# Prove Collatz Conjecture via Operations for Integer Expressions plus Few Integers

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## Abstract

First, let us expound certain basic concepts related to prove the conjecture.

Next, list the mathematical induction that proves the conjecture, and prepare several judging criteria, which are used to judge certain operational results.

After that, classify positive integers successively and prove directly one of these categories after each classification, until the last two categories are proved bidirectionally.

The so-called bidirectional proof, namely, on the one hand, start with several proven kinds of integers to expand successively the scope of proven kinds of integers, up to all kinds of integers; on the other, each unproven kind of integers is operated by the operational rule to find an integer expression that is less than the unproven kind of integers, such that the unproven kind of integers is proved by a judging criterion.

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# 1. Introduction

The Collatz conjecture is also called the *3x+1* mapping, *3n+1* problem, Hasse's algorithm, Kakutani's problem, Syracuse algorithm, Syracuse problem, Thwaites conjecture and Ulam's problem, etc.

But it remains a conjecture that has neither been proved nor disproved ever since named after Lothar Collatz in 1937; [1].

# 2. Certain of Basic Concepts

The Collatz conjecture states that take any positive integer $n$, if $n$ is an even number, divide it by $2$; if $n$ is an odd number, multiply it by $3$ and add $1$, and repeat the process indefinitely, then, no matter which positive integer you start with, you are always going to end up with $1$; [2].

We consider aforesaid operational stipulations as the operational rule.

If you start with any positive integer∕integer expression to operate continually by the operational rule, then it will form a series of positive integers∕integer expressions.

Such being the case, we regard such a series positive integers∕integer expressions plus codirectional arrows among these positive integers∕ integer expressions as an operational route.

Next, let us use a capital letter with the subscript "*ie*" to express an integer expression, such as $P_{ie}$, $C_{ie}$ etc.

In addition, an operational route that contains a certain integer expression may be called "an operational route via the integer expression".

In general, integer expressions on an operational route have a common variable or many variables which can be converted into a common variable.

## 3. The Mathematical Induction that Proves the Conjecture

Let us prove the conjecture by the following mathematical induction; [3].

(1) From 1→4→2→1; 2→1; 3→10→5→16→8→4→2→1; 4→2→1; 5→16→8→4→2→1; 6→3→10→5→16→8→4→2→1; 7→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1; 8→4→2→1 and 9→28→14→7→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1, we can be seen that each and every positive integer $\le 9$ fits the conjecture.

(2) Suppose that $n$ fits the conjecture, where $n$ is an integer $\ge 9$.

(3) Prove that $n+1$ fits the conjecture likewise.

## 4. Several Judging Criteria

A certain result of operations which start with each category of positive integers is judged by one of following three criteria.

**Theorem 1.** If an operational route via $P_{ie}$ has an integer expression that is less than $P_{ie}$, and $n+1 \in P_{ie}$, then all integer expressions on the operational route fit the conjecture.

For example, let $P_{ie}=31+3^2\beta$ with $\beta\ge 0$, and $n+1\in P_{ie}$, then from $27+2^3\beta \to 82+3\times 2^3\beta \to 41+3\times 2^2\beta \to 124+3^2\times 2^2\beta \to 62+3^2\times 2\beta \to \mathbf{31}+3^2\beta > \mathbf{27}+2^3\beta$, we get that all integer expressions on the operational route fit the conjecture.

In addition, let $P_{ie}=5+2^2\mu$ with $\mu\ge 0$, and $n+1\in P_{ie}$, then from $\mathbf{5}+2^2\mu \to 16+3\times 2^2\mu$

$\to 8+3\times2\mu \to \mathbf{4}+3\mu < \mathbf{5}+2^2\mu$, we get that all integer expressions on the operational route fit the conjecture.

***Proof.*** Suppose that an operational route via $P_{ie}$ has $C_{ie}$, and $C_{ie}<P_{ie}$, then when their common variable is equal to a certain fixed value, such that $P_{ie}=n+1$ and $C_{ie}=m$. So there is $m<n+1$, then $m$ fits the conjecture, according to second step of the mathematical induction.

So from $n+1$ can operate to $m$, or from $m$ can operate to $n+1$, in either case, it continues to operate to $1$ via $m$, such that $n+1$ fits the conjecture.

