{(1)})^{\mu\nu}_S + \sqrt{-\det(\mathbf{A}_{(2)})} (\mathbf{A}^{-1}_{(2)})^{\mu\nu}_S\right]. \tag{6.160}$$

Another surprising feature of both the kink and the vortex solutions is that the world-volume theory on the soliton has exactly the DBI form without any higher derivative corrections. This means that all such corrections must come from higher derivative corrections to the original actions (5.89) and (6.153). This may seem accidental, but may be significant for the following reasons. This result suggests that there is a close relation between the systematic derivative (of field strength) expansion of the world-volume action of the non-BPS D-p-brane (D-p-brane - $\overline{\text{D}}$-p-brane pair) and that of the BPS soliton solution representing D-$(p-1)$ brane (D-$(p-2)$-brane). It will be interesting to explore this line of thought to see if one can establish a precise connection between the two. Since the derivative expansion on the world-volume of BPS D-branes is well understood, finding a connection of the type mentioned above will provide a better understanding of the derivative expansion of the world-volume action of a non-BPS D-brane / brane-antibrane system. We considered DBI-type effective action of a complex tachyon and gauge fields of U(1)×U(1) symmetry, describing brane-antibrane system with fundamental strings. In the coincidence limit of $(D2, \overline{D2})$, static vortex solutions are obtained. Without DBI electromagnetic field, there exist only singular static global and local D-vortex solutions. When the radial component of electric field is turned on, we found regular static global and local D-vortex solutions. The obtained point-like D-vortex configurations are naturally embedded in straight stringy solutions in $(D3, \overline{D3})$ brane system, and are identified with D-strings (D1-branes). If the obtained macroscopic D-strings are gravitating, they become naturally candidates of cosmic D-strings in the early Universe.

7 The $(Dp, \overline{Dp})$ Brane System in Dual Effective Actions

Although the complete effective action describing the $(Dp, \overline{Dp})$ brane-antibrane system has not been derived from first principles, it is known to satisfy a set of consistency conditions. In the context of our discussion in these proceedings this action describes the Higgs phase for the relative BI vector field. We will work to second order in α', ignoring tachyonic couplings to C_{p-1} and taking the other RR potentials to zero. This truncated action will contain however the relevant couplings for describing the most important aspects of the dynamics of the $(Dp, \overline{Dp})$ system, both in the Higgs and in the confining phases. Our starting point is then the action

$$S(\chi, A) = \int d^{p+1}x \Big\{ e^{-\phi}\Big(\frac{1}{2}F^+ + B_2\Big) \wedge *\Big(\frac{1}{2}F^+ + B_2\Big) + \frac{1}{4}e^{-\phi}F^- \wedge *F^- + \\ + |T|^2(d\chi - A^-) \wedge *(d\chi - A^-) + d|T| \wedge *d|T| - V(|T|) + C_{p-1} \wedge F^- \Big\}. \qquad (7.161)$$

Here we have set $2\pi\alpha' = 1$, A^+ and A^- are the overall and relative BI vector fields: $A^+ = A + A'$, $A^- = A - A'$, and the complex tachyon is parametrized as $T = |T|e^{i\chi}$. $V(|T|)$ is the tachyon potential, whose precise form will be irrelevant for our analysis. The pullbacks of the spacetime fields into the worldvolume are implicit.

The last coupling shows that when the tachyon condenses in a vortex-like configuration a $D(p-2)$-brane is generated as a topological soliton. In this process the relative $U(1)$ vector superfield eats the scalar superfield χ, gets a mass and is removed from the low energy spectrum. The overall $U(1)$ vector superfield, under which the tachyon is neutral, remains unbroken, but it is believed to be confined.

Note that since A^- is massive it cannot be dualized in the standard way. We can however use the standard procedure to dualize the phase of the tachyon and A^+. These fields are dualized, respectively, into a $(p-1)$-form, W_{p-1}, and a $(p-2)$-form, that we denote by A^-_{p-2} given that due to the opposite orientation of the antibrane the relative and overall gauge potentials should be interchanged under duality. The intermediate dual action that is obtained after these two dualizations are carried out is such that, up to a total derivative term, A^- becomes massless and can therefore be dualized in the standard way into A^+_{p-2}.

