Novel Propulsion Method Inspired by Forced Vibrating Systems

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Abstract

Purpose In this paper, we investigate the cause of vibration behind a system with uneven mass distribution around the axis of rotation. Such a system has a defined response and is described by the particular solution of a second-order inhomogeneous differential equation. However, the differential equation and the response (particular solution) cannot justify why the system should vibrate (or move in one direction) from the moment Newton’s third law of motion holds.

Methods The answer to this apparent contradiction comes from the study of the eccentric mass momentum transfer to the rest of the system through a pair of non-rectilinear inertial forces.

Conclusions It reveals that the acceleration of an isolated system may become feasible when the momentum transfer comprises changes in internal angular momenta.

Keywords Newton’s 3rd law · Momentum conservation · Non-rectilinear inertial forces

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1 Introduction

In classical mechanics, any realization of the action-reaction principle presupposes two bodies to exert equal and opposite forces on each other. The same applies to an isolated system where its internal parts interact with each other and the system itself. According to Newton’s third law [1] [2], an internal action force exerted upon a part of an isolated system creates an opposite and of equal magnitude reaction force exerted upon the rest of the system. Consequently, the isolated system cannot acquire momentum through internal forces. However, classical mechanics and daily experience have taught us; a rotating unbalance [3] may vibrate, although the forces that act upon the system are internal. Today’s literature and applications address vibration in terms of the system’s response by ignoring the actual mechanism that causes it. This work reveals the mechanism for accelerating an isolated system [4] [5] using non-rectilinear internal inertial forces [6] [7] arising from the conservation of angular momentum.

2 Methods

Suppose the center of mass is out of alignment with the center of rotation of a system. In that case, an unbalance is created because of the uneven distribution of mass around the axis of rotation. When we force a part of the system (eccentric mass) to rotate, an excitation force arises that makes the system vibrate (see FIG. 1). A rotating unbalance system is described by an inhomogeneous second-order differential equation. The system could be classified as an isolated system because the cause of the system’s vibration does not come from an external source, e.g., gravitational or other, but an internal one instead. In FIG. 1 we have a forced vibrating system excited by two contra-rotating eccentric masses. On the y-Axis, the differential equation that describes the system is,

\[
(M - 2m) \frac{d^2 y}{dt^2} + m \left( \frac{d^2 y}{dt^2} - r_e \omega^2 \sin(\pi - \omega t) \right) + m \left( \frac{d^2 y}{dt^2} - r_e \omega^2 \sin(\omega t) \right) = -c \frac{dy}{dt} - ky, \tag{1}
\]

\[
M \frac{d^2 y}{dt^2} + c \frac{dy}{dt} + ky = 2mr_e \omega^2 \sin(\omega t) = 2F_{\text{int}} \sin(\omega t), \tag{2}
\]
Furthermore, the particular solution is of the form:
\[ y = y_{\text{max}} \sin(\omega t - \phi), \]  
\[ (3) \]

Assuming same stiffness and damping coefficient on the x-Axis, yields
\[ (M - 2m) \frac{d^2x}{dt^2} + m \left( \frac{d^2x}{dt^2} - r_e \omega^2 \cos(\pi - \omega t) \right) + m \frac{d^2x}{dt^2} - r_e \omega^2 \cos(\omega t) = -c \frac{dx}{dt} - kx, \]  
\[ (4) \]
\[ M \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = 0. \]  
\[ (5) \]

The counter-rotating eccentric masses create a null net excitation force along the x-Axis, resulting in no system’s acceleration (no vibration),
\[ \frac{d^2x}{dt^2} = 0 \Rightarrow \frac{dx}{dt} = 0 \Rightarrow x = 0. \]  
\[ (6) \]

A system (see FIG. 1) that is forced to vibrate through an external force \( F_{\text{ext}} \) is described by the following equation
\[ M \frac{d^2y}{dt^2} + c \frac{dy}{dt} + ky = F_{\text{ext}} \sin(\omega t). \]  
\[ (7) \]

Describing the systems (see FIG. 1) relative to an inertial frame of reference; the rotating unbalance uses a pair of centripetal \( 2F_{\text{int}} \) that are internal to the system, and the other one uses an external \( F_{\text{ext}} \) as excitation force. Setting the amplitude of the external force equals to the magnitude of the centripetal forces pair, equations Eq. (2) and Eq. (7) become equivalent and result in similar system responses (Eq. (3)). Although the same model predicts the response of both systems, the rotating unbalance cannot justify its motion in the context of Newton’s 3rd law. Replacing the external with an internal force, the system (see right in FIG. 1) should normally never vibrate.