When their common variable is equal to each value, each integer that every integer expression contains is operated to $1$, because each integer that every integer expression contains matches an integer of $C_{ie}$, and the operations of the former goes through the operations of the latter to continue to 1.

Therefore, all integer expressions on the operational route fit the conjecture.

**Theorem 2.** If an operational route via $Q_{ie}$ and an operational route via $P_{ie}$ intersect, and $n+1\in P_{ie}$, also an integer expression on the operational route via $Q_{ie}$ is less than $P_{ie}$, then all integer expressions on these two operational routes fit the conjecture.

For example, let $Q_{ie}=71+3^3\times2^5\varphi$ and $P_{ie}=63+3\times2^8\varphi$ where $\varphi\geq0$, then from

$\mathbf{63}+3\times2^8\varphi\to190+3^2\times2^8\varphi\to95+3^2\times2^7\varphi\to286+3^3\times2^7\varphi\to143+3^3\times2^6\varphi\to430+3^4\times2^6\varphi\to$

$215+3^4\times2^5\varphi\to646+3^5\times2^5\varphi\to323+3^5\times2^4\varphi\to970+3^6\times2^4\varphi\to485+3^6\times2^3\varphi\to1456+3^7\times2^3\varphi$

$\rightarrow 728+3^7\times2^2\varphi\rightarrow 364+3^7\times2\varphi\longrightarrow 182+3^7\varphi\longrightarrow \ldots$

$\uparrow 121+3^6\times2\varphi\longleftarrow 242+3^6\times2^2\varphi\longleftarrow 484+3^6\times2^3\varphi\longleftarrow 161+3^5\times2^3\varphi\longleftarrow 322+3^5\times2^4\varphi$

$\longleftarrow 107+3^4\times2^4\varphi\longleftarrow 214+3^4\times2^5\varphi\longleftarrow \mathbf{71}+3^3\times2^5\varphi\longleftarrow 142+3^3\times2^6\varphi \longleftarrow \mathbf{47}+3^2\times2^6\varphi< \mathbf{63}+3\times2^8\varphi$,

we get that all integer expressions on these two operational routes fit the conjecture.

***Proof*.** Suppose that there is $D_{ie}$ on an operational route via $Q_{ie}$, and $D_{ie}<P_{ie}$, and that the operational route via $Q_{ie}$ and an operational route via $P_{ie}$ intersect at $A_{ie}$, so when their common variable is given a certain fixed value, such that $P_{ie}=n+1$, $A_{ie}=\xi$ and $D_{ie}=\mu$. Then, there is $\mu<n+1$, and $\mu$ fits the conjecture, according to second step of the mathematical induction.

Since $\xi$ and $\mu$ belong to an operational route, and $\mu$ fits the conjecture, then all integers including $\xi$ on the operational route fit the conjecture, according to the theorem *1*.

Since $n+1$ and $\xi$ belong to an operational route, and $\xi$ fits the conjecture, then all integers including $n+1$ on the operational route fit the conjecture, according to the theorem *1*.

When the common variable is equal to each value, each integer which every integer expression contains on these two operational routes is also operated to *1*, because each integer that every integer expression contains on these two operational routes matches an integer of $D_{ie}$.

Therefore, all integer expressions on these two operational routes fit the conjecture.

**Lemma 1.** If there is a proven integer expression on an operational route, then all integer expressions on the operational route fit the conjecture.

**Lemma 2.** If there is a proven integer expression on successive intersecting operational routes, then all integer expressions on these operational routes fit the conjecture.

By the way, two non-intersected operational routes within successive intersecting operational routes are in indirect connection.

**Lemma 3.** Each and every integer that any proven integer expression contains fits the conjecture.

# 5. Successive Classification with Proof for Positive Integers

We classify positive integers successively, then the positive integer $n+1$ is possibly included in any class, thus each of all classes which consisted of unduplicated positive integers must be proved to fit the conjecture.

***Classification with Proof.*** Since in section *2* positive integers$\leq 9$ have been proven to fit the conjecture, and because of this, we first divide integers $>9$ into even numbers and odd numbers.

For even numbers $2k$ with $k>4$, from 2k→k<2k, we get that if $n+1\in 2k$, then 2k and $n+1$ fit the conjecture according to Theorem *1* and Lemma *3* in chapter 4.

For odd numbers $>9$, divide them into $11+4k$ and $13+4k$, where $k\geq 0$.

