On the geometrical optics and the Atiyah-Singer index theorem

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We assume that the curvature in the Atiyah-Singer index theorem is related with, a function of the Riemann-Christoffel curvature tensor, so $I_M$ is also a function of the Riemann-Christoffel curvature tensor.

I. GEOMETRICAL OPTICS

In our previous work on the geometrical optics, we obtain the result as below\textsuperscript{1}

\[
g N_\sigma \partial_\rho \ln |\partial_\nu \left\{ \frac{c}{w} \arccos \left( A^\mu (U^{(1)}) \hat{m}^\mu (U^{(1)}) - \frac{1}{g} \hat{m}^\mu (U^{(1)}) \times \partial_\mu \hat{m} (U^{(1)}) \right) a^{-1}_\mu + ct \right\} | = R_{\mu \nu \rho \sigma} \]

where $R_{\mu \nu \rho \sigma}$ is the Riemann-Christoffel curvature tensor.

II. THE ATIYAH-SINGER INDEX THEOREM

Roughly, the Atiyah-Singer index theorem can be written as\textsuperscript{2}

\[
\text{Index} = \int_M I_M \cdot \text{ch} (\sigma) \quad (2)
\]

where $I_M$ is a differential form determined by the curvature of the manifold, $M$, on which the equation is defined and $\text{ch} (\sigma)$ is a differential form obtained from the symbol of the equation.

We assume that the curvature in eq. (2) is related with, a function of the Riemann-Christoffel curvature tensor, so $I_M$ is also a function of the Riemann-Christoffel curvature tensor

\[
I_M = f(R_{\mu \nu \rho \sigma}) \quad (3)
\]

where

\[
g N_\sigma \partial_\rho \ln |\partial_\nu \left\{ \frac{c}{w} \arccos \left( A^\mu (U^{(1)}) \hat{m}^\mu (U^{(1)}) - \frac{1}{g} \hat{m}^\mu (U^{(1)}) \times \partial_\mu \hat{m} (U^{(1)}) \right) a^{-1}_\mu + ct \right\} | = R_{\mu \nu \rho \sigma} \quad (4)
\]

\textsuperscript{1}Miftachul Hadi, Magnetic symmetry of geometrical optics, \url{https://vixra.org/abs/2104.0188}, 2021.

\textsuperscript{2}Nigel Higson, John Roe, The Atiyah-Singer Index Theorem.