The final dual action reads:

$$S(W_{p-1}, A_{p-2}) = \int d^{p+1}x \Big\{ e^{\phi}\Big(\frac{1}{2}F^+_{p-1} + W_{p-1} + C_{p-1}\Big) \wedge *\Big(\frac{1}{2}F^+_{p-1} + W_{p-1} + C_{p-1}\Big) \\ + \frac{1}{4}e^{\phi}F^-_{p-1} \wedge *F^-_{p-1} + \frac{1}{4|T|^2}dW_{p-1} \wedge *dW_{p-1} + d|T| \wedge *d|T| - V(|T|) - B_2 \wedge F^-_{p-1} \Big\} \qquad (7.162)$$

The action (7.162) is an extension of the actions proposed, and it will become clear later that it describes the confining phase for the overall $(p-2)$-form dual potential. This phase arises after the condensation of zero-dimensional topological defects which originate from the end-points of open strings stretched between the branes. The interpretation of the low energy mode W_{p-1} is as describing the fluctuations of these defects, and is such that away from the defects $W_{p-1} = dA^+_{p-2}$. It can be seen that the original gauge invariance has been mapped into a gauge transformation of W_{p-1} and A^+_{p-2}. This symmetry can be gauge fixed by absorbing F^+_{p-1} into W_{p-1}, which becomes then massive. The overall A^+_{p-2} gauge potential is then removed from the low energy spectrum through the so-called Julia-Toulouse mechanism, which we will discuss further in the next section and is, essentially, the contrary of the more familiar Higgs mechanism. The Julia-Toulouse mechanism is therefore the responsible for the removal of the relative U(1) at strong coupling. However it clearly sheds no light on the removal of A^+.

When comparing the action (7.162) to the actions describing the confining phases of anti-symmetric field theories presented one sees that the modulus of the tachyon plays the role of the density of condensing topological defects, as can be expected since the instability in the confining phase is originated by the presence of the topological defects. In the confining models of Quevedo and Trugenberger a consistency requirement is that the antisymmetric field theory in the Coulomb phase is recovered for zero density of defects. This is indeed satisfied by our action (7.162) for vanishing tachyon, since the $|T| \to 0$ limit forces the condition that W_{p-1} must be exact and can therefore be absorbed through a redefinition of A^+, recovering the Coulomb phase in dual variables. Quevedo and Trugenberger made explicit in the framework of antisymmetric field theories an old idea in solid-state physics due to Julia and Toulouse. These authors argued that for a compact tensor field of rank $(h-1)$ in $(p+1)$-dimensions a confined phase might arise after the condensation of $(p-h-1)$-dimensional topological defects. The fluctuations of the continuous distribution of topological defects generate a new low-energy mode in the theory which can be described by a new h-form, W_h, such that away from the defects $W_h = dA_{h-1}$, where A_{h-1} is the original tensor field. The effective action describing the confining phase of the antisymmetric tensor field then depends on a gauge invariant combination of the antisymmetric tensor field, A_{h-1}, and the extended h-form, W_h. This combination is such that when the density of topological defects vanishes the original action describing the antisymmetric tensor field theory in the Coulomb phase is recovered. As discussed, the finite condensate phase is a natural generalization of the confinement phase for a four dimensional vector gauge field to arbitrary $(h-1)$-forms in d dimensions.

Given that the worldvolume theory of a $(Dp, \overline{Dp})$ system is a vector field theory, the results for $h=2$ can be applied to this case. In this case the Coulomb phase is the phase with zero tachyon, and it is therefore described by the Lagrangian:

$$L(A) = e^{-\phi}\Big(\frac{1}{2}F^+ + B_2\Big)\wedge *\Big(\frac{1}{2}F^+ + B_2\Big) + \frac{1}{4}e^{-\phi}F^-\wedge *F^- + C_{p-1}\wedge F^- \, . \tag{7.163}$$