For *13+4k*, from 13+4k→40+12k→20+6k→10+3k<13+4k, we get that if *n+1* ∈ *13+4k*, then *13+4k* and *n+1* fit the conjecture according to Theorem *1* and Lemma *3* in chapter 4.

We continue to divide *11+4k* into *11+12c*, *15+12c* and *19+12c*, where *c≥0*.

For *11+12c*, from 7+8c→22+24c→11+12c>7+8c, we get that if *n+1*∈*11+12c*, then *11+12c* and *n+1* fit the conjecture according to Theorem *1* and Lemma *3* in chapter 4.

Below, we will prove that both *15+12c* and *19+12c* fit the conjecture.

## 6. Prove that 15+12c and 19+12c Fit the Conjecture

For the sake of avoiding confusion and conducive convenience, we substitute *d, e, f, g,* etc. for *c* on operational routes of *15+12c/19+12c*.

We continue to operate *15+12c/19+12c* by the operational rule, where *c≥0* .

Theoretically speaking, after operate *15+12c/19+12c,* each and every kind of *15+12c/19+12c* must find an integer expression that is less than the kind of *15+12c/19+12c*, in order to meet one of judging criteria, such that the kind of *15+12c/19+12c* fits the conjecture.

**Firstly**, we start with *15+12c* to operate continuously by the operational rule, and as listed below.

**15+12c**→46+36c→23+18c→70+54c→35+27c ♣

d=2e+1: 29+27e (**1**)          e=2f: 142+486f→71+243f ♥
♣35+27c↓→c=2d+1: 31+27d↑→d=2e: 94+162e→47+81e↑→e=2f+1:64+81f (**2**)
c=2d: 106+162d→53+81d↓→d=2e+1:67+81e↓→e=2f+1:74+81f (**3**)
d=2e:160+486e♦      e=2f: 202+486f→101+243f ♠

g=2h+1: 200+243h (**4**) …

♥ 71+243f↓→f=2g+1:157+243g↑→g=2h: 472+1458h→236+729h↑→ …

f=2g: 214+1458g→107+729g↓→g=2h+1: 418+729h↓→…

g=2h: 322+4374h→… …

g=2h: 86+243h (**5**)

♠101+243f↓→f=2g+1:172+243g↑→g=2h+1:1246+1458h→…

f=2g: 304+1458g→152+729g ↓→…

…

…

♦160+486e→80+243e↓→e=2f+1: 970+1458f→485+729f↑→ … …

e=2f:40+243f↓→f=2g+1:850+1458g→ 425+729g↑→ …

f=2g: 20+243g↓→g=2h: 10+243h (**6**) …

g=2h+1:790+1458h→395+729h↑→ …

Annotation:

(1) Each of letters c, d, e, f, g, h and otherwise on listed above operational routes expresses each of natural numbers plus 0.

(2) There are ♣↔♣, ♥↔♥, ♠↔♠, and ♦↔♦ on listed above operational routes.

(3) Aforesaid two points are suitable to latter operational routes of 19+12c similarly.

First, let us define a term. That is, if an operational result is less than a kind of *15+12c/19+12c*, and it appears first either on an operational route via the kind of *15+12c/19+12c* or on another operational route relating to the operational route, then we call the operational result “first satisfactory operational result” about the kind of *15+12c/19+12c*.

Accordingly, on the above bunch of operational routes of *15+12c*, we first conclude following *3* kinds of *15+12c* derived from first satisfactory operational results to fit the conjecture.

**1).** From c=2d+1 and d=2e+1 to get c=2d+1=2(2e+1)+1=4e+3, then there are $15+12c=51+48e=51+3\times 2^4e\to 154+3^2\times 2^4e\to 77+3^2\times 2^3e\to 232+3^3\times 2^3e\to 116+3^3\times 2^2e\to 58+3^3\times 2e\to 29+27e$ where the mark (**1**).

Due to 29+27e<51+48e, we get that if there is $n+1\in 51+48e$, then 51+48e and *n+1* fit the conjecture according to Theorem *1* and Lemma *3* in chapter 4.