Assuming all interactions are rectilinear and apply for the same time interval, the proof that an isolated system cannot accelerate via internal forces can be stated as the action force upon a part of the system with mass \( m \) creates a reaction force upon the rest of the system \( (M - m) \). Simultaneously (same time interval), the latter becomes the action force exerted upon another part of the system with mass \( m \) located in the opposite direction, creates a reaction force upon the rest of the system \( (M - m) \) opposing the last action force. Hence,
\[ \vec{F}_{A \rightarrow m} dt/2 - \vec{F}_{R \rightarrow (M-m)} dt/2 = \vec{F}_{A \rightarrow m} dt/2 + \vec{F}_{R \rightarrow (M-m)} dt/2 = 0, \]  
\[ (8) \]
\[ d\vec{p}_m/2 - d\vec{p}_{(M-m)}/2 - d\vec{p}_{m}/2 + d\vec{p}_{(M-m)}/2 = 0, \]  
\[ (9) \]
\[ d\vec{p}_{m} \cdot \vec{0} = -d\vec{p}_{(M-m)} \cdot \vec{0} = d\vec{p}_{m} \cdot \vec{0} = 0 \Rightarrow d\vec{p}_m = \vec{0} \Rightarrow d\vec{p}_M = \vec{0}. \]  
\[ (10) \]

The question that now arises is that since the pair of centripetal forces are internal to the rotating unbalance, what is the cause of its vibration and how it complies with Newton’s 3rd law? Two crucial but straightforward observations may help us answer this question. First, the system \( (M) \) is not anchored in a surface (it is free to move), and the eccentric masses are parts intertwined with the system itself. Any attempt of the eccentric masses to tangentially accelerate, the change in their angular momentum will inevitably be imparted to the rest of the system, leading to the system’s acceleration in the opposite direction. Secondly, the transfer of angular momentum from the parts to the rest of the system corresponds to opposing inertial forces (Newton’s 3rd law) that are not rectilinear. Thus, according to FIG. 2 and the conservation of angular momentum implies
\[ \vec{F}_{\text{ext}} \perp \vec{F}_{\text{y}} \Rightarrow \vec{F}_{\text{y}} = \vec{F}_{\text{ext}} - \vec{F}_{\text{x}} + \vec{F}_{\text{A}}, \]  
\[ (11) \]
\[ \vec{F}_{\text{R}x} \perp \vec{F}_{\text{R}y} \Rightarrow \vec{F}_{\text{R}} = \vec{F}_{\text{R}x} + \vec{F}_{\text{R}y} \text{ and } \vec{F}_{\text{R}} = 2\vec{F}_{\text{y}}, \]  
\[ (12) \]
\[ \sum \vec{F}_{\text{ext}} = \vec{0}, \]  
\[ (13) \]
Novel Propulsion Method Inspired by Forced Vibrating Systems

\[
\sum \vec{\tau}_{\text{int}} = \vec{0} \Rightarrow (\vec{r}_e \times \vec{F}_R) + (\vec{F}_R \times \vec{r}_e) + (\vec{r}_e \times (-\vec{F}_R)) + (-\vec{r}_e \times -\vec{F}_R) = \vec{0},
\]

Eq. (14)

\[
\vec{0} + \frac{d}{dt} (\vec{r}_e \times (\vec{p}_{Rt} + (-\vec{r}_e)) = \vec{0} \Rightarrow \frac{d\vec{p}_{Rt}}{dt} = \vec{0},
\]

Eq. (15)

\[
\sum \vec{\tau}_{\text{int}} = \vec{0} \Rightarrow (\vec{r}_e \times \vec{F}_A) + (-\vec{r}_e \times -\vec{F}_A) + (\vec{r}_e \times 2\vec{F}_A) + (-\vec{r}_e \times -2\vec{F}_A) = \vec{0},
\]

Eq. (16)

\[
\vec{r}_e \times (2\vec{F}_A + (-\vec{F}_R)) = \vec{0},
\]

Eq. (17)

\[
\vec{r}_e \times (2\vec{F}_A + (-\vec{F}_R)) = \vec{0},
\]

Eq. (18)

\[
\vec{0} \Rightarrow \frac{d\vec{p}_{Rt}}{dt} \neq \vec{0},
\]

Eq. (19)

\[
\frac{d}{dt} (\vec{r}_e \times (2\vec{p}_A + (-\vec{r}_e)) = \vec{0} \Rightarrow \frac{d\vec{p}_A}{dt} = 0
\]

Eq. (20)

\[
\frac{d\vec{p}_A}{dt} < 0 \Rightarrow \frac{d\vec{p}_R}{dt} > 0 \text{ or } \frac{d\vec{p}_A}{dt} > 0 \Rightarrow \frac{d\vec{p}_R}{dt} < 0,
\]

Eq. (21)

\[
\frac{d\vec{p}_R}{dt} = -2\frac{d\vec{p}_A}{dt} = -2m_\varepsilon \frac{d\omega}{dt} = -2ma_{\lambda} = -2dF_{\text{int}}.
\]

Eq. (22)

3 Discussion and Conclusions

In a system with uneven distribution of mass around the axis of rotation, the conservation of angular momentum enabled us to reveal an internal angular momentum transfer mechanism between the eccentric masses and the system itself. The changes in angular momenta for the eccentric masses and the system do not cancel themselves, contrary to what we saw with Eqs. [9][10], because they are acting in a non-rectilinear manner. The conservation of momentum and energy is a direct consequence of Newton’s 3rd law; however, the conservation of linear and angular momentum is the one that may determine whether a system or object should accelerate or not. Conclusively, this work provides a paradigm shift of how motion can be conducted using non-rectilinear internal inertial forces arising from the conservation of angular momentum that could potentially revolutionize transportation technologies.

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Conflict of Interest

The author declares that he has no conflict of interests.

References