Developping now on the ideas in this rseaerch article we have that the topological defects whose condensation will give rise to the confining phase are $(p-3)$-branes, which originate in this case from the end-points of $D(p-2)$-branes stretched between the Dp and the $\overline{Dp}$. The new mode associated to the fluctuations of the defects is described by a 2-form, W_2, which will couple in the action through a gauge invariant combination with the overall $U(1)$ vector superfield. The action should depend as well on the density of topological defects, such that when this density vanishes the original action in the Coulomb phase, given by (7.163), is recovered. We will see that, contrary to the actions constructed, where the density of topological defects entered as a parameter which was interpreted as a new scale in the theory, in the $(Dp, \overline{Dp})$ case it must be a dynamical quantity because it is related through duality to the modulus of the tachyonic excitation of the open $D(p-2)$-branes in the dual Higgs phase. We will denote this field by $|\widetilde{T}|$ and, moreover, we will use the duality with the Higgs phase to include in the action its kinetic and potential terms.

The dual effective action that we propose for describing the confining phase of the $(Dp, \overline{Dp})$ brane system is then given by

$$\begin{aligned} S(W_2, A) \; &= \; \int d^{p+1}x\Big\{e^{-\phi}\Big(\frac{1}{2}F^+ + W_2 + B_2\Big)\wedge *\Big(\frac{1}{2}F^+ + W_2 + B_2\Big) + \frac{1}{4}e^{-\phi}F^-\wedge *F^- + \\ &+ \frac{1}{4|\widetilde{T}|^2}dW_2\wedge *dW_2 + d|\widetilde{T}|\wedge *d|\widetilde{T}| - V(|\widetilde{T}|) + C_{p-1}\wedge F^-\Big\} \, . \end{aligned} \tag{7.164}$$

This action has been constructed under four requirements. One is gauge invariance, both under gauge transformations of the BI vector fields and under $W_2 \to W_2 + d\Lambda_1$, which ensures that only the gauge invariant part of W_2 describes a new physical degree of freedom. This transformation must be supplemented by $A^+ \to A^+ - 2\Lambda_1$, a symmetry that has to be gauge fixed. The second is relativistic invariance. The third requirement is that the original action describing the Coulomb phase must be recovered when $|\widetilde{T}| \to 0$. Indeed, when $|\widetilde{T}| \to 0$ we must have that $dW_2 = 0$, so that $W_2 = d\psi_1$ for some 1-form ψ_1. This form can then be absorbed by A^+, and the original action (7.163) is recovered. On the other hand, consistency with the duality symmetries of superstring theory will later on imply that W_2 must couple only to the overall $U(1)$ vector field.

Now, in (7.164) F^+ can be absorbed by W_2, fixing the gauge symmetry, and the action can then be entirely formulated in terms of W_2 and the relative vector field:

$$\begin{aligned} S(W_2, A^-) \; &= \; \int d^{p+1}x\Big\{e^{-\phi}\Big(W_2 + B_2\Big)\wedge *\Big(W_2 + B_2\Big) + \frac{1}{4}e^{-\phi}F^-\wedge *F^- + \\ &+ \frac{1}{4|\widetilde{T}|^2}dW_2\wedge *dW_2 + d|\widetilde{T}|\wedge *d|\widetilde{T}| - V(|\widetilde{T}|) + C_{p-1}\wedge F^-\Big\} \, . \end{aligned} \tag{7.165}$$

In this process the original gauge field A^+ has been eaten by the new gauge field W_2, and has therefore been removed from the low energy spectrum. This solves the puzzle of the unbroken overall $U(1)$ at weak string coupling through the Julia-Toulouse mechanism. Let us now see how the fundamental superstring arises from this dual effective action.

Inspired by Mandelstam-'t Hooft duality we expect that the dual of the action (7.164) describes the Higgs phase for the $(p-2)$-form field dual to the overall BI vector. The dualization of the BI vector fields in (7.164) takes place in the standard way, given that they only couple through their derivatives. In turn, the 2-form W_2 is massive, but it can still be dualized in the standard way from the intermediate dual action that is obtained after dualizing the BI vector fields, in which it only couples through its derivatives. Let us call the dual of this form, a $(p-3)$-form, χ_{p-3}. The final dual action reads:

$$S(\chi_{p-3}, A_{p-2}) = \int d^{p+1}x \Big\{ e^{\phi}\Big(\frac{1}{2}F^+_{p-1} + C_{p-1}\Big) \wedge *\Big(\frac{1}{2}F^+_{p-1} + C_{p-1}\Big) + \frac{1}{4}e^{\phi}F^-_{p-1} \wedge *F^-_{p-1}$$
$$+|\widetilde{T}|^2\Big(d\chi_{p-3} - A^-_{p-2}\Big) \wedge *\Big(d\chi_{p-3} - A^-_{p-2}\Big) + d|\widetilde{T}| \wedge *d|\widetilde{T}| - V(|\widetilde{T}|) - B_2 \wedge F^-_{p-1}\Big\} \quad (7.166)$$

where once again the overall and the relative gauge superfields are interchanged.