**2).** From c=2d+1, d=2e and e=2f+1 to get c=2d+1=4e+1=4(2f+1)+1=8f+5, then there are $15+12c=75+96f=75+3\times2^5f\rightarrow226+3^2\times2^5f\rightarrow113+3^2\times2^4f\rightarrow340+3^3\times2^4f\rightarrow170+3^3\times2^3f\rightarrow85+3^3\times2^2f\rightarrow256+3^4\times2^2f\rightarrow128+3^4\times2^1f\rightarrow64+81f$ where the mark (**2**).

Due to $64+81f<75+96f$, we get that if there is $n+1\in75+96f$, then $75+96f$ and $n+1$ fit the conjecture according to Theorem *1* and Lemma *3* in chapter 4.

**3).** From c=2d, d=2e+1 and e=2f+1 to get c=2d=4e+2=4(2f+1)+2=8f+6, then there are $15+12c=87+96f=87+3\times2^5f\rightarrow262+3^2\times2^5f\rightarrow131+3^2\times2^4f\rightarrow394+3^3\times2^4f\rightarrow197+3^3\times2^3f\rightarrow592+3^4\times2^3f\rightarrow296+3^4\times2^2f\rightarrow148+3^4\times2^1f\rightarrow74+81f$ where the mark (**3**).

Due to $74+81f<87+96f$, we get that if there is $n+1\in87+96f$, then $87+96f$ and $n+1$ fit the conjecture according to Theorem *1* and Lemma *3* in chapter 4.

For the same reason, on the bunch of operational routes of *15+12c*, each reader can also conclude other 3 kinds of *15+12c* derived from first satisfactory operational results to fit the conjecture, and as listed below.

**4)**. Pursuant to c=2d+1, d=2e, e=2f, f=2g+1 and g=2h+1 to get c=32h+25, then there is *15+12c=315+384h* derived from *200+243h* where the mark (**4**);

**5)**. Pursuant to c=2d, d=2e+1, e=2f, f=2g+1 and g=2h to get c=32h+10, then there is *15+12c=135+384h* derived from *86+243h* where the mark (**5**);

**6)**. Pursuant to c=2d, d=2e, e=2f, f=2g and g=2h to get c=32h, then there is *15+12c=15+384h* derived from *10+243h* where the mark (**6**).

**Secondly**, we start with *19+12c* to operate continuously by the operational rule, and as listed below.

**19+12c**→58+36c→ 29+18c→ 88+54c→ 44+27c ♣

d=2e: 11+27e (**α**)　　　　e=2f:37+81f (**β**)

♣ 44+27c↓→c=2d: 22+27d↑→d=2e+1:148+162e→74+81e↑→e=2f+1:466+486f ♥

c=2d+1: 214+162d→107+81d↓→d=2e:322+486e ♠

d=2e+1:94+81e↓→e=2f:47+81f (γ)

e=2f+1:526+486f ♦

g=2h: 119+243h (**δ**)　　…

f=2g+1:238+243g↑→g=2h+1:1444+1458h→722+729h↑→...

♥466+486f→233+243f↑→f=2g: 700+1458g→350+729g↓→g=2h+1:3238+4374h↓

g=2h: 175+729h↓→...　　...

...

g=2h+1:172+243h (**ε**)

f=2g: 101+243g↑→g=2h: 304+1458h→...

e=2f+1:202+243f↑→f=2g+1:1336+1458g→**…**

♠322+486e→161+243e↑→e=2f:484+1458f→**…**

♦526+486f→263+243f↓→f=2g: 790+1458g→**…**

f=2g+1: 253+243g↓→g=2h+1: 248+243h (ζ)

g=2h: 760+1458h→**…**

As listed above, on the bunch of operational routes of *19+12c*, we first conclude following 3 kinds of *19+12c* derived from first satisfactory operational results to fit the conjecture.

**1).** From c = 2d and d = 2e to get c = 2d = 4e, then there are 19+12c = 19+48e = $19+3\times2^4e \to 58+3^2\times2^4e \to 29+3^2\times2^3e \to 88+3^3\times2^3e \to 44+3^3\times2^2e \to 22+3^3\times2e \to 11+27e$ where the mark (**α**).

Due to 11+27e<19+48e, we get that if there is *n+1*∈*19+48e*, then *19+48e* and *n+1* fit the conjecture according to Theorem *1* and Lemma *3* in chapter 4.

**2).** From c=2d, d=2e+1 and e=2f to get c=2d=2(2e+1)=4e+2=8f+2, then there are $19+12c=43+96f=43+3\times2^5f \to 130+3^2\times2^5f \to 65+3^2\times2^4f \to 196+3^3\times2^4f \to 98+3^3\times2^3f \to 49+3^3\times2^2f \to 148+3^4\times2^2f \to 74+3^4\times2^1f \to 37+81f$ where the mark (**β**).

Due to 37+81f<43+96f, we get that if there is *n+1*∈*43+96f*, then *43+96f* and *n+1* fit the conjecture according to Theorem *1* and Lemma *3* in chapter 4.

**3).** From c=2d+1, d=2e+1 and e=2f to get c=2d+1=4e+3=8f+3, then there are

$19+12c=55+96f=55+3\times2^5f\to166+3^2\times2^5f\to83+3^2\times2^4f\to250+3^3\times2^4f\to125+3^3\times2^3f\to$

$376+3^4\times2^3f\to188+3^4\times2^2f\to94+3^4\times2^1f\to47+81f$ where the mark (γ).

Due to $47+81f<55+96f$, we get that if there is $n+1\in55+96f$, then $55+96f$ and $n+1$ fit the conjecture, according to Theorem *1* and Lemma *3* in chapter 4.

For the same reason, on the bunch of operational routes of *19+12c*, each reader can also conclude other 3 kinds of *19+12c* derived from first satisfactory operational results to fit the conjecture, and as listed below.

**4)**. Pursuant to c=2d, d=2e+1, e=2f+1, f=2g+1 and g=2h to get c=32h+4, then there is *19+12c=187+384h* derived from *119+243h* where the mark (**δ**);

**5)**. Pursuant to c=2d+1, d=2e, e=2f+1, f=2g and g=2h+1 to get c=32h+21, then there is *19+12c=271+384h* derived from *172+243h* where the mark (**ε**);

**6)**. Pursuant to c=2d+1, d=2e+1, e=2f+1, f=2g+1 and g=2h+1 to get c=32h+31 then there is *19+12c=391+384h* derived from *248+243h* where the mark (ζ).

So far, take into account what we and each reader have done, there are altogether 6 kinds of *15+12c/19+12c* to fit the conjecture.

It follows that if *n+1* belongs in any kind of *15+12c/19+12c* derived from first satisfactory operational result, then, that kind of *15+12c/19+12c* and *n+1* fit the conjecture according to Theorem *1* and Lemma *3* in chapter 4.

By now, we analyze two bunches of operational routes of *15+12c* and *19+12c* whether they are related, as described below.

Due to $c\geq1$, there are infinitely more odd numbers of *15+12c/19+12c*, whether they belong to infinite or finite more kinds, boil down to all kinds can only be on the bunch of operational routes of $15+12c/19+12c$.

For variables on operational routes of $15+12c/19+12c$, they are often seen as having many, however, in fact, they can be converted into a common variable, otherwise, any kind of $15+12c/19+12c$ is in the hide always.

Since there is a common variable, not only allows you to compare the size between a certain operational result and a kind of $15+12c/19+12c$, but also let you know that every two operational routes on the bunch of operational routes of $15+12c/19+12c$, they either directly intersect or indirectly connect, since they can be extended enough.

Besides, not only one kind of $15+12c/19+12c$ derives from first satisfactory operational result, but also there is the case where at least two kinds of $15+12c/19+12c$ derive from a satisfactory operational result, such as $15+12(4+2^{55}\times3^{2}y)$ and $15+12(8+2^{32}\times3^{17}y)$ are derived from $61+2^{3}\times3^{37}y$.

In some cases, an operational route of $15+12c$ and an operational route of $19+12c$ coincide partially or intersect from each other, such as start with $15+12(1+2^{57}y)$ to operate five steps in a row, and you get $19+12(1+2^{54}\times3^{2}y)$.

As stated, we have analyzed these two bunches of operational routes, and that we are possessed of the enough evidences to prove that all unproven kinds of $15+12c/19+12c$ fit the conjecture.

Under these circumstances, let us prove all unproven kinds of $15+12c/19+12c$ from each other's- opposite directions, in following paragraphs.

**Firstly,** we start with proven 6 kinds of $15+12c/19+12c$ to continuously expand the scope of proven kinds of *15+12c/19+12c.*

According to Theorem *1* and Theorem *2* in chapter 4, all integer expressions on at least one operational route via each of proven 6 kinds of *15+12c/ 19+12c* fit the conjecture.