The action (7.166) describes an Abelian Higgs model for the relative $(p-2)$-form field, with the dual $(p-3)$-form χ_{p-3} playing the role of the associated Goldstone boson. That an effective mass gauge invariant term of this kind could drive the dual Higgs mechanism was suggested, although it could not be explicitly derived from the action describing the Higgs phase at weak coupling, i.e. from Sen's action. The Goldstone boson χ_{p-3} is associated to the fluctuations of the $(p-3)$-dimensional topological defects that originate from the end-points of the $D(p-2)$-branes stretched between the Dp and the $\overline{Dp}$. This is consistent with the fact that this field is the worldvolume dual of the field W_2, which was accounting for these fluctuations in the confining action (7.164). Moreover, we can identify for $p=3$ the condensing Higgs scalar as the modulus of the tachyonic mode associated to open D-strings stretched between the $D3$ and the $\overline{D3}$. Indeed when $p=3$ the action (7.166) turns out to be the S-dual of the original action (7.161) describing the perturbative Higgs phase of the $(D3, \overline{D3})$ system. This is an important consistency check, although strictly speaking S-duality invariance would only be expected for zero tachyon. In this duality relation the modulus of the perturbative tachyon is mapped into $|\widetilde{T}|$, which can then be interpreted as the modulus of the tachyonic excitation associated to the open D-strings. Since $\widetilde{\chi}$ has also an interpretation as the phase of the dual tachyon we can think of $\widetilde{T}$ as the complex tachyonic mode associated to the D-strings stretched between the $D3$ and the $\overline{D3}$. For $p \neq 3$, since the tachyonic condensing charged object is a $(p-3)$-brane, the phase of the tachyon is replaced by a $(p-3)$-form. It would be interesting to clarify the precise way in which these fields arise as open $D(p-2)$-brane modes.

As we have seen, a $(Dp, \overline{Dp})$ system admits two types of topological defects: particles and $(p-3)$-branes, which are, respectively, perturbative and non-perturbative in origin. The combined electric and magnetic Higgs mechanisms introduce mass gaps to both U(1) vector potentials, being the only remnants $D(p-2)$-branes and fundamental strings, realized as solitons on the common $(p+1)$-dimensional worldvolume. We have seen that it is possible to incorporate the non-perturbative degrees of freedom associated to the extended topological defects in the weak coupling regime, using Julia and Toulouse's idea, introducing a new form describing the fluctuations of these defects and imposing a set of consistency conditions. In fact, one can combine the weakly coupled action presented in section 3 with Sen's action in order to incorporate the degrees of freedom associated to both the zero dimensional and extended topological defects:

$$\begin{aligned} S(\chi, W_2, A) \;=\; & \int d^{p+1}x \Big\{ e^{-\phi}\Big(\frac{1}{2}F^+ + W_2 + B_2\Big) \wedge *\Big(\frac{1}{2}F^+ + W_2 + B_2\Big) + \frac{1}{4}e^{-\phi}F^- \wedge *F^- + \\ & +|T|^2(d\chi - A^-) \wedge *(d\chi - A^-) + d|T| \wedge *d|T| + \frac{1}{4|\widetilde{T}|^2}dW_2 \wedge *dW_2 + \\ & +d|\widetilde{T}| \wedge *d|\widetilde{T}| - V(|T|) - V(|\widetilde{T}|) + C_{p-1} \wedge F^- \Big\}. \end{aligned} \quad (7.167)$$

This action describes both the perturbative and the non-perturbative Higgs mechanisms simultaneously at weak coupling, and it admits both a magnetic vortex solution, which by charge conservation is identified with the $D(p-2)$-brane, and an electric vortex solution, identified as the fundamental string.

The total higher-dimensional dual effective action of extremal $(Dp, \overline{Dp})$ brane systems is constructed in the special form

$$
\begin{aligned}
S_T(\chi, A_-^+, W_-^+) &= \int d^{p+1}x \Big\{ e^{-\phi}\Big(\frac{1}{2}F^+ + B_2\Big) \wedge *\Big(\frac{1}{2}F^+ + B_2\Big) + \frac{1}{4}e^{-\phi}F^- \wedge *F^- \\
&+ |T|^2(d\chi - A^-) \wedge *(d\chi - A^-) + d|T| \wedge *d|T| - V(|T|) + C_{p-1} \wedge F^- \Big\} \\
&+ \Big\{ e^{\phi}\Big(\frac{1}{2}F^+_{p-1} + W_{p-1} + C_{p-1}\Big) \wedge *\Big(\frac{1}{2}F^+_{p-1} + W_{p-1} + C_{p-1} + \frac{1}{4}e^{\phi}F^-_{p-1} \wedge *F^-_{p-1}\Big) \\
&+ \frac{1}{4|T|^2}dW_{p-1} \wedge *dW_{p-1} + d|T| \wedge *d|T| - V(|T|) - B_2 \wedge F^-_{p-1} \Big\} \\
&+ \Big\{ e^{-\phi}\Big(\frac{1}{2}F^+ + W_2 + B_2\Big) \wedge *\Big(\frac{1}{2}F^+ + W_2 + B_2\Big) + \frac{1}{4}e^{-\phi}F^- \wedge *F^- \\
&+ \frac{1}{4|\widetilde{T}|^2}dW_2 \wedge *dW_2 + d|\widetilde{T}| \wedge *d|\widetilde{T}| - V(|\widetilde{T}|) + C_{p-1} \wedge F^- \Big\} \\
&+ \Big\{ e^{-\phi}\Big(W_2 + B_2\Big) \wedge *\Big(W_2 + B_2\Big) + \frac{1}{4}e^{-\phi}F^- \wedge *F^- \\
&+ \frac{1}{4|\widetilde{T}|^2}dW_2 \wedge *dW_2 + d|\widetilde{T}| \wedge *d|\widetilde{T}| - V(|\widetilde{T}|) + C_{p-1} \wedge F^- \Big\} \\
&+ \Big\{ e^{\phi}\Big(\frac{1}{2}F^+_{p-1} + C_{p-1}\Big) \wedge *\Big(\frac{1}{2}F^+_{p-1} + C_{p-1}\Big) + \frac{1}{4}e^{\phi}F^-_{p-1} \wedge *F^-_{p-1} \\
&+ |\widetilde{T}|^2\Big(d\chi_{p-3} - A^-_{p-2}\Big) \wedge *\Big(d\chi_{p-3} - A^-_{p-2}\Big) + d|\widetilde{T}| \wedge *d|\widetilde{T}| - V(|\widetilde{T}|) \Big\} \\
&- \Big\{ B_2 \wedge F^-_{p-1} + e^{-\phi}\Big(\frac{1}{2}F^+ + W_2 + B_2\Big) \wedge *\Big(\frac{1}{2}F^+ + W_2 + B_2\Big) + \frac{1}{4}e^{-\phi}F^- \wedge *F^- \\
&+ |T|^2(d\chi - A^-) \wedge *(d\chi - A^-) + d|T| \wedge *d|T| + \frac{1}{4|\widetilde{T}|^2}dW_2 \wedge *dW_2 \\
&+ d|\widetilde{T}| \wedge *d|\widetilde{T}| - V(|T|) - V(|\widetilde{T}|) + C_{p-1} \wedge F^- \Big\}. \qquad (7.168)
\end{aligned}
$$