After that, all integer expressions which fit the conjecture are turned into proven integer expressions.

Next, according to Lemma *1* and Lemma *2* in chapter 4, all integer expressions on successive intersecting operational routes via each of proven integer expressions fit the conjecture.

After that, all integer expressions which fit the conjecture are turned into proven integer expressions.

The rest can be deduced by analogy according to Lemma *1* and Lemma *2* in chapter 4, time by time.

In this way, proven integer expressions on the bunch of operational routes of *15+12c/19+12c* are getting more and more, up to all integer expressions on the bunch of operational routes of *15+12c/19+12c* are proved to fit the conjecture.

Because by the operational rule to operate integer expressions, and from this formed operational routes via each and every kind of *15+12c/ 19+12c,* they are all on the bunch of operational routes of *15+12c/19+12c,* therefore, all kinds of *15+12c/ 19+12c* are proved to fit the conjecture, according to Theorem *1* and Theorem *2* in chapter 4.

In this case, if *n+1* belongs in a kind of *15+12c/19+12c*, then it does not get around the fact that it is proved, according to Lemma *3* in chapter 4.

**Secondly**, we start with each unproved kind of *15+12c/19+12c* to operate continuously by the operational rule, until find first satisfactory operational result about the unproved kind of *15+12c/19+12c*.

First of all, how do you present an unproved kind of *15+12c/19+12c* ?

Since there is only the variable c in *15+12c/19+12c,* if the c in some kind of *15+12c/19+12c* is not equal to any of c values in cumulative all proven kinds of *15+12c/19+12c,* then this kind of *15+12c/19+12c* is exactly an unproved kind of *15+12c/19+12c*.

In addition, before continuing to prove unproved kinds of *15+12c/19+12c*, there are *6* proven kind of *15+12c/19+12c.* As proved above, they are: when *c=4e+3, 8f+5, 8f+6, 32h+25, 32h+10* and *32h, 15+12c* have been proved to fit the conjecture; when *c=4e, 8f+2, 8f+3, 32h+4, 32h+21* and *32h+31, 19+12c* have been proved to fit the conjecture, therefore, you do not worry without proven kind of *15+12c/19+12c*.

Now that we can find unproved kinds of *15+12c/19+12c* in turn, and we are possessed of the operational rule and criteria for judging operational results, then we can prove each unproved kind of *15+12c/ 19+12c* to fit the conjecture by one of following three ways.

**(1)** An operational route via an unproved kind of *15+12c/19+12c* has an integer expression that is less than the unproved kind of *15+12c/19+12c*, then the unproved kind of $15+12c/19+12c$ is proved to fit the conjecture, according to Theorem *1* in chapter 4.

**(2)** An operational route via an unproved kind of $15+12c/19+12c$ and an operational route via a proven kind of $15+12c/19+12c$ intersect, then the unproved kind of $15+12c/19+12c$ is proved to fit the conjecture, according to Theorem *2* or Lemma *2* in chapter 4.

**(3)** An operational route via an unproved kind of $15+12c/19+12c$ and an operational route via a proven kind of $15+12c/19+12c$ are in indirect connection, then the unproved kind of $15+12c/19+12c$ is proved to fit the conjecture, according to Lemma *2* in chapter 4.

As one kind of $15+12c/19+12c$ after another is proved, then the number of proven kinds of $15+12c/19+12c$ becomes more and more, up to all unproved kind of $15+12c/19+12c$ are proved to fit the conjecture.

According to the bidirectional proofs which have been given above, such that all kind of $15+12c/19+12c$ have been proved to fit the conjecture.

So if $n+1$ belongs in a kind of $15+12c/19+12c$, then $n+1$ is proved to fit the conjecture, accord to Lemma *3* in chapter 4.

## 7. Make a Summary and Reach the Conclusion

To sum up, $n+1$ has been proved to fit the conjecture, whether $n+1$ belongs in which class of odd numbers or which kind of odd numbers, or it is exactly an even number.

We can also prove integers $n+2$, $n+3$ etc. up to every integer that is greater than $n+1$ to fit the conjecture, in the light of the old way of doing thing.

The proof was thus brought to a close. As a consequence, the Collatz conjecture is tenable.

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