8 Conclusion

In this paper we have analyzed kink and vortex solutions in tachyon effective field theory by postulating suitable form of the tachyon effective action on the non-BPS D-brane and brane-antibrane system respectively. In both cases the topological soliton has all the right properties for describing a BPS D-brane. These properties include localization of the energy-momentum tensor on subspaces of the codimensions, as is expected of a D-brane and also the DBI form of the effective action describing the world-volume theory on the soliton. For the kink solution we have also done the analysis including the world-volume fermions, and shown the appearance of κ-symmetry in the world-volume theory on the kink. One feature of both the solutions is infinite spatial gradient of the tachyon field away from the core of the soliton. If we want to construct a solution describing tachyon matter in the presence of such a soliton, then the spatial gradient of the tachyon field represents local velocity of the tachyon matter. This shows that tachyon matter in the presence of such a solution will fall towards the core of the soliton. If this feature survives in the full superstring theory, then it will imply that any tachyon matter in contact with the soliton will be sucked in immediately. This is consistent with the general analysis where similar effect was found by analyzing the boundary state associated with the time dependent solutions. This might provide a very effective means of absorbing tachyon matter from the surrounding by a defect brane, and drastically modify the results for the formation of topological defects during the rolling of the tachyon field. The appearance of infinite slope during the dynamical process of defect formation has already been observed in present scientific research. We should note however that a different type of solution where a codimension soliton and tachyon matter coexist has been constructed. One question that we have not addressed is the analysis of the world-volume theories on multiple kink-antikink pairs and multivortex solutions. In a finite region around the location of each soliton the solution will have the form discussed, and we need to ensure that before taking the limit, the various fields match smoothly, keeping or order larger in the intervening space. Analysis of the world-volume theory around such a background will clearly yield the sum of the world-volume actions on the individual solitons, since essentially the field configurations around individual solitons do not talk to each other in the limit. The interesting question is whether we can see the excitations associated with the fundamental string stretched between the solitons. We believe these excitations must come from classical solutions describing fundamental superstring along the line. We can, for example, take the solutions in the DBI theory given in and lift them to solutions of the equations of motion derived from the current considerations. The spontaneously broken gauge symmetry that mixes the states of the open superstring living on individual D-branes with states of the open superstring stretched between different D-branes, exchanges perturbative states with solitonic states, and hence is analogous to the electric magnetic duality symmetry in gauge field theories. Tachyon potential with such highly restricted behavior has not been available when looked for in the context of the open superstring field theory constructions. In this sense, it is quite promising that the supergravity analogue of the tachyon potential obtained in the present work can serve as a successful candidate for such a flat potential since, as we have seen in this work, it possesses purely logarithmic dependence on the tachyon field and hence is slowly varying. Thus it will be the issue of one of our future works to explore the potentially successful role played by the tachyon field arising in the unstable brane-antibrane system and having the structure of potential discovered in the present work. To summarize, in this work, using an exact supergravity solution representing the brane-antibrane system, we demonstrated in a rigorous fashion that one can construct a supergravity analogue of the tachyon potential which may possess generic features of the genuine stringy tachyon potentials. In doing so, our philisophy was to evaluate the interaction energy between the brane and the antibrane for small but finite inter-brane separation and then identify it with the tachyon potential. This identification demands an appropriate suggestion to relate the parameter in the supergravity solution representing the inter-brane distance to the tachyon field expectation value and we proposed an ansatz given in the article. The general lesson that one could learn from the results of this paper is that for many purposes, it is useful to complement the supergravity action, describing low energy effective action of closed superstring theory, by coupling it to the tachyon effective action of the type described in this paper. In such a theory, BPS D-branes arise naturally as topological solitons rather than having to be added by hand, and we get the correct low energy effective action on these D-branes. Furthermore, we have seen earlier that this effective action is capable of describing certain time dependent solutions of open string theory, and solutions describing the fundamental string. Coupling the tachyon field to supergravity does not give rise to any new perturbative physical states, and hence does not violate any known result in string theory. Finally, as was argued in the research literature, coupling of the tachyon effective action to gravity may resolve some of the conceptual problems involving 'time' in quantum gravity. Brane-antibrane systems hold the divine key and open the magical gate of knowledge to the unraveling and direct entry of mankind into the extreme hyperspace where the branes live, swim and interact under extreme conditions in higher-dimensional moduli painting supercollection with hypermanifolds of membrane universes. The considered fundamental membrane models provide the unique opportunity for in-depth construction of multiversum doctrina dominum in the mirror of a fundamental membrane theory possessing the qualities to accomplished, summarizing and unifying the general theoretical framework with the beam penetrating deep into the depths of the magical world structure plus vigorously extremal multiverse.

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